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Focusing To Transform-Limited, Phase-Controlled, Few-Cycle Pulses With Lenses

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Abstract. We investigate the conditions under which lenses can be used to focus broadband, visible or near-infrared radiation to transform-limited few-cycle pulses, and to exercise also a control on the focal carrier-envelope phase shift for phase-sensitive interactions with matter, as high-order harmonic or attosecond pulse generation.

Keywords: Carrier-envelope phase, extreme nonlinear optics, focusing
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In experiments of high-harmonic generation [1], attosecond pulse generation [2, 3], and of other phase-sensitive interactions of few-cycle pulses with matter [4, 5], mirrors are usually used to focus the driving pulses because of the widespread belief that lenses would introduce strong pulse deterioration. In these phase-sensitive interactions, it is also desirable to maintain a constant pulse carrier-envelope phase (CEP), or at least to control its variation. Several methods have been proposed to tailor specific variation curves for the total CEP shift $-\pi$ in the focal region of a mirror due to Gouy phase, [6, 7, 8], but none of them is easily implementable.

In this work we study focusing of few-cycle pulses with lenses, and describe the conditions under which a) the dispersion introduced by the lens material can be pre-compensated by means of standard pulse shaping techniques, as done, for instance, for a dielectric slab, and b) the lens chromatic aberration does not introduce any of the undesirable broadening and distortion effects described in the early works on femtosecond pulse focusing [9, 10, 11, 12, 13, 14]. Contrary what is believed, we conclude that focused, transform-limited, non-reshaping, few-cycle (even single-cycle) pulses along focal depths of the order of a millimeter, can be obtained by using common lens material, focal lengths and input spot sizes of the order of those typically used in experiments. We also find that the choice of particular values of the focal length and input spot size result in pulses with partially frozen CEP in the focal region of the lens.

FOCUSING TO TRANSFORM-LIMITED FEW-CYCLE PULSES

Our study is based on an analytical-numerical model of focusing with lenses with spherical surfaces, that incorporates the effects of the dispersion due to the lens material (including its variable thickness), the lens aperture, and its spherical and chromatic

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aberrations [15]. From this model we have first concluded that focusing of pulsed Gaussian beams with lenses can be accurately described by the well-known formulas of the Gaussian beam formalism [16] provided that the spot size in front of the lens is small enough compared to the lens aperture so that truncation and spherical aberration effects are negligible. In particular, the electric field along the optical axis can be calculated, in the Debye approximation of negligible focal shift, from the inverse Fourier transform of

$$E(\omega,z) \simeq p(\omega)\exp\left(i\frac{\omega}{c}nD\right)\exp\left(i\frac{\omega}{c}z\right)\frac{-f}{Z-iL_R}, \tag{1}$$

where $p(\omega)$ is a broadband function about a carrier frequency ω_0, and represents the complex amplitude of each monochromatic Gaussian beam component at the entrance plane of the lens, D and n are the lens center thickness and its refractive index, c is the speed of light in vacuum,

$$f = \left[(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{(n-1)^2}{n}\frac{D}{R_1R_2}\right]^{-1}-\frac{R_2D}{n(R_1+R_2-D)+D}, \tag{2}$$

is the paraxial back focal length, with R_1 and R_2 the radii of the front and back lens surfaces, $Z=z-f$ is an axial coordinate with origin at the geometrical focus at the frequency ω, z with origin at the lens back vertex, $L_R=2cf^2/\omega s^2$ measures the half focal depth, and s is the Gaussian spot size at the frequency ω in front of the lens.

The first relevant point is that the validity of Eq. (1) under conditions of negligible truncation and spherical aberration implies that *the dispersion due to the lens material is substantially the same as that due to a dielectric slab of thickness equal to the lens center thickness.* A transform-limited, few-cycle pulse after the lens can then be obtained by dispersion pre-compensation using standard pulse shaping techniques, as diffraction gratings or chirped mirrors. Pre-compensation of the second- and third-order dispersion due to the lens center thickness generally suffices for typical lens thicknesses of the order of a few millimeters with the bandwidths of few-cycle pulses, and for specifically designed thin lenses down to the limit of single-cycle pulses.

