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Soft singularity crossing and transformation of matter properties

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Abstract. We investigate particular cosmological models, based either on tachyon fields or on perfect fluids, for which soft future singularities arise in a natural way. Our main result is the description of a smooth crossing of the soft singularity in models with an anti-Chaplygin gas or with a particular tachyon field in the presence of dust. Such a crossing is made possible by certain transformations of matter properties. Some of these cosmological evolutions involving tachyons are compatible with SNIa data. We compute numerically their dynamics involving a first soft singularity crossing, a turning point and a second soft singulatity crossing during recollapse, ending in a Big Crunch singularity.

Keywords: cosmology, singularities, tachyons
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INTRODUCTION

The general relativity connects the geometrical properties of the spacetime to its matter content. As they say the matter tells to the spacetime how to curve itself, the spacetime geometry tells to the matter how to move. The cosmological singularities constitute one of the main problems of modern cosmology. The discovery of the cosmic acceleration stimulated the development of "exotic" cosmological models of dark energy; some of these models possess the so called soft or sudden singularities characterized by the finite value of the radius of the universe and its Hubble parameter. Let us recall that "traditional" or "hard" singularities are associated with the zero volume of the universe (or of its scale factor), and with an infinite values of the Hubble parameter, of the energy density and of the pressure –Big Bang and Big Crunch. In our paper [1] it was shown that in some models interplay between the geometry and the matter forces the matter to change some of its basic properties, such as equation of state for fluids and even the form of the Lagrangian. The most interesting from these models is that based on tachyons. The so called tachyons (Born-Infeld fields) is a natural candidate for a dark energy. The particular tachyon model [2], proposed in 2004, has two particular features: tachyon field transforms itself into a pseudo-tachyon field, the evolution of the universe can encounter a new type of singularity - the Big Brake singularity. The Big Brake singularity is a particular type of "soft" cosmological singularity - the radius of the universe is finite, the velocity of expansion is equal to zero, the deceleration is infinite. It was shown also [3, 4] that the predictions of the model do not contradict observational data on supenovae of the type Ia. The Big Brake singularity is a particular one - it is possible to cross it [4].

Let us ask ourselves an open questions: other soft singularities - is it possible to cross them ?

DESCRIPTION OF THE TACHYON MODEL

We consider a flat Friedmann universe

$$ds^2 = dt^2 - a^2(t)dl^2$$

The tachyon Lagrange density is

$$L = -V(T)\sqrt{1-\dot{T}^2}$$

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The energy density

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}}$$

The pressure

$$p = -V(T)\sqrt{1-\dot{T}^2}$$

The Friedmann equation is as usual

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \rho$$

The equation of motion for the tachyon field is

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V_{,T}}{V} = 0.$$

In our model [2]

$$\begin{aligned} V(T) = {} & \frac{\Lambda}{\sin^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\,T\right]} \\ & \times\sqrt{1-(1+k)\cos^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\,T\right]}\,, \end{aligned}$$

where k and $\Lambda > 0$ are the parameters of the model. The case $k > 0$ is more interesting. In this case some trajectories (cosmological evolutions) finish in the infinite de Sitter expansion. In other trajectories the tachyon field transforms into the pseudotachyon field with the Lagrange density, energy density and positive pressure given by

$$\begin{aligned} & L = W(T)\sqrt{\dot{T}^2-1}, \\ & \rho = \frac{W(T)}{\sqrt{\dot{T}^2-1}}, \\ & p = W(T)\sqrt{\dot{T}^2-1}, \\ & W(T) = \frac{\Lambda}{\sin^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\,T\right]} \\ & \times\sqrt{(1+k)\cos^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\,T-1\right]}. \end{aligned}$$

What happens with the Universe after the transformation of the tachyon into the pseudotachyon ? It encounters the Big Brake cosmological singularity.

