THE PARADOX OF SOFT SINGULARITY CROSSING AVOIDED BY DISTRIBUTIONAL COSMOLOGICAL QUANTITIES.

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A flat Friedmann universe filled with a mixture of anti-Chaplygin gas and dust-like matter evolves into a future soft singularity, where despite infinite tidal forces the geodesics can be continued. In the singularity the pressure of the anti-Chaplygin gas diverges, while its energy density is zero. The dust energy density however does not vanish, neither does the Hubble parameter, which implies further expansion, if its evolution is to be continuous. If so, the energy density and the pressure of the anti-Chaplygin gas would become ill-defined, hence only a contraction would be allowed. Paradoxically, the universe in this cosmological model would have to expand and contract simultaneously. The paradox can be avoided by redefining the anti-Chaplygin gas in a distributional sense. Then the Hubble parameter could be mirrored to have a jump at the singularity, allowing for a subsequent contraction. With this modification the set of Friedmann, Raychaudhuri and continuity equations are all obeyed both at the singularity and in its vicinity.

Keywords: singularity; anti-Chaplygin gas; distributions

1. Introduction

The discovery of the cosmic acceleration stimulated the development of "exotic" cosmological models of dark energy; some of these models possess the so called soft or sudden singularities characterized by the finite value of the radius of the universe and of its Hubble parameter. One of examples of such singularities is the Big Brake singularity arising in a specific tachyon model.[1] The toy tachyon model,[1] proposed in 2004, has two particular features: *i*) the tachyon field transforms into a pseudo-tachyon field; *ii*) the evolution of the universe can encounter a new type of singularity - the Big Brake singularity. When a universe encounter the Big Brake singularity its scalefactor is finite, the velocity of expansion is equal to zero, the deceleration is infinite. The predictions of the model match observational data on supenovae of the type Ia[2,3] and the Big Brake singularity is a special one - it is

possible to cross it.[3]

One of the simplest cosmological models revealing the Big Brake singularity is based on the anti-Chaplygin gas[1] with an equation of state

$$p = \frac{A}{\rho}, \quad A > 0,$$

which through the continuity equation leads to

$$\rho(a) = \sqrt{\frac{B}{a^6} - A}, \; B > 0.$$

At $a = a_* = \left(\frac{B}{A}\right)^{1/6}$ the universe encounters the Big Brake singularity. This singularity is traversable, because all the Christoffel symbols are finite and hence the geodesics equations are well-defined.

2. The model with the anti-Chaplygin gas and dust

Let us consider now the model of the flat Friedmann universe filled with the anti-Chaplygin gas and dust.[4] The energy density and the pressure are

$$\rho(a) = \sqrt{\frac{B}{a^6} - A} + \frac{M}{a^3}, \quad p(a) = \frac{A}{\sqrt{\frac{B}{a^6} - A}}.$$

Due to the dust component, the Hubble parameter has a non-zero value at the singularity, therefore the presence of the dust implies further expansion. With continued expansion however, the energy density and the pressure of the anti-Chaplygin gas would become ill-defined.

We solve the paradox by redefining the anti-Chaplygin gas in a distributional sense. Then a contraction could follow the expansion phase at the singularity at the price of a jump in the Hubble parameter. Although such an abrupt change is not common in any cosmological evolution, we show that the set of Friedmann, Raychaudhuri and continuity equations are all obeyed both at the singularity and in its vicinity.

The jump in the Hubble parameter

$$H \to -H$$

leaves intact the first Friedmann equation $H^2 = \rho$, the continuity equations and the equations of state, however, it breaks the validity of the second Friedmann (Raychaudhuri) equation $\dot{H} = -\frac{3}{2}(\rho + p)$. This is, because in the vicinity of the singularity the Hubble parameter can be expanded as

$$\begin{aligned} H(t) = {} & H_S sgn(t_S - t) \\ & + \sqrt{\frac{3A}{2H_S a_S^4}} sgn(t_S - t)\sqrt{|t_S - t|}\,, \end{aligned}$$

leading to

$$\dot{H} = -2H_S\delta(t_S - t) - \sqrt{\frac{3A}{8H_S a_S^4}}\frac{sgn(t_S - t)}{\sqrt{|t_S - t|}} \ .$$

To restore the validity of the Raychaudhuri equation we add a singular δ -term to the pressure of the anti-Chaplygin gas

$$p = \sqrt{\frac{A}{6H_S|t_S - t|}} + \frac{4}{3}H_S\delta(t_S - t).$$

To preserve the equation of state we also modify the expression for its energy density:

$$\rho = \frac{A}{\sqrt{\frac{A}{6H_S|t_S - t|}} + \frac{4}{3}H_S\delta(t_S - t)} \ .$$

In order to prove that p and ρ represent a self-consistent solution of the system of cosmological equations, we employed the following distributional identities:

$$\begin{aligned} [sgn\,(\tau)\,g\,(|\tau|)]\,\delta\,(\tau) &= 0 \ , \\ [f\,(\tau) + C\delta\,(\tau)]^{-1} &= f^{-1}\,(\tau) \ , \\ \frac{d}{d\tau}\,[f\,(\tau) + C\delta\,(\tau)]^{-1} &= \frac{d}{d\tau}f^{-1}\,(\tau) \ . \end{aligned}$$

3. Conclusion

The use of generalized functions (distributions) is not uncommon in physics. They appear naturally whenever there are lower-dimensional localized sources (including branes in higher dimensional theories), but also when in quantum field theory the product of distributions becomes well-defined by a renormalization procedure. The addition of a δ-function centred on a point where the pressure already diverges can be considered as a similar procedure.

Finally, we formulate questions for further related studies: How general is the paradox of the soft singularity crossing? Is it possible to find other ways out from the paradox of the soft singularity crossing?

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