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Markov chains and entropy tests in genetic-based lithofacies analysis of deep-water clastic depositional systems

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Abstract: The aim of this study was to examine the relationship between structural elements and the so-called genetic lithofacies in a clastic deep-water depositional system. Process-sedimentology has recently been gaining importance in the characterization of these systems. This way the recognized facies attributes can be associated with the depositional processes establishing the genetic lithofacies. In this paper this approach was presented through a case study of a Tertiary deep-water sequence of the Pannonian-basin.

Of course it was necessary to interpret the stratigraphy of the sequences in terms of „general" sedimentology, focusing on the structural elements. For this purpose, well-logs and standard deep-water models were applied.

The cyclicity of sedimentary sequences can be easily revealed by using Markov chains. Though Markov chain analysis has broad application in mainly fluvial depositional environments, its utilization is uncommon in deep-water systems. In this context genetic lithofacies was determined and analysed by embedded Markov chains. The randomness in the presence of a lithofacies within a cycle was estimated by entropy tests (entropy after depositional, before depositional, for the whole system). Subsequently the relationships between lithofacies were revealed and a depositional model (*i.e.* modal cycle) was produced with 90% confidence level of stationarity. The non-randomness of the latter was tested by chi-square test.

The consequences coming from the comparison of „general" sequences (composed of architectural elements), the genetic-based sequences (showing the distributions of the genetic lithofacies) and the lithofacies relationships were discussed in details. This way main depositional channel has the best, channelized lobes have good potential hydrocarbon reservoir attributes, with symmetric alternation of persistent fine-grained sandstone (Facies D) and muddy fine-grained sandstone with traction structures (Facies F).

Keywords: Markov chains; Pannonian Basin; deep-water systems; genetic lithofacies; depositional process

1 Introduction

Vertical variations of lithofacies have an important role within a sedimentary sequence in recognition of depositional environment. According to Walther's facies correlation law [1], only those facies can be settled on each other which can exist next to each other at a given time. Thus a quasi-gradual transition from one facies to another represents that the two facies were adjacent laterally once.

In deep-water depositional systems (*i.e.* submarine fan complex [2]) the distribution of the facies, the coarsening- and fining upward successions, the geometries and sand/mud contents of each parts lead to detect the so-called structural elements. Structural elements (*i.e.* sedimentary subenvironments) have been emphasized for decades in the major regional-scale models, according to the works of Normark [3], Mutti and Ricci Lucchi [4], Mutti [5], Reading and Richards [6], Bouma [7] etc. By revealing these, potential hydrocarbon stratigraphy traps of these systems can be understood more efficiently.

Sediment deposition occurs mainly from gravity-driven processes (such as slumps, slides, cohesive debris flow, sandy debris flow, turbidiy currents) in these systems. Sediment concentration and deposit thickness as fraction of flow thickness decreases from slumps and slides to turbidity currents. Redeposition by bottom currents also has an important role in these systems [8]. By application of the principles of process sedimentology (it is concerned with establishing connectivity between the deposit and the physics of the depositional process), one can attach the identified faciological attributes (grain-size, textural and compositional maturity, sediment structures

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etc.) to the related depositional process. Thus genetic (corresponding to the depositional process) lithofacies (GLF) can be obtained.

In order to determine regularity and cyclicity in sedimentary sequences, Markov-chains are common and useful methods. By using Markov-chains with the established genetic lithofacies, modal cycles (*i.e.* depositional model) can be developed.

Through the case study from a Tertiary (Late Miocene and Lower Pliocene) deep-water sedimentary sequence of the Pannonian Basin, the following methods are applied:

- recognizing the GLFs (on core samples), and deducing the significant vertical lithologic transitions in the examined sequences;
- analysing cyclicity and significance of successions by chi-square and entropy tests (post-depositional, pre-depositional, whole-system entropies), respectively;
- interpreting the "general" sedimentary sequences by the help of well-logs and regional-scale models focusing on structural elements;
- consequently, revealing connectivity between structural elements with potential hydrocarbon reservoir character and modal cycles, cyclic patterns (sets of post- versus pre-depositional entropies [9]).

2 Available data and the General geological setting of the region of the case study

The case study is located in one of the deep subbasins of the Pannonian Basin in the Great Hungarian Plain. Related to the research, the data comes from a single well, which is labelled as WELL-A hereinafter. The available and applied data were composed of calibrated spontaneous potential and gamma-ray logs. They are the most responsive to the real lithological attributes. Approximately 35 metres of core sample (total interval: 35.5 metres) were investigated.

