Acta Sci. Math. (*Szeged*)
71 (2005), 427–455

Book Reviews

Communicated by P. Hajnal

Steven R. Finch, **Mathematical constants** (Encyclopedia of Mathematics and Its Applications, Vol. 94), XIX+602 pages, Cambridge University Press, Cambridge, 2003.

This excellent book contains 136 "stories", each devoted to a mathematical constant or a class of constants, from the well known and famous (π) to the highly exotic and obscure ones. Topics covered include the statistics of continued fractions, chaos in nonlinear systems, prime numbers, sum-free sets, isoperimetric problems, approximation theory, self-avoiding walks and the Ising model (from statistical physics), binary and digital search trees (from theoretical computer science), complex analysis, geometric probability, and the traveling salesman problem.

The exact list of the chapters is: 1. Well-known Constants, 2. Constants Associated with Number Theory, 3. Constants Associated with Analytic Inequalities, 4. Constants Associated with the Approximation of Functions, 5. Constants Associated with Enumerating Discrete Structures, 6. Constants Associated with Functional Iteration, 7. Constants Associated with Complex Analysis, 8. Constants Associated with Geometry.

The book is very useful and interesting for all mathematicians looking for information about a lot of specific constants, because it brings together all significant mathematical constants in one place.

J. Németh (Szeged)

Stephen G. Krantz, **A Mathematician's Survival Guide: Graduate School and Early Career Development**, XV+222 pages, American Math. Soc., Providence R.I., 2003.

A graduate student easily becomes discouraged. How should I start a research project? How to find a good advisor and problem? What to do if I can not solve my problem? What to do if I can? What am I supposed to do at the department? What shall I do in the next 3-5 years? And after? Am I gifted enough to be a mathematician? Do my mathematics make any sense at all? Who can help me and how? Such questions arise, and it may be difficult to find the answers. In this frequent situation A Mathematician's Survival Guide is really a need.

Steven G. Krantz is an acclaimed mathematician, author of 150 papers and dozens of books, he has worked in several institutions, supervised many students, served as a member of committees and editorial boards. He has comprehensive experience in every aspect of this walk of life, and lets the reader in on the secrets of being a mathematician, with a wondrous and charming style. He successfully avoids the generalities and the stock phrases, the advices are always clear and informative.

The first part of the book tells you most of the basic things of becoming a mathematician: preparing for and getting into graduate school, passing exams, finding a good advisor, duties at the department, writing a thesis, getting a degree. Then the everyday practical and moral difficulties are concerned, from taking out a loan to love affairs with colleagues and fellow students. The remaining part of the book is devoted to the post-graduate life: looking for a job, teaching, researching, publishing, getting tenure. Money matters are also touched.

The last chapter is completely different from the previous ones: that gives a nice overview of the mathematics one has to know in a graduate school, such as the basic concepts of real and complex analysis, geometry, topology and algebra. The book contains a detailed glossary and several appendices describing the administrative structure and the composition of universities and departments, the academic ranks and a checklist for graduate school.

This book can easily be read in one sitting, which is just what I did, but later I found myself frequently rereading various parts. None the less the book covers widespread subjects in so many words, there are some topics I miss, for example balancing between career and family. However the European graduate schools vary a bit from the ones in the U.S., most of the chapters can be utilized by a continental reader as well, and it is also good to know about the differences. The essential issue of a guidance book is always the usefulness. I find the book very helpful for everybody, who is an apprentice in this profession like me, and joyful at the same time. It's definitely worth reading, even just for fun. For a young mathematician, a great help to plan ahead.

Gergely Röst (Szeged)

Burkard Polster, **The Mathematics of Juggling** (Undergraduate Texts in Mathematics), XIII+226 pages, Springer-Verlag, New York – Berlin – Heidelberg, 2003.

This book is an interesting and comprehensive introduction to the mathematics of juggling. The target audience includes mathematics educators, readers of popular mathematics, mathematically minded jugglers and mathematicians interested in unusual applications.

It is not easy to communicate to the general public that mathematics can be a lot of fun and very useful. Mathematicians investigate models of the real world, and the results of these investigations often lead to measurable benefits; however, the details are usually too difficult for outsiders. This book offers a possible solution to this problem and helps in making mathematics more popular. For example, based on this book, it takes only an hour to outline to an outsider what is a simple juggling series, the model, what are the basic mathematical results for them, and how they lead to the invention of new and attractive juggling tricks, like 441. This trick and many others can easily be demonstrated by a computer juggling program; the book gives the address of a web site where such programs are freely available.

Of course, the book contains much more than easy results on simple juggling sequences. Juggling cards, necklaces in state graphs, estimations for the maximal length of prime juggling series, the number of juggling sequences with a given number of balls and period are just a few keywords from the chapter devoted to single juggling series. The popular nature of this list and the content of the book could be misleading: some seemingly easy problems or statements lead to serious mathematics, mainly combinatorics, and sometimes space considerations allow only a sketch or a reference for the proof. Generalizations from simple to multiplex and multihand juggling are also included. The first juggling-related mathematical theorems, due to Claude Shannon, obtain a proper emphasis in the middle of the book.

The book surveys many interesting juggling-related parts of mathematics and life. For example, it points out that bell-ringing (or change ringing), which is a traditional part of exceptionally great state celebrations mainly in English speaking countries, has a close connections with juggling. Indeed, any bell ringing pattern corresponds to some simple juggling series performed with a set of so-called bell balls. A bell ball is a ball that makes a bell-sound tuned to a given pitch when it hits the juggler's hand. It is the book under review that contains the most comprehensive introduction to the mathematics of bell ringing and the corresponding toss juggling.

The book contains anecdotes on famous juggler-mathematicians, many useful hints for practical jugglers, and juggling related aspects of various things like irrational numbers and braids. My experience is that it contains material sufficient to hold a one-semester course for interested graduate students of mathematics.

I am pleased to recommend this nice book to everyone who is interested in the topic.

Gábor Czédli (Szeged)

Gerald E. Sacks, **Mathematical logic in the 20th century**, XIV+693 pages, Singapore University Press, National University of Singapore World Scientific, New Jersey – London – Singapore – Hong Kong, 2003.

How to get acquainted with the main 20th century achievements of mathematical logic? The best way is reading the classics. This volume is a great help for those who choose this hard but rewarding path. The author selected those classical papers that he thinks the most influential ones. We quote the introduction to see the intentions of the author.

"I have not read all the logic of the last century – far from it. And only a fraction of what was read was understood. The choices made were personal in nature. Who knows that 'personal' means? The selection was certainly not based on an Olympian view of mathematical logic derived from a long and scholarly life of pondering the subject. Perhaps the choices cohere, if only because it is hard to see how it could be otherwise.

"The selection rules were: (R1) No papers from before World War II, (R2) No long papers, (R3) At most one paper by any author, (R4) The paper was (and is) intellectually exciting then (and now)..."

We give the complete list of papers reprinted in this volume: *Cohen, Paul J.*, The independence of the Continuum Hypothesis, *Cohen, Paul J.*, The Independence of the Continuum Hypothesis II, *Devlin, K.I. and Jensen, R.B.*, Marginalia to a Theorem of Silver, *Friedberg, Richard M.*, There Theorems on Recursive Enumeration. I. Decomposition. II. Maximal Set. III. Enumeration without Duplication, *Girard, Jeans-Yves*, Introduction to Π^1_2-Logic, *Gödel, Kurt*, Consistency-Proof for the Generalized Continuum-Hypothesis, *Hrushovski, Ehud*, The Mordell-Lang Conjecture for Function Fields, *Kreisel, G.*, Model-Theoretic Invariants: Applications to Recursive and Hyperarithmetic Operations, *Kleene, S.C.*, Recursive Functionals and Quantifiers of Finite Types I *Lachlan, A.H.*, A Recursively Enumerable Degree which will not Split over all Lesser Ones, *Martin, Donald A.*, Measurable Cardinals and Analytic Games, *Matijasevic, Ju.V.*, Enumerable Sets are Diophantine, *Morley, Michael*, Categoricity in Power, *Moschovakis, Y.N.*, Hyperanalytic Predicates, *Mucnik, A.A.*, Solution of Post's Reduction Problem and Some Other Problems of the Theory of Algorithms, *Post, Emil L.*, Recursively Enumerable Sets of Positive Integers and Their Decision Problems, *Robinson, Abraham*, Non-Standard Analysis, *Sacks, Gerald E.*, The Recursively Enumerable Degrees are Dense, *Scott, Dana*, Measurable Cardinals and Constructible Sets, *Shelah, S.*, Stable Theories, *Shoenfield, J.R.*, The Problem of Predicativity, *Silver, Jack*, On the Singular Cardinals Problem, *Soare, Robert*, Automorphisms of the Lattice of Recursively Enumerable Sets Part I: Maximal Sets, *Solovay, Robert M.*, A Model of Set-Theory in which Every Set of Reals is Lebesgue Measurable, *Spector, Clifford*, On Degrees of Recursive Unsolvability, *Tarski, Alfred*, A Decision Method for Elementary Algebra and Geometry, *Vaught, R.L.*, Denumerable Models of complete Theories, *Wilkie, A.J.*, Model Completeness Results for Expansions of the Ordered Field of Real Numbers by Restricted Pfaffian Functions and the Exponential Function, *Woodin, W. Hugh*, Supercompact Cardinals, Sets of Reals, and Weakly Homogeneous Trees, *Zil'ber, B.I.*, Structural Properties of Models of $\aleph_1$-Categorical Theories.