THE BIG BRAKE COSMOLOGICAL SINGULARITY AND OTHER SOFT SINGULARITIES

The Big Brake singularity is characterized by the following formulae:

$$t \to t_{BB} < \infty$$

$$a(t \to t_{BB}) \to a_{BB} < \infty$$

$$\dot{a}(t \to t_{BB}) \to 0$$

$$\ddot{a}(t \to t_{BB}) \to -\infty$$

$$R(t \to t_{BB}) \to +\infty$$

$$T(t \to t_{BB}) \to T_{BB},\ |T_{BB}| < \infty$$

$$|\dot{T}(t \to t_{BB})| \to \infty$$

$$\rho(t \to t_{BB}) \to 0$$

$$p(t \to t_{BB}) \to +\infty$$

If $\dot{a}(t_{BB}) \neq 0$ it is a more general soft singularity.

CROSSING THE BIG BRAKE SINGULARITY AND THE FUTURE OF THE UNIVERSE

At the Big Brake singularity the equations for geodesics are regular, because the Christoffel symbols are regular (moreover, they are equal to zero). Is it possible to cross the Big Brake ? Let us study the regime of approaching the Big Brake. Analyzing the equations of motion we find that approaching the Big Brake singularity the tachyon field behaves as

$$T = T_{BB} + \left(\frac{4}{3W(T_{BB})}\right)^{1/3} (t_{BB} - t)^{1/3}.$$

Its time derivative $s \equiv \dot{T}$ behaves as

$$s = -\left(\frac{4}{81W(T_{BB})}\right)^{1/3} (t_{BB} - t)^{-2/3},$$

the cosmological radius is

$$a = a_{BB} - \frac{3}{4}a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{4/3},$$

its time derivative is

$$\dot{a} = a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{1/3}$$

and the Hubble variable is

$$H = \left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{1/3}.$$

All these expressions can be continued in the region where $t > t_{BB}$,which amounts to crossing the Big Brake singularity. Only the expression for s is singular but this singularity is integrable and not dangerous. Once reaching the Big Brake, it is impossible for the system to stay there because of the infinite deceleration, which eventually leads to the decrease of the scale factor. This is because after the Big Brake crossing the time derivative of the cosmological radius and Hubble variable change their signs. The expansion is then followed by a contraction, culminating in the Big Crunch singularity.

CROSSING OF THE SOFT SINGULARITY IN THE MODEL WITH THE ANTI-CHAPLYGIN GAS AND DUST

One of the simplest cosmological models revealing the Big Brake singularity is the model based on the anti-Chaplygin gas with an equation of state

$$p = \frac{A}{\rho}, \ \ A > 0.$$

Such an equation of state arises, for example, in the theory of wiggly strings [5, 6]. It was called "anti-Chaplygin gas" in paper [2] in analogy this the Chaplygin gas cosmological model, where the pressure is negative [7]. The energy density of the anti-Chaplygin gas is

$$\rho(a) = \sqrt{\frac{B}{a^6} - A}$$

It is easy to see that at $a = a_* = \left(\frac{B}{A}\right)^{1/6}$ the universe encounters the Big Brake singularity.

Let us consider now a more complex cosmological model, including not only anti-Chaplygin gas, but also dust. In this case the energy density and the pressure are

$$\rho(a) = \sqrt{\frac{B}{a^6} - A} + \frac{M}{a^3}, \ \ p(a) = \frac{A}{\sqrt{\frac{B}{a^6} - A}}.$$

Due to the dust component, the Hubble parameter has a non-zero value at the encounter with the singularity, therefore the dust implies further expansion. With continued expansion however, the energy density and the pressure of the anti-Chaplygin gas would become ill-defined.

In principle, one can solve the paradox by redefining the anti-Chaplygin gas in a distributional sense [8]. Then a contraction could follow the expansion phase at the singularity at the price of a jump in the sign of the Hubble parameter. Although such an abrupt change is not common in any cosmological evolution, we explicitly show that the set of Friedmann, Raychaudhuri and continuity equations are all obeyed both at the singularity and in its vicinity. The jump in the Hubble parameter

$$H \rightarrow -H$$

leaves intact the first Friedmann equation

$$H^2 = \rho,$$

the continuity equations and the equations of state, however, it breaks the validity of the second Friedmann (Raychaudhuri) equation

$$\dot{H} = -\frac{3}{2}(\rho + p).$$

Indeed,

$$\begin{aligned} H(t) &= H_S sgn(t_S - t) \\ &+ \sqrt{\frac{3A}{2H_S a_S^4}} sgn(t_S - t)\sqrt{|t_S - t|}\ , \end{aligned}$$

$$\dot{H} = -2H_S\delta(t_S - t) - \sqrt{\frac{3A}{8H_S a_S^4}}\frac{sgn(t_S - t)}{\sqrt{|t_S - t|}}\ .$$