At the beginning of Late Miocene the Central Paratethys had become a hydrologically isolated large lake (Lake Pannon) [10], until it was completely filled (Pliocene). The process of filling up showed progradational feature and was controlled particularly by fluvial and deltaic systems during Late Miocene and Lower Pliocene. The sediment-supply was derived from north-west and north-east, east [11]. Flora and fauna of Lake Pannon reached great endemic diversities [12].

The following main depositional environments characterized Lake Pannon [13]: (1) fluvio-lacustrine and deltaic plain (2) delta front and delta slope (3) prodelta (4) deep-water systems (5) basin plain.

The growth of deep-water systems belong to Szolnoki Formation [14]. In the Great Hungarian Plain its thickest sequences (approx. < 1000 m) take place in deep-subbasins (Jászság Basin, Derecske Trough, Makó Trough, Békés Basin) [11].

3 Methods

As previously mentioned, the analytical procedure consists of three steps: (1) interpreting the "general" sedimentary sequence (2) recognizing GLSs (3) quantitative stratigraphical analysis based on GLSs (embedded Markov-chains, entropy and chi-square tests).

3.1 Interpretation of general sedimentary sequence

Average grain-size distribution, regularity of alternations of litofacies in each parts and well-logs (calibrated spontaneous potential, gamma-ray) are applied to reveal structure elements. The well-known log motifs refer to the structural elements in deep-water systems after [15]:

- bell: fining-upward succession, channel-levee complex, unchannelized lobe, abandonment of any channel or lobe;
- funnel: coarsening-upward succession, development of distal lobe;
- cylindrical: wedge-bodies, proximal main depositional channel, channelized lobe, lobe without channel-levee complex;
- irregular: zone of slides and slumps, zone of sand sheets (inactive or distal part);
- symmetric: development then abandonment of channelized/unchannelized lobe.

3.2 Characterization of genetic lithofacies

The main gravity-driven processes dominating in deep-water systems are: (1) slides and slumps (2) cohesive debris flows and sandy debris flows (3) turbidity currents. Reworking bottom currents also operate.

- (1) ”A *slide* is a coherent mass of sediment that moves along a planar glide plane and shows no internal de-

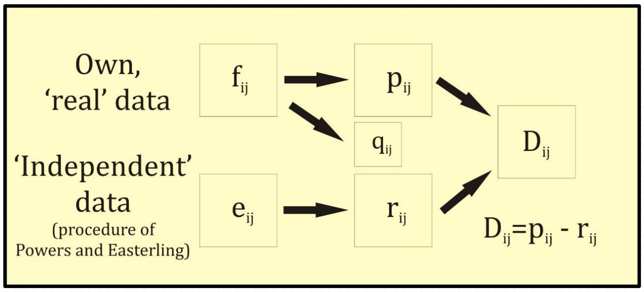

Figure 1: The algorithm of analysis by embedded Markov-chain; f_{ij} = tally count matrix, p_{ij}, q_{ij} = upward and downward probability matrices, respectively, e_{ij} = expected transition frequency, r_{ij} = independent trials probability matrix, D_{ij} = normalized difference matrix

formation" ([8, p. 49]). "A *slump* is a coherent mass of sediment that moves on a concave-up glide plane and undergoes rotational movements causing internal deformation" ([8, p. 52]). General features of deposits of slide and slump: (1) gravel to mud lithofacies (2) basal zone of shearing (3) tension faults (4) clastic and sand injections (5) secondary internal glide planes (slides) (6) alternations of contorted and uncontorted layers, chaotic bedding (slumps);

- (2) "*Debris flow* is a sediment flow with plastic rheology and laminar state from which deposition occurs through freezing en masse" ([8, p. 59]). *Sandy debris flow* is a transformation between cohesive debris flow and turbidity current, with a lower laminar and an upper turbulent part. The term "high-density turbidity current" is also used for this type of process (*e.g.* Lowe [16]). It is misleading, because the bigger volume of the deposited sediment is derived from the lower part of flow, which is clearly not in turbulent state [8]. In this work, deposits of cohesive debris flow and of sandy debris flow is handled altogether. General features of deposits of cohesive or sandy debris flow: (1) gravel to mud lithofacies (2) floating mudstone clasts near top of the beds (3) projected, brecciated mudstone clasts (4) inverse, normal, inverse to normal, and no grading;
- (3) "*Turbidity current* is a sediment flow with Newtonian rheology and turbulent state in which sediment is supported by turbulence and from which deposition occurs through suspension settling" ([8, p. 77; 17]). General features of deposits of turbidity current: (1) fine-grained sand to mud lithofacies (2) normal grading without any other structures (3) erosional (flute marks) basal contact (4) thin layers, mainly few centimetres;
- (4) *Bottom currents* (induced by tidal, thermohaline or wind forces [18]) are responsible particularly for traction structures in deep-water systems [8]. Their deposits can be characterized by: (1) fine-grained sand and mud lithofacies (2) thin-bedded, laminated sand with mud, and its rhythmic occurence (3) low-angle cross laminae, ripple-cross laminae (4) flaser, lenticular bedding.