The longitudinal chromatic aberration of the lens is taken into account in Eq. (1) by the dependence of the focal length f in Eq. (2) with frequency. Strong pulse reshaping and broadening upon focusing is experienced by pulses uniformly illuminating the lens, or with spot sizes comparable with the lens aperture, and for short carrier wave lengths [9, 10, 11, 12, 13, 14], but may be negligible under conditions close to those typically used in phase-sensitive, light-matter interaction experiments. Specifically, chromatic-aberration-induced reshaping and broadening is negligible if the propagation time difference from the entrance plane to the focus between a "marginal" ray at radius s and an axial ray is much smaller than the pulse duration. This condition can be conveniently expressed as $|\gamma|\equiv|(f_0'/L_{R,0})\omega_0|\simeq|[n_0'/(n_0-1)](f_0/L_{R,0})\omega_0|\ll\omega_0\Delta T/2.77$, where prime signs denote differentiation with respect to ω and subscripts 0 evaluation at ω_0, where the approximate equality is obtained from the thin lens approximation $f_0'\simeq-[n_0'/(n_0-1)]f_0$ to the derivative of f in Eq. (2) at ω_0, and ΔT is the bandwidth-limited pulse duration.

A realistic example that illustrates the conditions of pre-compensable lens material dispersion and negligible focal pulse reshaping and broadening is shown in Fig. 1. Pre-compensation of the second- and third-order dispersion due to 1 mm center thickness of a CaF_2 lens of aperture radius 1 cm and focal length $f_0 = 15.44$ cm suffices to produce after the lens a pulse with the bandwidth-limited duration $\Delta T = 5.4$ fs (two-cycles) at 800 nm carrier wave length, from an input pulsed Gaussian beam of carrier spot size $s_0 = 1.42$ mm (see caption for more details about the lens and input pulse). With this lens and input pulse, truncation is indeed negligible, and the spherical aberration coefficient, defined and evaluated as in [17], is also very small. Also, from the given input spot size, focal length, and CaF_2 dispersion properties, $|\gamma| = |-1| \ll \omega_0 \Delta T/2.77 = 4.585$. The pulse is then seen to propagate without appreciable envelope broadening or reshaping along the entire focal region of half focal depth $L_{R,0} = 3$ mm [Figs. 1 (a), (b) and (c)].

CONTROLLING THE FOCAL CEP VARIATION

If envelope reshaping in the focal region is negligible, the CEP variation in the focal region can then be evaluated following a similar procedure to that used in previous works [7, 8] for spherical mirrors under similar condition, with the only difference that the chromatic aberration must be taken into account. The result of our calculations starting from Eq. (1) is

$$\Delta\Phi(Z_0) = -\tan^{-1}\left(\frac{Z_0}{L_{R,0}}\right) + \frac{1}{1+(Z_0/L_{R,0})^2}\left[g\left(\frac{Z_0}{L_{R,0}}\right) + \gamma\left(\frac{Z_0}{L_{R,0}}\right)^2\right], \tag{3}$$

where $Z_0 = z - f_0$, $\Delta\Phi(Z_0)$ is the CEP shift taking the geometrical focus at the carrier frequency as the reference point, and $g = -(L'_{R,0}/L_{R,0})\omega_0 \simeq 1 + 2(s'_0/s_0)\omega_0$ is determined by the (possible) dependence on frequency of the input Gaussian spot size. Previous measurements of the CEP shift in absence of chromatic aberration [5, 6], i.e., for $\gamma = 0$, suggest that $g \sim 0$ for the input pulse, since the observed CEP shift is compatible with Gouy phase shift $-\tan^{-1}(Z_0/L_{R,0})$ [Fig. 1(d, dashed curve)]. According to Eq. (3), chromatic aberration ($\gamma \neq 0$) influences the focal CEP evolution, even if it does not cause envelope reshaping ($|\gamma| \ll \omega_0 \Delta T/2.77$). In particular, the value $\gamma = -1$ results in a nearly constant CEP in the first half of the focal region [Fig. 1(d, solid curve)]. In practice, one may need this effect in a given depth of focus $L_{R,0}$. Frozen CEP ($\gamma = -1$) then requires the use of the focal length and input spot size

$$f_0 = \frac{n_0 - 1}{n'_0}\frac{L_{R,0}}{\omega_0}, \quad s_0 = \left(\frac{2c}{\omega_0 L_{R,0}}\right)^{1/2} f_0 \,. \tag{4}$$