To restore the validity of the Raychaudhuri equation we add a singular δ -term to the pressure of the anti-Chaplygin gas

$$p = \sqrt{\frac{A}{6H_S|t_S - t|}} + \frac{4}{3}H_S\delta(t_S - t).$$

To preserve the equation of state we also modify the expression for its energy density:

$$\rho = \frac{A}{\sqrt{\frac{A}{6H_S|t_S-t|}} + \frac{4}{3}H_S\delta(t_S - t)}\ .$$

In such a way the system of equations describing the crossing the singularity is self-consistent. However, the abrupt transition from the expansion to the contraction of the universe does not look natural. There is an alternative way of resolving the paradox. One can try to change the equation of state of the anti-Chaplygin gas at passing the soft singularity.

There is some analogy between the transition from an expansion to a contraction of a universe and an absolutely elastic bounce of a ball from a wall in classical mechanics. In this case there is also an abrupt change of the direction of the velocity (momentum). However, we know that really the velocity is changed continuously due to the deformation of the ball and of the wall.

The pressure of the anti-Chaplygin gas

$$p = \frac{A}{\sqrt{\frac{B}{a^6} - A}}$$

tends to $+\infty$ when the universe approaches the soft singularity. Requiring the expansion to continue into the region $a > a_S$, while changing minimally the equation of state, we assume

$$p = \frac{A}{\sqrt{|\frac{B}{a^6} - A|}},$$

$$p = \frac{A}{\sqrt{A - \frac{B}{a^6}}},\ \text{for } a > a_S.$$

It implies the energy density

$$\rho = -\sqrt{A - \frac{B}{a^6}}.$$

The anti-Chaplygin gas transforms itself into Chaplygin gas with negative energy density. The pressure remains positive and the expansion continues. The spacetime geometry remains continuous. The expansion stops at $a = a_0$, where

$$\frac{M}{a_0^3} - \sqrt{A - \frac{B}{a_0^6}} = 0.$$

Then the contraction of the universe begins. At the moment when the energy density of the Chaplygin gas becomes equal to zero (again a soft singularity), the Chaplygin gas transforms itself into the anti-Chaplygin gas and the contraction continues to culminate in the encounter with the Big Crunch singularity at $a = 0$.

CROSSING THE BIG BRAKE SINGULARITY AND THE FUTURE OF THE UNIVERSE IN THE TACHYON MODEL IN THE PRESENCE OF DUST

What happens with the Born-Infeld type pseudo-tachyon field in the presence of a dust component ? Does the universe still run into a soft singularity? Yes, it is true.

$$T = T_S \pm \sqrt{\frac{2}{3H_S}}\sqrt{t_S - t},\ H_S = \sqrt{\frac{\rho_{m,0}}{a_S^3}}.$$

How can the universe cross this singularity ? Let us first consider a pseudo-tachyon field with a constant potential, which is equivalent to the anti-Chaplygin gas. To the change of the equation of state of the anti-Chaplygin gas corresponds the following transformation of the Lagrangian of the pseudo-tachyon field:

$$L = W_0\sqrt{g^{tt}\dot{T}^2 + 1},$$

$$p = W_0\sqrt{\dot{T}^2 + 1}$$

$$\rho = -\frac{W_0}{\sqrt{\dot{T}^2 + 1}}.$$

It is a new type of Born-Infeld field, which we may call "quasi-tachyon".