3.3 Embedded Markov-chains, entropy analysis and chi-square test

The idea of cyclicity in sedimentary systems implies that one state (*i.e.* lithology) determine the succeeding state. In the case of the method of embedded Markov-chain, only lithologic changes (abrupt change in character) are recorded, regardless of the thickness of each lithology member (or bed). Counting the transitions in the sequence, one step embedded tally count matrix (f_{ij}) is structured (where i, j corresponds to row and column number). By means of it, upward (p_{ij}) (*i.e.* transition probability matrix) and downward (q_{ij}) probability matrices are calculated [19]. For establishing the expected transition frequency (e_{ij}) and therefrom the independent trials probability (r_{ij}) matrices, iterative procedure of Powers and Easterling [19] is applied. Normalized difference matrix (D_{ij}) is obtained by subtracting the value of each cell in the independent trials probability matrix (r_{ij}) from the corresponding cell in the transition probability matrix (p_{ij}) (Figure 1). The cells where positive values are present (at given limiting value), show those transitions which have Markovian property (*i.e.* cyclicity). Hattori (1976) introduced the entropy analysis in Markov-chains and general cyclic patterns in sedimentary successions [9]. Post- E(post), pre- E(pre) and whole depositional system E(sys) entropy values are calculated from upward and downward probability matrices, respectively, by application of modified Shannon-entropy (entropy value gives the rate of uncertainty of the occurrence of a facies). Generally, entropy is likely to increase with the number of states. Thus, entropy values must be normalized (by dividing both of E(post) and E(pre) by E(max), where E(max) is the maximum possible entropy in the system). The non-randomness of the obtained modal cycles are tested by chi-square test [19]. The calculations and the whole procedure followed Hattori's concept.

