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ON ESTIMATE OF THE NORM OF THE HOLOMORPHIC COMPONENT OF A MEROMORPHIC FUNCTION IN FINITELY CONNECTED DOMAINS

Abstract. In this paper we extend Gonchar-Grigorjan type estimate of the norm of holomorphic part of meromorphic functions in finitely connected Jordan domains with C^2 smooth boundary when the poles are in a compact set. A uniform estimate for Cauchy type integral is also given.

§1. Introduction

Landau investigated holomorphic functions in the unit disk $\mathbb{D}$ with $\|f\|_{\partial\mathbb{D}} \leqslant 1$ where $\|.\|_{\partial\mathbb{D}}$ denotes the sup norm over the boundary $\partial\mathbb{D}$ of $\mathbb{D}$. He showed that the absolute value of the sum of first n coefficients of Maclaurin series for such functions has order of growth $\log n$ (see [11], pp. 26-28). L.D. Grigorjan generalized this in the following sense, see [7]. Consider meromorphic functions in the unit disk with poles in some fixed compact subset of the unit disk and with total order n. Then the growth of the norm on the unit circle of sum of the principal parts is $\log n$. It is easy to see that the case when the origin is the only pole yields Landau's result. More generally, on simply connected domains with smooth boundary, when there is no restriction on the location of the poles, then we get linear growth for the norm (instead of $\log n$; see [6]).

Let us introduce the sup norm of meromorphic functions f on a domain D as follows:

$$\|f\|_{\partial D} := \sup \left\{ \limsup_{\zeta\to z} |f(\zeta)| : \ z \in \partial D \right\}.$$

In [5] A.A. Gonchar and L.D. Grigorjan proved the following theorem.

Theorem. *Let $D \subset \mathbf{C}$ be a simply connected domain and its boundary be C^1 smooth. Let $f : D \to \mathbf{C}_\infty$ be a meromorphic function on D such that it has m different poles. Denote by f_r the sum of principal parts of f (with*

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