This list proves the excellent taste and wise overview of the author on logic and assures everyone that this volume makes an indispensable addition to the numerous lecture notes and monographs on mathematical logic lying on our bookshelves.

Péter Hajnal (Szeged)

DOUGLAS R. STINSON, **Combinatorial Designs** (Constructions and Analysis), XVI+300 pages, Springer, New York – Berlin – Heidelberg – London – Paris – Tokyo – Hong Kong – Barcelona – Budapest, 2004.

This excellent introductory textbook is the work of one of the top experts of the field. After introducing the most important notions and techniques more advanced topics (pairwise balanced designs, t-designs and t-wise balanced designs, orthogonal arrays and codes) are touched and some of the most important and interesting applications are discussed.

The content of the book is well described by the chapter headings: Contents, Foreword, Preface, 1. Introduction to Balanced Incomplete Block Designs, 2. Symmetric BIBDs, 3. Difference Sets and Automorphisms, 4. Hadamard Matrices and Designs, 5. Resolvable BIBDs, 6. Latin Squares, 7. Pairwise Balanced Designs I, 8. Pairwise Balanced Designs II, 9. t-Designs and t-wise Balanced Designs, 10. Orthogonal Arrays and Codes, 11. Applications of Combinatorial Designs, A. Small Symmetric BIBDs and Abelian Difference Sets, B. Finite Fields, References, Index.

To see the possible audience of the textbook we quote the introduction where the author talks about his goals: "This book is intended primarily to be a textbook for study at the senior undergraduate or beginning graduate level. Courses in Mathematics or computer science can be based on this book. Regardless of the audience, however, it requires a certain amount of 'mathematical maturity' to study design theory. The main technical prerequisites are some familiarity with basic abstract algebra (group theory, in particular), linear algebra (matrices and vector spaces), and some number-theoretic fundamentals (e.g., modular arithmetic and congruences).

"As mentioned above, this book is primarily intended to be a textbook. In addition, all of the material in this book is suitable for self-study by graduate students, who will find it provides helpful background information concerning research topics in design theory. Researchers may also find that some of the sections on advanced topics provide a useful reference for material that is not easily accessible in textbook form."

Péter Hajnal (Szeged)

K. Denecke, M. Erné and S. L. Wismath (Eds.), **Galois Connections and Applications** (Mathematics and its Applications, Vol. 565), XV+501 pages, Kluwer Academic Publishers, Dordrecht – Boston – London, 2004.

Since the classical Galois theory of algebraic equations over fields, the following situation has frequently appeared in various mathematical contexts: given partially ordered sets A and B, we have order-reversing maps $\rho\colon A \to B$ and $\lambda\colon B \to A$ such that $\rho\lambda$ and $\lambda\rho$ are closure operators on A and B, respectively. This pair of maps is a *Galois connection* between A and B. The notion comes from Oystein Ore (Trans. Amer. Math. Soc. **55** (1944), 493-513), who also introduced the close concept of the *adjunction* (ρ and λ are order-preserving, $\rho\lambda$ is a closure, and $\lambda\rho$ is a kernel operator), which was extended later to the basic categorical notion of an adjoint functor. A slogan due to Saunders Mac Lane says that adjoint functors arise everywhere. So do Galois connections, too: they are in the background of such diverse concepts as, e.g., Hilbert's nilradical ideals, Dedekind cuts, and Garrett Birkhoff's equational theories.

The book under review is the first one fully dedicated to Galois connections and adjunctions. It contains fifteen papers on the topic, mostly general surveys or reports on recent results. The first of them is M. Erné's large, comprehensive essay, *Adjunctions and Galois Connections: Origins, History and Development*, an attractively written, extremely instructive introduction. After having read this paper, the subsequent ones can be read independently. They are written by well-known experts in general algebra. Here is the list of them (I take the liberty to abbreviate "Galois Connections" to GC): *Categorical Galois Theory: Revision and Some Recent Developments* (by G. Janelidze), *The Polarity Between Approximation and Distribution* (by M. Erné), *GC and Complete Sublattices* and *Complexity of Terms and the Galois Connection* Id-Mod (by K. Denecke and S. L. Wismath), *GC for Operations and Relations* (by R. Pöschel), *GC and Polynomial Completeness* (by K. Kaarli), Q-*Independence and Weak Automorphisms* (by K. Głazek and St. Niwczyk), *A Survey of Clones Closed Under Conjugation* (by Á. Szendrei), *GC for Partial Algebras* (by P. Burmeister), *Iterated GC in Arithmetic and Linguistics* (by Joachim Lambek, presenting, among others, how Beatty sequences are generated by adjunctions), *Deductive Systems and GC* (by I. Chajda and R. Halaš), *A Galois Correspondence for Digital Topology* (by J. Šlapal), *GC in Category Theory, Topology and Logic* (by W. Gähler).

The concluding essay of the volume, *Dyadic Mathematics — Abstractions from Logical Thought*, is contributed by Rudolf Wille. It is a gentle introduction to Formal Concept Analysis, developed by the author and his group, starting with the basic remark that, within a given sector of our knowledge, assigning to any set of objects the set of attributes they all share, and to any set of attributes the set of objects that have all of them, we obtain a Galois connection.

I recommend this valuable collection to everybody involved in algebraic research and/or teaching algebra in higher education.

Béla Csákány (Szeged)

P. M. Cohn, **Basic Algebra** (Groups, Rings and Fields), XII+465 pages, Springer-Verlag, London, 2003.

For more than twenty years, generations of algebraists have learnt the fundamentals of their subject from P. M. Cohn's three volume book Algebra. The book under review is a new version of Volumes 2 and 3. From the Preface: 'The present book is based on both these volumes, complemented by the definitions and basic facts on groups and rings. Thus the volume is addressed to students who have some knowledge of linear algebra and who have met groups and fields, though all the essential facts are recalled here. My overall aim has been to present as many of the important results in algebra as would conveniently fit into one volume.' The topics discussed are well illustrated by the titles of the chapters: Sets, Groups, Lattices and Categories, Rings and Modules, Algebras, Multilinear Algebra, Field Theory, Quadratic Forms and Ordered Fields, Valuation Theory, Commutative Rings, Infinite Field Extensions.

'On a first encounter some readers may find the style of this book somewhat concise, but they should bear in mind that mathematical texts are best read with paper and pencil, to work out the full consequences of what is being said and to check examples.' The book is a wonderful piece of work that condenses and transmits the long experience of an excellent teacher. The reader enjoys the structure of the precisely designed volume and the beautiful combination of clear arguments and well-chosen examples. A lot of exercises helps the reader to deepen his knowledge.

This updated and improved introduction to abstract algebra must be on the bookshelves of all algebraists and of all students interested in algebra. Such an important work would have deserved a more perceptive technical editor to correct the misuse of the typesetting system. (For an extreme example, see the typographic quality of page 29.)

Mária B. Szendrei (Szeged)

J. Stillwell, **Elements of Number Theory** (Undergraduate Text in Mathematics), XII+254 pages, Springer-Verlag, New York – Berlin – Heidelberg, 2003.

From the Preface of the book: "In the present book we seek *integer solutions*, and this leads to the concepts due to Kummer and Dedekind.

Solving equations in integers is the central problem of number theory, so this book is truly a number theory book, with most of the results found in standard number theory courses. However, numbers are best understood through their algebraic structure, and the necessary algebraic concepts — rings and ideals — have no better motivation than number theory.

The first nontrivial examples of rings appear in the number theory of Euler and Gauss. The concept of ideal — today as routine in ring theory as the concept of normal subgroup is in group theory — also emerged from number theory, and in quite heroic fashion. Faced with failure of unique prime factorization in the arithmetic of certain generalized "integers", Kummer created in the 1840s a new kind of number to overcome the difficulty. He called them "ideal numbers" because he did not know exactly what they were, though he knew how they behaved. Dedekind in 1871 found that these "ideal numbers" could be realized as *sets* of actual numbers, and he called these sets *ideals*.

Dedekind found that ideals could be defined quite simply; so much so that a student meeting the concept today might wonder what all the fuss is about. It is only in their role as "ideal numbers", where they realize Kummer's impossible dream, that ideals can be appreciated as a genuinely brilliant idea."

The author published in 2002 the book "Mathematics and Its History". The present book also shows his outstanding knowledge in the history of Mathematics: at each example or problem the historical connections are well demonstrated.

The book is divided into twelve Chapters: 1. Natural numbers and integers, 2. The Euclidean algorithm, 3. Congruence arithmetic, 4. The RSA cryptosystem, 5. The Pell equation, 6. The Gaussian integers, 7. Quadratic integers, 8. The four square theorem, 9. Quadratic reciprocity, 10. Rings, 11. Ideals, 12. Prime ideals.

Each chapter begins with a simple motivating example, and — while maintaining the constant interest of the reader — gradually advances from simple to complicated, from the concrete example to the general theory. At the end of almost every section there are carefully chosen exercises which make possible an immediate reinforcement of the new proofs or ideas.

The book is clearly written, well organized and is a very pleasurable reading: it is an excellent and very useful undergraduate textbook. However, thanks to the many examples and exercises, it is well suitable for independent study, as well.

László Megyesi (Szeged)

Piotr Pragacz, **Topics in Cohomological Studies of Algebraic Varieties** (Impanga Lecture Notes), XXVIII+297 pages, Birkhäuser Verlag, Basel – Boston – Berlin, 2005.