For an arbitrary potential its Lagrangian reads

$$L = W(T)\sqrt{g^{tt}\dot{T}^2 + 1}, \qquad a > a_S,$$

The Klein-Gordon type equation is

$$\frac{\ddot{T}}{\dot{T}^2 + 1} + 3H\dot{T} - \frac{W_{,T}}{W} = 0,$$

the energy density is

$$\rho = -\frac{W(T)}{\sqrt{\dot{T}^2 + 1}},$$

while the pressure is

$$p = W(T)\sqrt{\dot{T}^2 + 1}.$$

In the vicinity of the soft singularity the friction term $3H\dot{T}$ in the equation of motion dominates over the potential term $W_{,T}/W$. Hence, the dependence of $W(T)$ on its argument is not essential and a pseudo-tachyon field approaching this singularity behaves like one with a constant potential. Thus, it is reasonable to assume that upon crossing the soft singularity the pseudo-tachyon transforms itself into a quasi-tachyon for any potential $W(T)$.

Now let us consider the dynamics of the model with trigonometric potential in the presence of dust. After the soft singularity crossing the absolute value of the negative contribution to the energy density of the universe induced by the quasi-tachyon grows while the energy density of the dust decreases due to the expansion of the universe. Thus, at some moment the total energy density vanishes and the universe reaches the point of maximal expansion, after which the expansion is replaced by a contraction and the Hubble variable changes sign. At some finite moment of time the universe hits again the soft singularity. Upon crossing this singularity the quasi-tachyon transforms back to pseudo-tachyon. After this the universe continues its contraction until it hits the Big Crunch singularity.

NUMERICAL SIMULATIONS FOR THE TACHYON MODEL

Comparing the prediction of our model with the Supernovae Ia Union2 Dataset, we have found the subset of accessible initial conditions $(T, \dot{T}, \Omega_m)$. Starting from this initial conditions we have simulated future evolutions of the universe. Some of the trajectories go towards de Sitter attractive node. Other trajectories go towards the transformation tachyon-pseudo-tachyon, the first crossing the soft singularity, the turning point, the second soft singularity crossing, and finally, the encounter with the Big Crunch.

CONCLUSIONS AND DISCUSSION

Before summing up we would like to mention some papers, which were important for the development of the line of research, presented in this work. First of all, it is the paper by Feinstein [9], where a cosmological tachyon model with a potential inversely proportional to the squared tachyon field was studied. This paper has stimulated the elaboration of more complicated model [2], where the phenomena of the tachyon-pseudo-tachyon transformation was first noticed and the Big Brake singularity was discovered. Then, in the papers by Fernández-Jambrina and Lazkoz [10, 11] it was shown that aome soft singularities can be crossed by extended bodies and, hence, by a universe itself.

Soft cosmological singularities known since the eighties [12], have been attracting growing attention during the last few years. Here we have presented the investigations of particular cosmological models based on tachyon fields or perfect fluids (introduced in paper [2]), for which soft singularities arise in a natural way. The main result of our investigations is the description of a smooth crossing of soft singularities, arising in models with anti-Chaplygin gas or of a particular tachyon field in the presence of dust. Such a crossing is accompanied by certain transformations of matter properties, embodied in a change either of equation of state or of Lagrangian.

The interesting feature of the tachyon model is that there exist cosmological evolutions whose past is compatible with the supernova data and whose future reveals “exotic phase transitions” which are here described in detail. We have performed a detailed numerical analysis of these evolutions.

All our studies, both theoretical and numerical, were performed assuming a a spatially-flat universe. Next interesting step for the study of dark energy models possessing soft future singularities is the inclusion of spatially closed universes. Indeed, observations do allow for a tiny spatial curvature, a positive curvature being slightly preferred. While a tiny viable curvature will not change the situation for most models studied in this paper, a larger number of situations can arise in the presence of spatial curvature for the tachyon models because of their rich dynamics. Indeed, if the universe reaches the point of maximal expansion before occurence of the soft future singularity, the latter will not occur at all. In the case of our tachyon model this can happen for specific initial conditions. If for some peculiar initial conditions the turning point and the soft singularity coincide the latter retains its character of a Big Brake singularity.

Another interesting direction of development of the present work is the consideration of cosmological perturbations and their possible influence on the structure of sudden singularities and on the conditions of their crossing. To our knowledge no systematic study of this kind appeared yet in the literature.

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