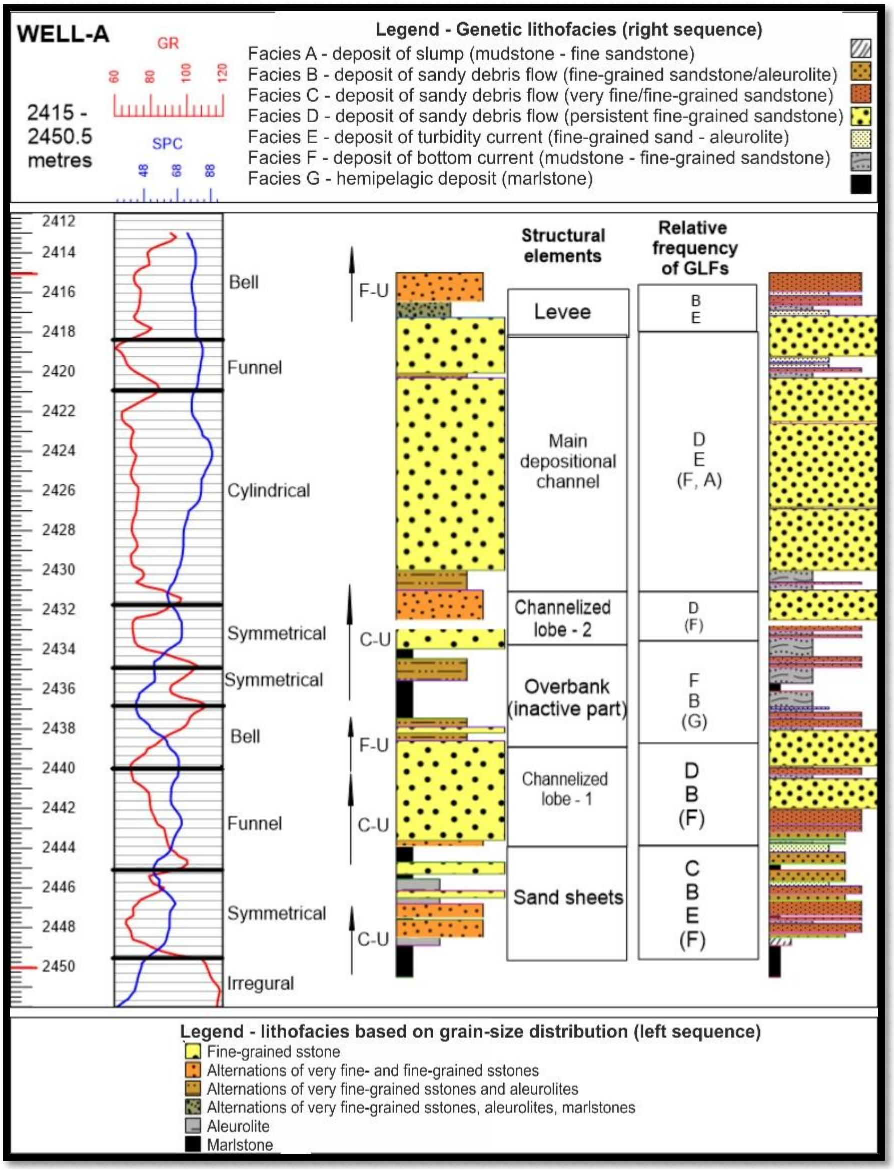

Figure 2: Composite figure of WELL-A. Left part: General sedimentological sequence, emphasizing structural elements. Right part: sequence based on GLFs. Connection between the two parts is showed by relative frequencies of GLFs in each structural elements: facies in upper position denotes that the facies occurs with higher frequency

4 Qualitative and quantitative results

Figure 2 shows the general sedimentological explanation used in WELL-A. Structural elements are revealed. Potential hydrocarbon reservoirs could occur in "channelized lobe 1-2", and "main depositional channel". The recognized genetic lithofacies are: (1) Facies A - *deposit of slump* – chaotic bedding, mud- to fine-sandstone (ss) (2) Facies B-D – *deposits of sandy debris flows* – graded bedding, floating clasts; B – fine-ss/aleurolite, C – very fine-/fine-ss, D – persistent (thickness > 1 m) fine-ss) (3) Facies E - *deposit of turbidity current* – normal grading, fine-ss and aleurolite (4) Facies F – *deposit of reworking bottom current* – laminated bedding, ripple-cross lamination, cross lamination, flaser and lenticular bedding – mudstone/fine-ss (5) Facies G – *hemipelagic settling* – marlstone. Based on transitions of the GLFs, another sequence is structured (Figure 2). Approximately relative frequencies of GLFs in each zone of structural elements are deduced.

In aspect of quantitative stratigraphical analysis, all matrices mentioned above and entropy values are calculated. Transition tally count, difference matrices, entropy

Table 1: WELL-A: transition count matrix, difference matrix with positive values (limiting value is average of the non-negative differences: 0.1357), normalized entropies of each GLF, entropy of the whole system, and stationarity test by chi-square test)

	Transition tally count matrix (p_{ij})							Difference matrix with positive* values (D_{ij})						
	A	B	C	D	E	F	G	A	B	C	D	E	F	G
A	0	1	1	1	0	0	0			0.226	0.226			
B	0	0	1	1	3	10	2					0.155	0.168	
C	1	2	0	0	2	1	1							
D	1	0	0	0	1	5	0						0.370	
E	0	1	3	0	0	4	0			0.258			0.150	
F	0	12	1	5	2	0	1		0.188					
G	1	2	1	0	0	1	0	0.153						

	Norm. E(pre)		Norm. E(post)	Non-randomness test: chi-square test	
A	0.613		0.613	Degree of freedom = $(N - 1)^2$ - N	
B	0.603		0.672	where N = observed states	
C	0.823		0.865	DOF = 29, χ^2 = 39.87	
D	0.444		0.444	Limiting confidence level values at 29 DOF	
E	0.737		0.544	90%	95%
F	0.726		0.656	39.09	42.56
G	0.58		0.744	Markov-process: stationary	
	E(sys) = 9.133		E(max) = 2.585		