The editor describes the origins of the present volume as follows: "The articles in this volume are an outgrowth of seminars and schools of Impanga." "Impanga is an algebraic geometry group operating since 2000 at the Institute of Mathematics of Polish Academy of Sciences in Warsaw. Besides seminars Impanga organized the following schools at the Banach Center in Warsaw: *Characteristic classes of singular varieties*, April. 2002, *Stratifications of moduli spaces*, May 2002, *Schubert varieties*, May 2003, and *Hommage à Grothendieck*, January 2004."

The topics covered in this volume are characteristic classes of singular varieties, geometry of flag varieties, cohomological computations for homogeneous spaces, K-theory of algebraic varieties, quantum cohomology and Gromov-Witten theory. The authors targeted graduates and post graduate students as possible audience with friendly presentation, exhibiting many examples. The present book is also useful for researchers since starting from elementary topic, through classical results it reaches many results not published in monographs.

Finally we list the presented papers: *Contents*, *Preface*, *Priotr Pragacz* Notes on the Life and Work of Alexander Grothendieck, *Paolo Aluffi* Characteristic Classes of Singular Varieties, *Michel Brion* Lectures on the Geometry of Flag Varieties, *Anders Skovsted Buch* Combinatorial K-theory, *Haibao Duan* Morse Functions and Cohomology of Homogeneous Spaces, *Ali Ulas Ozgur Kisisel* Integrable Systems and Gromov-Witten Theory, *Piotr Pragacz* Multiplying Schubert Classes, *Jörg Schürmann* Lectures on Characteristic Classes of Constructible Functions, *Marek Szyjewski* Algebraic K-theory of Schemes, *Harry Tamvakis* Gromov-Witten Invariants and Quantum Cohomology of Grassmannians.

Péter Hajnal (Szeged)

Denis Serre, **Matrices, Theory and applications** (Translated from the 2001 French original, Graduate Texts in Mathematics, 216), XVI+202 pages, Springer-Verlag, New York, 2002.

The origin of the book is explained by the author in the preface.

"This text was conceived during the years 1998-2001, on the occasion of a course that I taught at the École Normale Supériuere de Lyon. As such every result is accompanied by a detailed proof. During this course I tried to investigate all the principal mathematical aspects of matrices: algebraic, geometric, and analytic. ...

This text was first published in French by Masson (Paris) in 2000, under the title *Les Matrices: théorie et practique*. I have taken the opportunity during the translation process to correct typos and errors, to index a list of symbols, to rewrite some unclear paragraphs, and to add a modest amount of material and exercises. In particular, I added three sections, concerning alternate matrices, the singular value decomposition, and the Moore-Penrose generalized inverse. Therefore, this edition differs from the French one by about 10 percent of the contents."

This book is organized into ten chapters. The chapter titles are: Preface, List of Symbols, 1 Elementary Theory, 2 Square Matrices, 3 Matrices with Real or Complex Entries, 4 Norms, 5 Nonnegative Matrices, 6 Matrices with Entries in a Principal Ideal Domain; Jordan Reduction, 7 Exponential of a Matrix, Polar Decomposition, and Classical Groups, 8 Matrix Factorizations, 9 Iterative Methods for Linear Problems, 10 Approximation of Eigenvalues, References, Index.

As the author writes in the introduction "The first three chapters summarize the basics. The next six chapters can be read of matrix theory and should be known by almost every graduate student in any mathematical field. The other parts can be read more or less independently of each other. However, exercises in a given chapter sometimes refer to the material introduced in another one."

The main motivation of the author is to unify the different approaches, possible presentations for matrix theory and obtain a textbook that is suitable for the wild range of applications and hence for the different audiences deeply interested in matrices. The main goal is certainly achieved. The algorithm driven presentation is useful for those who are mainly interested in practical applications, but suitable for theoreticians too. The carefully chosen exercises give good chance to see deeper connections and see further lines of research.

The complexity theoretical language, the presentation of modern numerical techniques and the classical algebraic basics make this textbook an excellent source for graduate students in each field using matrices. It can be recommended both for use in the classroom and for independent study.

Péter Hajnal (Szeged)

Adi Ben-Israel and Thomas Nall Eden Greville (deceased), **Generalized Inverses** (Theory and Applications. Second Edition (CMS Books in Mathematics)), XV+420 pages, Springer-Verlag, New York – Berlin – Heidelberg – London – Paris – Tokyo – Hong Kong – Barcelona – Budapest, 2003.

The notion of invertibility plays a significant role in the study of linear algebra. Briefly, if A is a complex $m \times n$ matrix of rank r, then A is invertible if and only if $m =$

$n = r$, and in that case, the inverse of A is $A^{-1} = |A|^{-1} A^{ad}$, where $|A|$ is the determinant and A^{ad} is the classical adjoint of A. Extensions of the idea of invertibility may be traced back over several centuries, but the major contribution to today's understanding of the topic was provided by R. Penrose in 1955 [Proc. Cambridge Philos. Soc. **51** (1955), 406–413]. In the article entitled "A Generalized Inverse for Matrices" he showed that there exists one and only one solution to the following list of equations: (1) $AXA = A$, (2) $XAX = X$, (3) $(AX)^* = AX$, and (4) $(XA)^* = XA$, where C^* denotes the conjugate transpose of C. The following year, R. Rado noted in the same journal [Proc. Cambridge Philos. Soc. **52** (1956), 600–601] that the solution to these equations was identical to the solution of a list of equations given 36 years earlier by E. H. Moore [Bull. Amer. Math. Soc. **26** (1920), 394–395]. The unique solution to these systems of equations, now called the Moore–Penrose inverse (some writers use the term "pseudoinverse") of A and usually denoted by $A^\dagger$, has since found application in several areas of mathematics and related natural scientific areas.

The book under review which is the second edition of the 30 years ago published one provides a detailed survey of generalized inverses and their main properties, illustrating the theory with applications. The second author, T.E.G. Greville, passed away before the project started. The book itself is the Volume 15 in the series of *Canadian Mathematical Society Books in Mathematics (Ouvrages de mathématiques de la Société mathématique du Canada)* published by Springer-Verlag of New York – Berlin – Heidelberg. It contains the two prefaces by the authors, the introduction, ten chapters, the appendix, and an extensive annotated bibliography.

In the introduction there are some about motivation of the study of generalized inverses (generalized inverse exists for a class of matrices larger than the class of nonsingular matrices; has some of the properties of the usual inverse; and reduces to the usual inverse when A is nonsingular), indication of their diversity, and some historical notes. The newly added preliminary chapter (Chapter 0) provides a collection of the facts, definitions, and notations that are used in successive chapters. The Introduction and the Preliminary chapters can be accessed freely on the internet: http://rutcor.rutgers.edu/~bisrael/BIG-0.pdf.

For any $m \times n$ complex matrix, let $A\{i, j, \cdots, l\}$ denote the set of all matrices which satisfy equations $(i), (j), \ldots, (l)$ from among the Penrose equations (1), (2), (3), (4). A matrix $X \in A\{i, j, \ldots, l\}$ is called an $\{i, j, \ldots, l\}$-inverse of A. Chapter 1 deals with existence, construction and algebraic properties of $\{1\}$-, $\{1, 2\}$-, $\{1, 2, 3\}$-, $\{1, 2, 4\}$- and $\{1, 2, 3, 4\}$-inverses and includes an explicit formula for the Moore-Penrose $A^\dagger$ based on full rank factorization of A. A method is described for the construction of $\{2\}$-inverses having a prescribed rank between r, the rank of A, and $\min(m, n)$.

In Chapter 2 a consistency condition for the system $AXB = D$ in terms of $\{1\}$-inverses of A and B is described, and development of characterizations of the sets $A\{1, 3\}, A\{1, 4\}, A\{2\}, A\{1, 2\}$ and other subsets of $A\{2\}$ is provided. Special emphasis has been given to the relationship between generalized inverses and idempotent matrices and projectors, and to generalized inverses with prescribed range and null space. Description of constrained linear equation and its solution is given by using restricted

$\{1\}$-inverse, and application of $\{1\}$-inverse for solving interval linear programming and of $\{1,2\}$-inverse for integral solution of linear equations are described. The Bott-Duffin inverse is developed and applied to electrical networks.

In Chapter 3, minimal properties of generalized inverses are introduced. For the system $Ax = b$ the smallest Euclidean norms are given and used when $X \in A\{1,3\}$ and $X \in A\{1,4\}$ in the case of inconsistent linear systems and of existent of multiplicity of solutions, respectively. Some examples of use of Tikhonov regularization are mentioned: constrained least squares problem, ridge estimation, more stable solution if A is ill-conditioned. After that weighted and oblique generalized inverses, essentially strictly convex norms and the associated projectors and generalized inverses are developed. In the end of this chapter an extremal property of the previously investigated Bott-Duffin inverse is treated with application to electrical network.

Chapter 4 deals with spectral generalized inverses. If λ is a nonzero eigenvalue of a singular square matrix A, it is not necessarily true that λ^{-1} is an eigenvalue of the generalized inverse $A^\dagger$. Generalized inverses having some of the spectral properties of the inverse of a nonsingular matrix are studied. To this end, the four Penrose equations are supplemented further by the following equations applicable only to square matrices: $(1^k) A^k X A = A^k$, (5) $AX = XA$, (6) $A^k X = X A^k$, (7) $AX^k = X^k A$, where k is a given positive integer. The authors defined and clearly developed the concept of a spectral inverse. The unique $\{1,2,5\}$-inverse called the group inverse had particular attention and its spectral properties are developed in detail (the group inverse exists if and only if the range and null space of A are complementary subspaces). A similar treatment is also given to the Drazin pseudoinverse (equivalently to the $\{1^k,2,5\}$-inverse, where k is the index of A), which exists for every square matrix. The spectral properties of the Drazin pseudo-inverse are the same as those of the group inverse with regard to nonzero eigenvalues and the associated eigenvectors, but weaker for 0-vectors.