For example, nearly constant CEP in $L_{R,0} = 3$ mm at 800 nm carrier wave length with a fused silica lens requires $f_0 = 9.8$ cm and $s_0 = 0.91$ mm. In the example of the CaF_2 lens in Fig. 1, the focal length $f_0 = 15.44$ cm and spot size $s_0 = 1.42$ mm also result in constant CEP ($\gamma = -1$) in $L_{R,0} = 3$ mm. As a verification of this effect, the open dots in Fig. 1(d) represent the values of the CEP shift, obtained from numerically computed pulse shapes along the focal region taking into account the lens variable thickness, aperture, spherical and chromatic aberrations, which are seen to fit accurately to the flattened CEP variation predicted by the approximate Eq. (3).

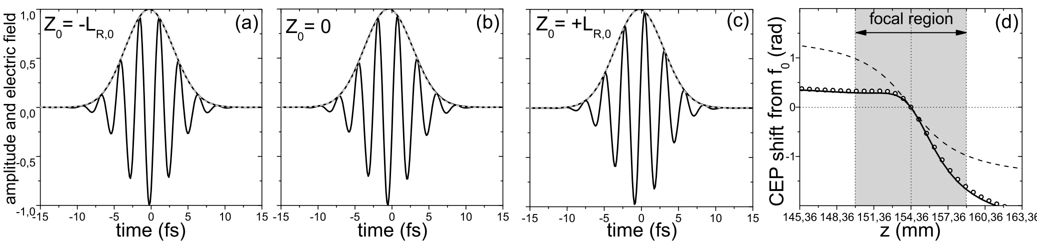

FIGURE 1. (a), (b) and (c) Nearly transform-limited (nearly Gaussian) focused pulses of duration $\Delta T \simeq 5.4$ fs at carrier wave length $\lambda_0 = 800$ nm (carrier frequency $\omega_0 = 2.355$ fs^{-1}) with undistorted envelope from $-L_{R,0}$ to $+L_{R,0}$, with $L_{R,0} = 3$ mm. This focused pulse is obtained with a CaF_2 lens of surface radii $R_1 = R_2 = 133.06$ cm, center thickness $D = 1.0$ mm, focal length $f_0 = 15.436$ cm, and lens aperture radius $a_L = 1$ cm, that focuses an input pulse of spectrum $p(\omega) = \exp[-\Delta T^2(\omega - \omega_0)^2/8\ln 2]\exp[-i\varphi_0''(\omega-\omega_0)^2/2 - i\varphi_0'''(\omega-\omega_0)^3/6]$, where $\Delta t = 5.4$ fs, and where $\varphi_0'' = 28.028$ fs^2 and $\varphi_0''' = 17.705$ fs^3 compensate for the lens center thickness second- and third-order dispersion, and of spot size at the carrier frequency $s_0 = 1.422$ mm. The dependence on frequency of the input spot size is described by $s = s_0(\omega_0/\omega)^{0.5}$, corresponding to an standard iso-diffracting, input pulsed Gaussian beam ($g = 0$). The dashed gray curves are the exactly transform-limited (Gaussian) pulse of duration $\Delta T = 5.4$ fs, which is suitably time-shifted in each figure for a better comparison with the focused pulse.
(d) Dashed curve: Gouy phase shift, or CEP shift for $g = 0$ and $\gamma = 0$ (mirror). Solid curve: CEP shift with the CaF_2 lens predicted by Eq. (3). Open circles: CEP shift evaluated numerically.

In conclusion, we suggest using lenses to focus femtosecond, few-cycle, even single-cycle pulses. Under focusing conditions close to those used in phase-sensitive interactions with matter, a lens can focus to transform-limited, propagation-invariant, few-cycle pulses, and allows to exercise control over the intra-focus CEP variation.

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