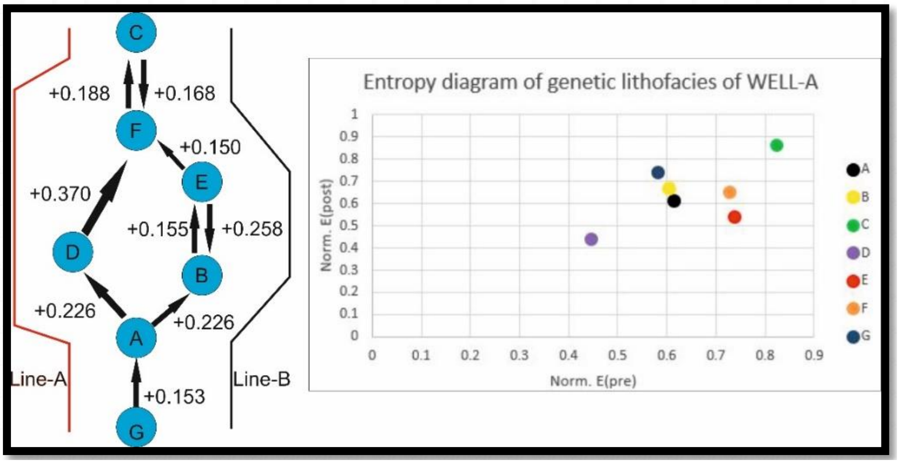

Figure 3: Facies relationship diagram of WELL-A. Line-A is chosen for modal cycle. Entropy diagram shows symmetrical shape, A = "Facies A", B = "Facies B", C = "Facies C", D = "Facies D", E = "Facies E", F = "Facies F", G = "Facies G"

values and chi-square test can be seen in Table 1. Facies relationship diagram (FRD) based on positive difference values is also constructed (Figure 3).

Facies A can be succeeded by Facies D or Facies B with the same probability. So, there are two possible ways in FRD (heteropic facies), but in one well (*i.e.* sequence) they cannot appear simultaneously, nevertheless geologically both are equally good. Therefore the line with higher probability value (multiplying the difference values in each way) – line-A – is chosen for developing a modal cycle. The higher justification of Line-A is valid only in WELL-A. Post- and pre-depositional diagram shows most closely symmetrical figure (Hattori's type-B diagram, Figure 3). The modal cycle has a pattern of GADFCF with 90% confidence level.

Modal cycle suggests that if persistent deposit of sandy debris flow (Facies D, potential HC-reservoir) appears once, it is followed by deposit of bottom current (Facies F), which has lower permeability because of its traction structures and finer grain-fraction. It means possible capping attributes. Unrestricted alternation of Facies F and C can denote additional reservoir-capping relationship with thinner sandstone reservoir (Facies C). Furthermore, Facies C has the highest value of normalized post depositional entropy which implies that its successor varies widely, hence marlstone can overlie on it as the best caprock.

The geological interpretation of the cyclical sequence is: (1) initial state is Facies G, which denotes the hemipelagic settling on basin plain (2) Facies A may denote slumps and slides, related to undercutting of channels. It is followed by (3) channel-fills (Facies D). Facies D seizes relatively the thickest part of the whole sequence. It means that this part of the complex is dominated by channel-systems (as main supplier channel or distributary channel). (4) Facies F denotes functioning of bottom currents. It is related to the inactive zones, so the channels migrate. (5) Facies C may show the overflows over the margin of the channels. The symmetrical attribute refers to that the whole system migrates laterally.

5 Discussions

WELL-A reveals a part of a typical sand-rich submarine fan complex with quasi-inactive parts (zone of thin sand sheets and overbank), channelized lobes (persistent sandstones in them may denote distributary channels) and main depositional channel. Potential hydrocarbon reservoirs may take place in channelized lobes (1–2) and main depositional channel.

Channelized lobe-1 is composed of mainly Facies D, and then, of lower proportion of Facies B and F. In case of Line-1, occurrence of Facies B is random. Subsequently, this part of sequence has cyclic alternation of reservoir sandstone (D) and finer-grained rock with traction structures with ability of trapping (F).

Channelized lobe-2 is composed of mainly facies D and Facies F. The situation is the same as in channelized lobe-1.

Main depositional channel is composed of Facies D, E and in smaller part, Facies F, A. Occurrence of Facies E is random, according to line-A. Facies A usually behave as a reservoir [8]. Presence of Facies D and F denote the same former state. Maybe this is the best hydrocarbon reservoir zone because Facies D reaches its thickest developments here.

6 Conclusions

On the strength of qualitative and quantitative analyses of WELL-A, main depositional channel has petrophysically the best, and channelized lobes (1 and 2) have good potential hydrocarbon reservoir attributes, with symmetric alternation of facies D and F (a probably reservoir-caprock relationship). Occurrence of facies C is random, but its successor varies widely (it can be marlstone, facies G, as well) due to its high normalized post-depositional entropy.

Generally, linking the structural elements and modal cycles based on genetic lithofacies is a good contrivance to reveal the nature of stratigraphy traps in deep-water systems. By means of it, it is possible to analyse the internal structure of each structural elements. In addition, inferences can be concluded about which depositional processes dominate in the different structural elements.

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