Chapter 5 entitled "Generalized inverses of partitioned matrices" contains the study of linear equations and matrices in partitioned form, and the associated generalized inverses. It includes partitioning by columns and by rows to solve linear equations and to compute generalized inverses; studying intersections of linear manifolds and using the results to obtain common solutions of pairs of linear equations and to invert matrices partitioned by rows; various results on generalized inverses of bordered and partitioned matrices. In the introduction of this chapter Greville's method for computing $A^\dagger$ was rung in but it is missing.

We can agree that Chapter 6, "Spectral theory for rectangular matrices", is perhaps the most lucid and complete presentation of the topic to be found in any text. It treats singular value decompositions, the Schmidt approximation theorem, partial isometries, the polar decomposition theorem and other decompositions for rectangular matrices, briefly the behavior of the Moore–Penrose inverse of perturbed matrix $A + E$, generalization of the classical spectral theorem for normal matrices to rectangular ones, and the generalized singular value decompositions.

Chapter 7 deals with computational aspects of generalized inverses mentioning some direct methods for the computation of $\{1\}$-, $\{1,2\}$- and $\{1,3\}$-inverses and of $\{2\}$-inverses with prescribed range and null space, as well as with iterative methods for computing $A^\dagger$. Greville's method and related results appeared in this chapter. The treatment is very brief and no error analysis is attempted: "these topics have been studied extensively in the numerical analysis literature".

Chapter 8, "Miscellaneous applications", is another newly added chapter and it starts a little bit provocative question: "What can you do with generalized inverses that you could not do without?" Parallel sums with applications in electrical networks; linear statistical model with biased and unbiased estimation; application to Newton methods for solving systems of nonlinear equations, to linear system theory, of group inverse to finite Markov chains, of Drazin inverse for solving singular linear difference equations; and finally the concept of matrix volume applied to surface integrals and to probability distributions may be the answers to the question.

The final chapter is about generalized inverses of linear operator between Hilbert spaces. Generalized inverses of linear integral and differential operators, as well as minimal properties of generalized inverses are studied. Results on integral and series representations of generalized inverses, and iterative methods for their computation, are given without proof.

The Appendix is based on a recently published paper by the first author on Moore of the Moore–Penrose inverse: Moore's lecture to the AMS in 1920 and general reciprocal translated to plain English.

An important feature of this book is the over 600 exercises (many of which are solved in detail) which are primarily intended to impart information rather than to test understanding. These exercises contain several applications beside Chapter 8. Each chapter ends with the section "Suggested further reading". These sections provide excellent additional references on topics treated, as well as reading suggestions on topics which are not considered in the book. The bibliography contains exactly 901 references. The first author compiled a more completed bibliography on generalized inverse containing over 2100 items which can be accessed from the Internet address http://rutcor.rutgers.edu/pub/bisrael/GI.html.

The book is very recommended for reference and self-study or for use as a classroom text. Because only elementary knowledge of linear algebra is assumed it can be used profitably by graduate or advanced undergraduate students of mathematics and computer science, and by PhD students of related scientific area, e.g., chemometrics, chemo/bio-informatics, biometrics, bio/geo-statistics, physics etc.

Róbert Rajkó (Szeged)

Alejandro Adem, R. James Milgram, **Cohomology of finite groups** (Grundlehren der Mathematischen Wissenschaften, Fundamental Principles of Mathematical Sciences 309), VIII+327 pages, Springer-Verlag, Berlin, 1994.

This is the second edition of the book of the same title whose first edition appeared in 1994. This book was a welcome addition to the literature on the cohomology of groups in 1994 and it has become a classic since then. It emphasizes the computational aspects of the cohomology of finite groups with coefficients in a field, and this sets it apart from other treatises.

The book begins with a discussion on group extensions, simple algebras and cohomology in Chapter I. Chapter II gives an overview of classifying spaces, Eilenberg-Mac Lane space, and the Steenrod algebra, and it introduces group cohomology in Section II.3. Chapter III is concerned with invariants of cohomology groups. This chapter contains a proof of Serre's theorem, and the Cárdenas-Kuhn theorem for weakly closed subgroups. Chapter IV discusses spectral sequences and detection theorems, and Chapter V deals with G-complexes and equivariant cohomology. Chapters VI, VII and VIII contain extensive discussions of the cohomology of the symmetric and alternating groups, the finite groups of Lie type, and the sporadic simple groups. The book concludes with Chapter IX on the plus construction, and Chapter X on the Schur subgroup of the Brauer group.

We cite from the introduction: "In this new edition a number of corrections were made. In addition we modified Chapter VIII so that it now includes mod 2 cohomology calculations for several additional sporadic simple groups. The exposition in Chapter III (on invariant theory) was also expanded somewhat."

Ferenc Fodor (Szeged)

A. A. Ivanov, M. W. Liebeck, J. Saxl (Eds.), **Groups, Combinatorics & Geometry, Durham 2001** (Proceedings of the L. M. S. Symposium held in Durham, July 16–26, 2001), X+335 pages, World Scientific Publishing Co., Inc., River Edge NJ – London – Singapore – Hong Kong, 2003.

The content of this volume is described in the introduction: "This book contains the proceedings of the L.M. S. Durham Symposium on Groups, Geometry and Combinatorics, July 16-26, 2001, supported by the Engineering and Physical Sciences Research Council of Great Britain.

"The symposium brought together about 70 leading experts in these areas, as well as 15 postdoctoral fellows and research students.

"These proceedings contain 20 survey articles, most of which are expanded versions of lectures, or series of lectures, given at the symposium. Broadly speaking, the topics covered in the articles are: *Geometrices, amalgams and recognition, of simple groups*, *Groups of Lie type and representation theory*, *Probabilistic and asymptotic group theory*, *Algebraic combinatorics and permutation groups*, *Computational group theory and sporadic groups*, *Applications*."

Finally let us see the list of presented papers: *Tuna Altnel, Alexandre V. Borovik and G. Cherlin*, Classification of simple K^*-groups of finite Morley rank and even type: geometric aspects; *C.D. Bennett, R. Gramlich, C. Hoffman and S. Shpectorov*, Curtis-Phan-Tits theory; *Jonathan Brundan and Alexander Kleschev*, Representation theory of symmetric groups and their double covers; *Peter J. Cameron*, Coherent configurations, association schemes and permutation groups; *Persi Diaconis*, Mathematical developments from the analysis of riffle shuffling; *Jason Fulman and Robert Guralnick*, Derangements in simple and primitive groups; *William M. Kantor and Ákos Seress*, Computing with matrix groups; *Martin W. Liebeck and Gary M. Seitz*, A survey of maximal subgroups of exceptional groups of Lie type; *Martin W. Leibeck and Anaer Shalev*, Bases of primitive permutation groups; *Ulrich Meierfrankenfeld, Bernd Stellmacher and Gernot Stroth*, Finite groups of local characteristic p: an overview; *Thomas W. Müller*, Modular subgroup arithmetic; *Simon P. Norton*, Counting nets in the monster; *Cheryl E. Preager*, Overgroups of finite quasiprimitive permutation groups; *Lászó Pyber*, Old groups can learn new tricks; *Yoav Segev*, Shadows of elements, solvability of finite quotients and the Margulis-Platonov conjecture; *Aner Shalev*, Applications of random generation to residual properties of some infinite groups; *Pham Huu Tiep*, Low dimensional representations of finite quasisimple groups; *F.G. Timmesfeld*, Structure and presentations of Lie-type groups; *V.I. Trofimov*, Vertex stabilizers of locally projective groups of automorphisms of graphs: a summary; *Robert A. Wilson*, Computing in the monster.

Péter Hajnal (Szeged)

Alexander A. Mikhalev, Vladimir Shpilrain and Jie-Tai Yu, **Combinatorial Methods, Free Groups, Polynomials, and Free Algebras** (CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, volume 19), XII+315 pages, Springer-Verlag, New York, 2004.

The goal of the authors are well described in the preface: "The main purpose of this book is to show how ideas from combinatorial group theory have spread to the other two areas, with the main focus on the area of commutative algebra where the influence of these ideas has been especially spectacular.

"We would like to emphasize that we only consider here purely combinatorial methods and results; in particular, we leave out important interactions of group theory with topology and geometry that were recently used with great success in solving several difficult problems about free groups and their automorphisms. We also leave out geometric methods in affine algebraic geometry and concentrate on combinatorial ones, in particular on those that come from combinatorial group theory."

The chapter headings are: *Preface*, *Introduction*, *I. Groups*: Introduction, 1. Classical Techniques of Combinatorial Group Theory, 2. Test Elements, 3. Other Special Elements, 4. Automorphic Orbits, *II. Polynomial Algebra*: Introduction, 5. The Jacobian Conjecture, 6. The Cancellation Conjecture, 7. Nagata's Problem, 8. The Embedding Problem, 9. Coordinate Polynomials, 10. Test Polynomials, *III. Free Nielsen-Schreier Algebras*: Introduction, 11. Schreier Varieties of Algebras, 12. Rank Theorems and Primitive Elements, 13. Generalized Primitive Elements, 14. Free Leibniz Algebras, *References*, *Notation Index*, *Author Index*, *Subject Index*.

The book is targeted at research mathematicians as well as graduate students with an interest in the general area of algebra.

Péter Hajnal (Szeged)

Peter Walker, **Examples and Theorems in Analysis**, X+287 pages, Springer Verlag, London – Berlin – Heidelberg – New York, 2004.

This book is a unique and very practical contribution to the teaching of calculus for freshmen or sophomores at various kinds of colleges and universities. Dozens of bulky volumes are available at the textbook market, which offer a shorter or longer introduction to either Calculus or Mathematical Analysis. The former ones promise a rapid way to acquire the basic techniques without a genuine understanding (however, this level of knowledge is usually enough for those whose major is not mathematics). The latter ones guarantee an essentially deeper understanding or insight through the so-called epsilon-delta arguments. But the price is a much slower pace of acquiring the subject, and what is more problematic, that these epsilon-delta reasonings are rather abstract or intrinsic or sometimes boring, while the student may miss the point. The trouble with epsilons and deltas at the present stage are well known for every lecturer.

The aim of this book is to try to give the subject concreteness and immediacy by giving the well-chosen examples equal status with the theorems. The results are introduced and motivated by reference to examples which illustrate their use, as well as further examples to show how far the assumptions may be relaxed before the result fails.

The contents cover sequences, functions and continuity, differentiation, constructive integration, improper integrals, series, and applications. For example, the discussion of Newton's method in Chapter 3 is a little more extended than usual, partly due to the common misunderstanding that if two terms in an iterative scheme agree to some number of decimal places, then they must equal the value of the limit to that accuracy. In Chapter 4 instead of introducing the integral as an antiderivative, the author treats integration by using the set of regulated functions, that is, those which have finite left- and right-hand limits at every point. According to the author's intension, the technicalities are less than for the general Riemann approach, and at the same time this class of integrable functions is easily identified and sufficient for most uses at this level. For advanced applications, Lebesgue's theory and its generalizations are essential, but those are beyond the scope of this book. On the other hand, improper integrals are discussed in some detail in Chapter 5, since many of the most interesting examples, for instance

$$\int_0^{\pi/2} \ln(\sin x)dx = -\frac{\pi}{2}\ln 2, \quad \int_0^{\infty} \frac{\sin x}{x}dx = \frac{\pi}{2}$$

are of this type, and improper integrals motivate the discussion of distributions in the last Chapter 7.

A number of applications show what the subject is about, and what can be done with it. The applications in the theory of Fourier series and integrals, distributions, and asymptotics show how the results may be put to use. For instance, Euler's classical series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6},$$

or the limit

$$\lim_{n\to\infty} \sqrt{n}\sin_n(x) = \sqrt{3}, \quad 0 < x < \pi,$$

where the iterated sine $\sin_n(x)$ denotes the n-fold composition $\sin(\sin(\ldots\sin(x)))$, are proved in Chapter 7 devoted to applications.

Throughout the book functions of a single real (or occasionally complex) variable are considered, other quantities such as a in $\cos(ax)$ play the role of parameters. However, in some places the variables have a more equal role, notably when differentiating under the integral sign or interchanging the order of integration, both occurring in Chapter 7. In order to justify these results the author looks at functions of several real variables in the Appendix, where the proof of Fubini's theorem is also included in a special form, which is satisfactory for justifying the interchange mentioned just above.

The book entitled *Introductory Mathematics: Algebra and Analysis* by Geoff Smith (Springer Undergraduate Mathematics Series, Springer, London, UK, 1998) is strongly suggested by the present author as a useful preliminary to this one. In particular, there are references to it as for properties of the real and complex number systems.

The exercises at the ends of chapters are at all levels. Those marked with one star develop new ideas in some way, for instance, convexity or the use of one-sided derivatives in relation to the Mean Value Theorem. Those with two stars are open questions with the polite request "If you have solved one of these, please let the author know!"

To sum up, this excellent book is written primarily for first- and second-year undergraduates in mathematics, but it will also be of interest to students of statistics, computer science and engineering, as well as to professionals in these fields. We warmly recommend it as an entertaining and stimulating companion on your way of acquiring the basic elements of Calculus and Mathematical Analysis.

Ferenc Móricz (Szeged)

John P. D'Angelo, **Inequalities from Complex Analysis** (The Carus Mathematical Monographs, Number 28), XVI+264 pages, The Mathematical Association of America, Washington, DC, 2002.

This excellent book is a wonderful scientific exposition of inequalities and positivity conditions for a lot of mathematical objects arising in complex analysis.

The author begins by defining the complex number field, and then deals with enough mathematical analysis to reach recently published research on positivity conditions for functions of several complex variables. A real "pearl" in the book is the complete proof of a stabilization theorem relating two natural positivity conditions for real-valued polynomials of several complex variables.

The chapter headings are: 1. Complex Analysis, 2. Complex Euclidean Spaces and Hilbert Spaces, 3. Complex Analysis in Several Variables, 4. Linear Transforms and Positivity Conditions, 5. Compact and Integral Operators, 6. Positivity Conditions for Real-valued Functions, 7. Stabilization and Applications, 8. Afterword.

Numerous examples, exercises, and discussions of geometric reasoning appear along the way.

The Bibliography contains 36 items.

The book is recommended to instructors and students in mathematics, interested in complex analysis, but physicists and engineers may also find the topics and discussions useful.

J. Németh (Szeged)

Bernard Dacorogna, Paolo Marcellini, **Implicit Partial Differential Equations** (Progress in Nonlinear Differential Equations and Applications, Volume 37), xii+273 pages, Birkhäuser, Boston – Basel – Berlin, 1999.

This book deals with partial differential equations and systems which are nonlinear in the highest derivatives. Boundary value problems posed for these equations or systems are considered and the existence of almost everywhere solutions to these problems are proved. The main tool is a new functional analytic method based on the Baire category theorem. Comparison with other methods is given: essentially that of viscosity solutions, but also briefly that of convex integration. Results obtained by this new method have important applications to the calculus of variations, geometry, nonlinear elasticity, problems of phase transitions and optimal design. The book contains an Introduction and 4 Parts (10 Chapters), References (with 316 items) and an Index.

A typical result is the following:

Theorem 1.2. *Let Ω be an open set of $\mathbb{R}^n$. Let $F\colon \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ be a continuous function, convex with respect to the last variable and coercive (i.e., $\lim F(x,u,\xi) = +\infty$, if $|\xi| \to \infty$ uniformly with respect to (x,u)). Let $\varphi \in W^{1,\infty}(\Omega)$ be a function satisfying*

$$F(x,\varphi(x),D\varphi(x)) \le 0 \quad \text{a.e.} \quad \text{in} \quad \Omega. \tag{1}$$

Then for every $\varepsilon > 0$ there exists $u \in W^{1,\infty}(\Omega)$ such that $\|u-\varphi\|_{L^\infty} \le \varepsilon$ and

$$F(x,u(x),Du(x)) = 0 \quad \text{a.e.} \quad \text{in} \quad \Omega, \quad u = \varphi \quad \text{on} \quad \partial\Omega. \tag{2}$$

The Baire category theorem-technique mentioned above (in this simplest case) is working as follows: The functional space

$$V := \{u \in \varphi + W_0^{1,\infty}(\Omega) : F(x,u(x),Du(x)) \le 0 \quad \text{a.e.} \quad x \in \Omega\}$$

is introduced which is the set of subsolutions of (2) (V is nonempty since (1) holds). Next V is endowed with the C^0 metric. Then the completeness of the metric space V is proved in the following way: The coercivity condition ensures that any Cauchy sequence in V has uniformly bounded gradient and therefore has a subsequence that converges weak* in

$W^{1,\infty}$ to a limit. Since the convexity of F implies lower semicontinuity, we get that the limit is indeed in V. Further for every integer k, the subset V^k of V

$$V^k := \left\{ u \in V : \int_\Omega F(x, u(x), Du(x))dx > -\frac{1}{k} \right\}$$

is introduced. It is shown that V^k is open in V and dense in V. Finally from Baire category theorem it is concluded that

$$\begin{aligned} \bigcap_k V^k &= \left\{ u \in V : \int_\Omega F(x, u(x), Du(x))dx \geq 0 \right\} \\ &= \{ u \in \varphi + W_0^{1,\infty}(\Omega) : F(x, u(x), Du(x)) = 0 \quad \text{a.e.} \quad x \in \Omega \} \end{aligned}$$

is dense, and hence nonempty, in V.

The ideas described above ensure existence theorems for Dirichlet problems posed for the eikonal equation or system, moreover some generalizations lead to the existence of a dense set of solutions of Dirichlet problems posed for general first order systems as well as solutions of Dirichlet-Neumann problems posed for second order equations (or systems). The applications presented in the book are very important (variational problems, the complex eikonal equation etc.). We emphasize also the results on Piecewise polynomial approximations of classes C^N and Sobolev spaces $W^{N,\infty}$ explained in the Appendix. Finally we present the 9 unsolved problems from the Introduction: 1. Selection criterion, 2. Measurable Hamiltonians, 3. Lipschitz boundary data, 4. Approximation of Lipschitz functions by smooth functions, 5. Extension of Lipschitz functions and compatibility conditions, 6. Existence under quasiconvexity assumption, 7. Problems with constraints, 8. Potential wells, 9. Calculus of variations.

Summarizing: this is a well written book that contains actual problems with solutions (most of the results belong to the authors, and some of them are published first here). It is warmly recommended to specialists in PDE-s, analysts, and to anyone with basic knowledge in PDE-s.

Jenő Hegedűs (Szeged)

J. Kevorkian, **Partial Differential Equations, Analytical solution techniques** (Second Edition), Texts in Applied Mathematics (Vol. 35), XI+636 pages, Springer-Verlag, New York – Berlin – Heidelberg, 2000.

"This new edition has been substantially updated to include a wider scope of topics. New material has been added to all the chapters, and some of the derivations and discussions have been streamlined. Also much of the needed background is reviewed in the Appendix."

The book under review consists of a Preface, 8 Chapters, an Appendix, References with 43 items, and an Index.

Some features of the book are the following: 1) All of the equations and conditions of Problems are derived from physical laws. 2) Both single equations and systems of equations are considered. 3) The most of the material is related to one, two and three spatial dimensional Problems, but general n spatial dimensional Problems are also considered. 4) Problems on whole line, plane, space; on half line, plane, space; on compacts are considered and solved. 5) In many cases the effect of lower-derivative terms is also studied. 6) A rich variety of analytical techniques are presented: the separation of the variables, the reflection method, the perturbation method, integral transforms, complex variable methods, the Cole-Hopf Transformation for the Burger's Equation. 7) In many cases the complete asymptotic expansion is presented for the solutions of small-parameter problems. 8) One of the most precise description of the vibrating string is given for the couple of horizontal and vertical deflexions. 9) Fundamental solutions, Green functions are built for the most important operators in mathematical physics. 10) The sections of all Chapters contain Problems for the individual work. 11) Emphasize finally that the (second) half of the book deals with quasilinear and nonlinear partial differential equations or/and systems.

More complete information on the contents of the book serve the headlines of the Chapters and Sections.

Ch.1 *The Diffusion Equation*: Heat Conduction, The Fundamental Solution, Initial-Value Problem in the Infinite Domain; Superposition, Problems in the Semi-infinite Domain; Green's Functions, Problems in the Finite Domain; Green's Function, Higher-Dimensional Problems, Burger's Equation.

Ch.2 *Laplace's Equation*: Applications, The Two-Dimensional Problem; Conformal Mapping, Fundamental Solution; Dipole Potential, Volume, Surface, and Line Distributions of Sources and Dipoles, Green's formula and Applications, Green's and Neumann's Functions, Solutions in Terms of Integral Equations.

Ch.3 *The Wave Equation*: The Vibrating String, Shallow-Water Waves, Compressible Flow, The One-Dimensional Problem on $-\infty < x < \infty$, Initial - and Boundary-Value Problems on $0 \leq x < \infty$, Initial - and Boundary-Value Problems on $0 \leq x \leq 1$, Effect of Lower-Derivative Terms, Dispersive Waves on $-\infty < x < \infty$, The Three-Dimensional Wave Equation; Acoustics.

Ch.4 *Linear Second-Order Equations with Two Independent Variables*: Classification and Transformation to Canonical Form, The General Hyperbolic Equation, Hyperbolic Systems of Two First-Order Equations.

Ch.5 *The Scalar Quasilinear First-Order Equation*: Conservation Laws in Two Independent Variables, Strict Solutions in Two Independent Variables, Weak Solutions; Shocks, Fans, and Interfaces, The Scalar Quasilinear Equation in n Independent Variables.

Ch.6 *Nonlinear First-Order Equations*: Geometrical Optics; A Nonlinear Equation, Applications Leading to the Hamilton-Jacobi Equation, The Nonlinear Equation, The Complete Integral; Solutions by Envelope Formation.

Ch.7 *Quasilinear Hyperbolic Systems*: The Quasilinear Second-Order Hyperbolic Equation, Systems of n First-Order Equations, Systems of Two First-Order Hyperbolic Equations, Shallow-Water Waves, Compressible Flow Problems.

Ch.8 *Approximate Solutions by Perturbation Methods*: Regular Perturbations, Matched Asymptotic Expansions, Multi-Scale Expansions.

Appendix: Review of Green's Function for ODEs Using the Dirac Delta Function, Review of Fourier and Laplace Transforms, Review of Asymptotic Expansions.

Summarizing: the book provides a well chosen collection of analytical solution techniques by applications to a wide class of problems of mathematical physics. It will be useful for the researchers in PDE-s, physicists, engineers and also for students with basic knowledge in vector calculus, ODE-s and PDE-s.

Jenő Hegedűs (Szeged)

G. Bachman, L. Narici and E. Beckenstein, **Fourier and wavelet analysis** (Universitext), X+505 pages, Springer-Verlag, New York, 2000.

The book under review is intended primarily as an introduction to classical Fourier analysis, Fourier series and the Fourier transform.

The book is divided into 7 chapters. The first 3 chapters contain some background material from modern analysis: Ch. 1 Metric and normed spaces (definitions, inner products, orthogonality, linear isometry, the L_p and ℓ_p spaces ...); Ch. 2 Analysis (continuity, closed and open sets, completeness, compactness, ...); Ch. 3 Bases (best approximations, orthonormal bases and sequences, Haar base, unconditional convergence ...).

The heart of the book is the next 3 chapters: Ch. 4 Fourier series (sine and cosine series, Riemann-Lebesgue lemma, pointwise and uniform convergence, the Gibbs phenomenon, termwise differentiation and integration, summability, theorems of Fejér and Lebesgue, ...); Ch. 5 The Fourier transform (finite Fourier transform, Fourier transform, convolutions on $\mathbb{T}$ and $\mathbb{R}$, the Fourier map, inversion theorems, a sampling theorem, Mellon transform ...); and the relatively short Ch. 6 Discrete and fast Fourier transform (discrete Fourier transform, inversion theorem and the Cooley-Tukey "fast" algorithm).

The final Ch. 7 Wavelets consists of a sketchy introduction to the wavelet theory in a small amount of space.

The material of this book is easily accessible to senior undergraduate or graduate students or readers with a good undergraduate background in analysis; it doesn't assume deep knowledge in modern abstract analysis. The discussion is thorough and shows the material at a leisurely pace. There are many exercises that expand on the material, followed by hints or answers. The text includes many historical notes too. Unfortunately in the book there are no applications on physics or engineering, although they are mentioned.

Summarizing, this book is a reader-friendly, gentle introduction to the theory of Fourier analysis. I recommend this book to graduate or postgraduate mathematics and physics students, engineers, computer scientists and everybody who want to learn Fourier analysis by studying on his/her own.

Zoltán Németh (Szeged)

Alois Kufner and Lars-Erik Persson, **Weighted Inequalities of Hardy Type**, XVIII+357 pages, World Scientific Publishing Co., New Jersey – London – Singapore – Hong Kong, 2003.

This book is an excellent survey of the theory of weighted integral inequalities of Hardy type, including modifications concerning Hardy-Steklov operators, and some basic results about Hardy type inequalities and their limit (Carleman-Knopp type) inequalities.

The chapter headings are: 1. Hardy's Inequality and Related Topics, 2. Some weighted Norm Inequalities, 3. The Hardy-Steklov Operator, 4. Higher Order Hardy Inequalities, 5. Fractional Order Hardy Inequalities, 6. Integral Operators on the Cone of Monotone Functions.

The References contains more than 150 items. The book is warmly recommended to researchers dealing with inequalities and related areas of mathematics, but it can be useful for advanced graduate students, too.

J. Németh (Szeged)

Graeme Cohen, **A Course in Modern Analysis and its Applications** (Australian Mathematical Society Lectures Series, Vol. 17), XIII+333 pages, Cambridge University Press, Cambridge, 2003.

This book is aimed as a text book for a one-semester course at a senior undergraduate level, at the same time it is useful not only to mathematics undergraduates, but also to those who need to learn some mathematical analysis for use in other areas such as engineering, physics, biology or finance.

The list of the chapters is: 1. Prelude to Modern Analysis, 2. Metric Spaces, 3. The Fixed Point Theorem and its Applications, 4. Compactness, 5. Topological Spaces, 6. Normed Vector Spaces, 7. Mappings on Normed Spaces, 8. Inner Product Spaces, 9. Hilbert Spaces.

Every chapter ends with rich Exercises section (some of the problems covered here are worked out at the end of the book).

Bibliography contains 30 items.

The book can be warmly recommended to all instructors of graduate and undergraduate courses in analysis, as well as for interested students.

J. Németh (Szeged)

Bruce van Brunt, **The Calculus of Variation** (Springer Univeritext Series), XIV+290 pages, Springer-Verlag, New York, 2004.

The topics treated are mainly a small but complete 19^{th} century fashioned theory of the extremals of a functional of the form $(*)\ J(y) = \int_{x_0}^{x_1} f(x, y, y')\, dx\ (y \in C^2[x_0, x_1])$ with various constraints by means of quadrature methods. Outlooks are concerned with close generalizations to several independent variables respectively functionals of the same type with a single real variable but involving derivatives of higher order.

The content is divided into nine chapters preceded by an exhaustive Introduction (Chapter 1) exposing the basic ideas we encounter later. In Chapter 2 the author deduces the Euler-Lagrange equations of the extremals of $(*)$ and outlines some well-known quadrature methods for special cases. Chapter 3 deals with natural generalizations to several dependent respectively two independent variables, in particular here we can find the Euler-Lagrange formalism of classical mechanics. This chapter finishes with a glimpse to the inverse problem of finding variational principles for second order ordinary differential equations motivating later parts as e.g. the one about Noether's theorem. Chapter 4 treats generalizations of $(*)$ with multiple constraints as an extended Lagrange multiplier method. A whole chapter is devoted then to reformulations of eigenvalue problems in the framework of variational calculus. As a basic example the Sturm-Liouville theory of non-homogeneous vibrating chords is studied in detail with particular interest to the lowest eigenfrequency. The study of the Lagrangian approach is then continued and finished in Chapters 6-7 with a brief account of holonomic constraints and constraints with variable endpoints. The Hamiltonian formalism is introduced next in Chapter 9 with a well-written heuristics on the bases of the Legendre transformation of curves. The main problems touched here are constructions of symplectic transformations and the Hamilton-Jacobi equation. As illustrating examples, the generalized Fermat-principle in geometrical optics and the conservative systems in classical mechanics are presented. The chapter ends with a technical study of the method of additive separation of the Hamilton-Jacobi equation leading to Liouville's and Stäckel's theorems. Actually Chapter 9 finishes the main stream of the book: the natural correspondence between conservation laws and variational symmetries is developed concluding in Noether's theorem. The last chapter turns back to the real beginning: the analogs of the positive definiteness criteria for finite dimensional minimum are considered in the setting of functionals of type $(*)$. The classical Legendre and Jacobi conditions are proved and illustrated, also Morse' index theorem is explained without proof.

Nowadays the title seems to be misleading somewhat: the achievements of the second half of the 20^{th} century (think the results of Stampacchia) changed the character of the field with respect to the material selected by the author. Nevertheless, I find this book a very useful supplementary reading for undergraduate students and a good teaching aid for lecturers of topics involving traditional variational calculus (as e. g. mathematical physics). It is written with a deep pedagogical attention and contains carefully selected motivating examples as the problems of the brachistochrone, the catenary, the pendulum, the 2-dimensional classical isoperimetric problem, the geodesics on some simple 3-dimensional surfaces, the two body problem, the Sturm-Liouville problem with inhomogeneous chord etc., revisited several times from various view points corresponding to the pieces of theory developed. According to my classroom experience with undergraduate physicists, the presentation of the examples in the book may be very helpful to clarify the difficulties of the students to understand correctly the intuitive but often rather lacunary symbolism used in normal texts in physics instead of presenting them the formulas in all mathematical details. It can also be appreciated that the author tries to present the results showing motivation and heuristical ideas for each crucial theorem.

L. L. Stachó (Szeged)

Boris Aronov, Saugata Basu, János Pach and Micha Sharir (Eds.), **Discrete and computational geometry. The Goodman-Pollack Festschrift** (Algorithms and Combinatorics, 25), XII+853 pages, Springer-Verlag, Berlin, 2003.

This collection of papers is dedicated to Eli Goodman and Ricky Pollack. The volume contains a large number of state-of-the-art articles covering virtually every facet of discrete and computational geometry. Many know the names of Eli Goodman and Ricky Pollack, two New York mathematicians, but those certainly who know the journal *Discrete and Computational Geometry* published by Springer Verlag. Its first issue appeared in 1986, and the journal has thrived since then. It publishes 1300 pages per year currently. This volume celebrates the "founding fathers" and their achievements in the field.

Ferenc Fodor (Szeged)

Jiři Matoušek, **Using the Borsuk-Ulam Theorem** (Lectures on topological methods in combinatorics and geometry. Written in cooperation with Anders Björner and Günter M. Ziegler. Universitext), XII+196 pages, Springer-Verlag, Berlin, 2003.

A large number of important results in areas of mathematics such as combinatorics, discrete geometry and computer science have been proved applying methods of algebraic topology. One outstanding example is Lovász's proof of Kneser's conjecture. This book is intended to make elementary topological methods more accessible to those who work in other areas of mathematics. A number of combinatorial and geometric results are presented together with the necessary algebraic topological tools. The book is organized around one central theme, the Borsuk-Ulam theorem and its relatives. The first version of the text was based on lecture notes taken on the author's lectures in 1993 in Prague. Later, Günter Ziegler taught a course in Berlin which was partially based on this text and he also made corrections and additions. The present book is essentially a carefully rewritten version prepared during a predoctoral course the author taught in Zürich in 2001.

Chapters 1 and 2 basically introduce the fundamental topological tools used throughout the rest of the book followed by famous examples of applications of the Borsuk-Ulam theorem such as the ham sandwich theorem, Kneser's conjecture, Dol'nikov's theorem, Gale's lemma and Schrijver's theorem. Chapter 4 is a topological interlude which covers such concepts as quotient spaces, joins, k-connectedness and cell complexes. Chapter 5 is concerned with $\mathbb{Z}_2$-maps and nonembeddability, and Chapter 6 is about multiple points of coincidence. Each section is provided with a good selection of exercises ranging in difficulty from the easy to the very hard. The book concludes with a *Quick Summary* that collects the most important concepts and statements from each chapter. A *Hints to Selected Exercises* helps the reader by giving ideas for the solution of exercises.

This excellent book is useful for specialists in discrete geometry, combinatorics and computer science who want to learn how algebraic topology can be used in their discipline. It is also very suitable as a textbook for such a course.

Ferenc Fodor (Szeged)

Dennis Barden and Charles Thomas, **An Introduction To Differential Manifolds**, XII+218 pages, Imperial College Press, London, 2003.

This excellent book by two Cambridge mathematicians is intended as an introductory text into the theory of differential manifolds. It is based on more than twenty years of lectures at Cambridge University where this subject is part of the senior undergraduate year mathematics curriculum.

In Chapter 1 differential manifolds and basic techniques are introduced. Chapter 2 is wholly devoted to the infinitesimal theory introducing the reader to the tangent bundle. This knowledge is extended to general fibre bundles in Chapter 3 and used to provide examples of smooth manifolds in dimensions 3, 5 and 7. In Chapter 4 the globalisation of integration is discussed; in order to do this differential forms are introduced. Chapter 5 extends the idea of the differential of a function to a derivation of the full algebra of forms. Stokes' Theorem is proved here together with the generalization of the fundamental theorem of integral calculus. In Chapter 6 de Rham cohomology of differential manifolds is introduced. Chapter 7 discusses degrees, indices and other related topics. A concise introduction is provided to Lie groups in Chapter 8. Finally, the book closes with a short crash course in differential analysis which covers the prerequisites necessary to understand the material. Each chapter is followed by a number of exercises whose solutions are given at the end of the book. A true gem is the *Guide to the Literature* that gives advice on how one should want to continue reading by giving short outlines what one can expect from specific books recommended by the authors.

The book is an excellent introductory text into the theory of differential manifolds with a carefully thought out and tested structure and a sufficient supply of exercises and their solutions. It does not only guide the reader gently into the depths of the theory of differential manifolds but also careful on giving advice how one can place the information in the right context. It is certainly written in the best traditions of great Cambridge mathematics.

Ferenc Fodor (Szeged)

K. Borovkov, **Elements of Stochastic Modelling**, XIII+342 pages, World Scientific, New Jersey – London – Singapore – Hong Kong, 2003.

This is an introductory textbook on stochastic processes, for an undergraduate course that follows a calculus-based introductory course on probability theory. As a text for a second undergraduate course in stochastics, without measure theory, it may be viewed as a worthy competitor of S. M. Ross's well-known and deservedly popular book *Introduction to Probability Models* (Academic Press, New York, with several editions), and, on the long run, it even has a good chance to come out a winner.

Following an *Introduction*, pp. 1–8, on the nature of mathematical modelling, the second chapter *Basics of probability theory*, pp. 9–74, reviews necessary preliminaries, going in a nice descriptive manner from the St. Petersburg paradox to functionals of Brownian motion, as a prelude to a chapter on *Markov chains*, pp. 75–128, which in a technical sense serves as a foundation for the rest of the text. Then the chapter *Markov decision processes*, pp. 129–154, discusses first applications, focusing in part on discounted dynamic programming. The short fifth chapter *The exponential distribution and Poisson process*, pp. 155–170, then gives way to the one on *Jump Markov processes*, pp. 171–192, with various continuous-time inhomogeneous and birth-and-death processes, then to a traditional type of an exposition of the *Elements of queueing theory*, pp. 193–225, and finally to another short chapter on the *Elements of renewal theory*, pp. 227–236. Chapter 9 on the *Elements of time series*, pp. 237–273, is the only one with a statistical flavor, mostly on linear processes and filters, and the tenth and final chapter, *Elements of simulation*, pp. 275–305, gets as far as the Markov Chain Monte Carlo method.

Now then, given that most of us would agree that the curriculum of such a course is about right ($\pm\varepsilon$ depending on the personal taste of the instructor and the special needs of the audience), what does distinguish this book? The answer is very simple: its overall intelligence. It is a blend of the best traditions of honest scholarship and a no-nonsense modern style. The basic notions, problems and results are introduced through fine examples. Many of the results are exposed without formal rigorous proofs, but are always both rigorously stated and explained in an attractive and efficient manner. Most chapters suggest a carefully selected recommended literature, and all of Chapters 2–10 end with an excellent choice of theoretical and applied problems, complementing the material in an organic fashion, and an extra chapter *Answers to problems*, pp. 307–332, contains solutions, either just a numerical answer or a complete formal argument as needed, to all the problems. Lists of notations and abbreviations and an index also help the reader. Finally, a special feature is a large number of footnotes, in small print, that range from providing etymologies of the most frequently used technical terms to covering what this reviewer believes to be the minimal historical knowledge, at least for all the instructors, about the field of stochastics and its creators. A fine pedagogical touch and a good sense of proportion make the text extremely well balanced. A very intelligent book indeed.

Sándor Csörgő (Szeged)

Peter Knabner and Lutz Angermann, **Numerical Methods for Elliptic and Parabolic Partial Differential Equations** (Texts in Applied Mathematics 44), XV+424 pages, Springer-Verlag, New York – Berlin – Heidelberg, 2003.

Mathematics is playing a more and more important role in the natural, especially physical and biological sciences, while blurring the boundaries between pure and applied mathematics. The development of new courses is a natural consequence of an abundance of new techniques appeared on the research frontier. Thus, the purpose of this textbook is to meet the current and future needs of these advances and to encourage the teaching of new courses.

Shortly after the appearance of the German edition, the authors were asked by Springer to create an English version of their book published in the year 2000. They took this opportunity not only to correct some misprints and mistakes, but also to extend the text at various places.

The book consists of ten chapters, five appendices, References, and an Index.

Ch.0. *For Example: Modelling Processes in Porous Media with Differential Equations.* This chapter illustrates the scientific context in which differential equation models may occur in general and also in a specific example.

Ch.1. *For the Beginning: The Finite Difference Method for the Poisson Equation.* Generalizations and limitations of the finite difference method, maximum principles, and the stability problem are treated in detail.

Ch.2. *The Finite Element Method for the Poisson Equation.* The homogeneous Dirichlet boundary value problem for the Poisson equation forms the paradigm of this chapter, but more generally valid considerations are also emphasized. In this way the abstract foundation for the treatment of more general problems in the next chapter is provided.

Ch.3. *The Finite Element Method for Linear Elliptic Boundary Value Problems of Second Order.* Among others, Sobolev (the "correct" function) spaces and the condition number of finite element matrices are introduced, convergence rates are estimated, etc.

Ch.4. *Grid Generation and A Posteriori Error Estimation.* As one of the first steps, the implementation of the finite element method (and also of the finite volume method described in Ch.6) requires a "geometric discretization" of the domain; and this is presented here.

Ch.5. *Iterative Methods for Systems of Linear Equations*, with special emphasis on sparse matrices. Gradient and conjugate gradient methods, Krylov subspace methods in particular for nonsymmetric systems of equations, the multigrid method, and nested iterations are studied in great detail.

Ch.6. *The Finite Volume Method.* This method can be viewed as a discretization method of its own right, though it includes ideas from both finite difference and finite element methods. This chapter closes the presentation of numerical methods for elliptic partial differential equations.

Ch.7. *Discretization Methods for Parabolic Initial Boundary Value Problems.* Among others, semidiscretization by the method of vertical lines, fully discrete schemes, stability, the maximum principle for the one-step theta method are discussed.

Ch.8. *Iterative Methods for Nonlinear Equations.* In the same way as linear (initial-) boundary value problems by the discretization techniques lead to systems of linear equations, one gets nonlinear equations from nonlinear problems. The authors focus on Banach's fixed-point theorem, Newton's methods, and their variants.

Ch.9. *Discretization Methods for Convection-Dominated Problems.* The modelling of transport and reaction processes in porous media results in such differential equations. Standard methods, the streamline-diffusion method, and finite volume methods are presented in detail, and finally the Lagrange-Galerkin method is sketched.

In the *Appendices* the basic concepts of Analysis, Linear Algebra, Linear Functional Analysis, and Function Spaces are summarized.

The *References* comprises 39 items for textbooks and monographs, and 74 items for journal papers.

The *Index* is very detailed, occupies 10 pages with two columns on each of these pages.

At the end of each unit, a great variety of exercises challenge the reader's understanding. Detailed illustrations and 67 figures are also included.

To sum up, the book gives a carefully selected account of both the theory and implementation, and provides simultaneously both a rigorous and an inductive presentation of the technical details. We warmly recommend it for advanced undergraduate and beginning graduate courses. For students with mathematics major it is an excellent introduction to the theory and methods, guiding them in the selection of methods and helping them to understand and pursue finite element programming. For engineering and physics students it provides a general framework for the formulation and analysis of methods.

Ferenc Móricz (Szeged)

STIG LARSSON and VIDAR THOMÉE, **Partial Differential Equations with Numerical Methods** (Texts in Applied Mathematics 45), IX+259 pages, Springer-Verlag, Berlin – Heidelberg – New York, 2003.

The authors' purpose is to give an elementary, relatively short, and readable account of the basic types of linear partial differential equations, their properties, and the most commonly used methods for their numerical solution. The three major types of these equations are presented, namely elliptic, parabolic, and hyperbolic equations, and for each of them the text contains three chapters, which summarize the background material.

The existence, uniqueness, stability, and regularity of solutions, as well as the most important classes of numerical methods, namely finite difference methods and finite element methods have become most popular for elliptic and also for parabolic problems, whereas for hyperbolic equations the finite difference method continues to dominate.

To make the presentation more easily accessible, the elliptic chapters are preceded by a chapter of the two-point boundary value problem for an ordinary differential equation of second order, and the chapters on parabolic and hyperbolic evolution problems are preceded by a chapter on the initial value problem for a system of ordinary differential equations. A chapter on eigenvalue problems and eigenfunction expansions is also included, due to the fact that these are important tools in the analysis of partial differential equations. The last chapter provides a short survey of other classes of numerical methods of importance, namely collocation methods, finite volume methods, spectral methods, and boundary element methods.

The reading of the book does not assume a deep knowledge of the Classical and the Functional Analysis. In the first Appendix the basic elements of abstract linear spaces and function spaces, in particular Sobolev spaces, together with basic facts about Fourier transforms are collected, mostly without proofs. The solution of large systems of linear equations, which are necessary in the implementation of numerical methods, is included in the second Appendix.

The book consists of fourteen chapters and two appendices, a three pages Bibliography, and also a three pages Index. Each chapter ends with a problem section in which the reader is urged to prove some results only stated in the text, or to further develop some of the ideas presented, or simply to test some of the numerical methods on a computer, assuming that MATLAB or a similar software is available.

The table of Contents is the following: 1. Introduction, 2. A two-point boundary value problem, 3. Elliptic equations, 4. Finite difference methods for elliptic equations, 5. Finite element methods for elliptic equations, 6. The elliptic eigenvalue problem, 7. Initial value problems for ODEs, 8. Parabolic equations, 9. Finite difference methods for parabolic equations, 10. The finite element method for a parabolic problem, 11. Hyperbolic equations, 12. Finite difference methods for hyperbolic equations, 13. The finite element method for hyperbolic equations, 14. Some other classes of numerical methods, A1. Some tools from mathematical analysis, A2. Orientation on numerical linear algebra, Bibliography, and Index.

The book has developed from courses given by the authors over a long period of time at the Chalmers University of Technology and Göteborg University originally for third year engineering students, somewhat later also to beginning graduate courses for applied mathematics students. We warmly recommend it to advanced undergraduate and beginning graduate students of applied mathematics and/or engineering at every university of the world.

Ferenc Móricz (Szeged)

Mika Hirvensalo, **Quantum Computing** (Second Edition Natural Computing Series), XIII+214 pages, Springer, New York – Berlin – Heidelberg – London – Paris – Tokyo – Hong Kong – Barcelona – Budapest, 2004.

This year is the ten years anniversary of Peter Shor's algorithm for factorising large numbers on quantum computers. This famous algorithm, if implemented on a quantum computer, could factor integers in polynomial time. Currently there are no such algorithms known for conventional computers and there are no working quantum computers, able to perform Shor's algorithm. Still the result sent shock waves around the world, that is natural since fast factorization able us to break the wide-spread RSA method. Shor's results caused a boom at the borderline of physics and computer science, a completely new field, the so-called quantum informatics has developed. In 2001 Hirvensalo published the first edition of the present book. The success of this monograph is reflected in the fast appearance of the second edition. In the preface the author explains the changes made compared to the first edition, we quote his words.

"After the first edition of this book was published, I received much positive feedback from the readers. It was very helpful to have all those comments suggesting improvements and corrections. In many cases, it was suggested that more aspects on quantum information would be welcome. Unfortunately, I am afraid that an attempt to cover such a broad area as quantum information theory would make this book too scattered to be helpful for educational purposes."

"On the other hand, I admit that some aspects of quantum information should be discussed. The first edition already contained the so-called No-Cloning Theorem. In this edition, I have added a stronger version of the aforementioned theorem due to R. Jozsa, a variant which also covers the no-deleting principle. Moreover, in this edition, I have added some famous protocols, such as *quantum teleportation.*"

"The response to the first edition strongly supports the idea that the main function of this book should be educational, and I have not included further aspects of quantum information theory here..."

The chapter headings are: 1. Introduction, 2. Quantum Information, 3. Devices for Computation, 4. Fast Factorization, 5. Finding the Hidden Subgroup, 6. Grover's Search Algorithm, 7. Complexity Lower Bounds for Quantum Circuits, 8. Appendix A: Quantum Physics, 9. Appendix B: Mathematical Background, References, Index. The discussion on quantum information now is in a separate chapter and the topics involved are extended. Also Appendix A is a little bit extended compared to the first edition.

So the author does not attempt to compile an overview of all the results in quantum computing. One of his main driving goal is to write a book that is well usable in classroom. He has succeeded and this way he reaches a wide audience including computer scientists, theoretical mathematicians and physicists.

Péter Hajnal (Szeged)