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Research article

Development and differences in mathematical problem-solving skills: A cross-sectional study of differences in demographic backgrounds

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ABSTRACT

Problem-solving skills are the most applicable cognitive tool in mathematics, and improving the problem-solving skills of students is a primary aim of education. However, teachers need to know the best period of development and the differences among students to determine the best teaching and learning methods. This study aims to investigate the development and differences in mathematical problem-solving skills of students based on their grades, gender, and school locations. A scenario-based mathematical essay test was administered to 1067 students in grades 7–9 from schools in east Java, Indonesia, and their scores were converted into a logit scale for statistical analysis. The results of a one-way analysis of variance and an independent sample *t*-test showed that the students had an average level of mathematical problem-solving skills. The number of students who failed increased with the problem-solving phase. The students showed development of problem-solving skills from grade 7 to grade 8 but not in grade 9. A similar pattern of development was observed in the subsample of urban students, both male and female. The demographic background had a significant effect, as students from urban schools outperformed students from rural schools, and female students outperformed male students. The development of problem-solving skills in each phase as well as the effects of the demographic background of the participants were thoroughly examined. Further studies are needed with participants of more varied backgrounds.

1. Introduction

Problem-solving skills are a complex set of cognitive, behavioral, and attitudinal components that are situational and dependent on thorough knowledge and experience [1,2]. Problem-solving skills are acquired over time and are the most widely applicable cognitive tool [3]. Problem-solving skills are particularly important in mathematics education [3,4]. The development of mathematical problem-solving skills can differ based on age, gender stereotypes, and school locations [5–10]. Fostering the development of mathematical problem-solving skills is a major goal of educational systems because they provide a tool for success [3,11]. Mathematical problem-solving skills are developed through explicit training and enriching materials [12]. Teachers must understand how student profiles influence the development of mathematical problem-solving skills to optimize their teaching methods.

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Various studies on the development of mathematical problem-solving skills have yielded mixed results. Grissom [13] concluded that problem-solving skills were fixed and immutable. Meanwhile, other researchers argued that problem-solving skills developed over time and were modifiable, providing an opportunity for their enhancement through targeted educational intervention when problem-solving skills developed quickly [3,4,12]. Tracing the development of mathematical problem-solving skills is crucial. Further, the results of previous studies are debatable, necessitating a comprehensive study in the development of students' mathematical problem-solving skills.

Differences in mathematical problem-solving skills have been identified based on gender and school location [6–10]. School location affects school segregation and school quality [9,14]. The socioeconomic and sociocultural characteristics of a residential area where a school is located are the factors affecting academic achievement [14]. Studies in several countries have shown that students in urban schools demonstrated better performance and problem-solving skills in mathematics [9,10,15]. However, contradictory results have been obtained for other countries [6,10].

Studies on gender differences have shown that male students outperform female students in mathematics, which has piqued the interest of psychologists, sociologists, and educators [7,16,17]. The differences appear to be because of brain structure; however, sociologists argue that gender equality can be achieved by providing equal educational opportunities [8,16,18,19]. Because the results are debatable and no studies on gender differences across grades in schools have been conducted, it would be interesting to investigate gender differences in mathematical problem-solving skills.

Based on the previous explanations, teachers need to understand the best time for students to develop mathematical problem-solving skills because problem-solving is an obligatory mathematics skill to be mastered. However, no relevant studies focused on Indonesia have been conducted regarding the mathematical problem-solving skill development of students in middle school that can provide the necessary information for teachers. Further, middle school is the important first phase of developing critical thinking skills; thus relevant studies are required in this case [3,4]. In addition, a municipal policy-making system can raise differences in problem-solving skills based on different demographic backgrounds [10]. Moreover, the results of previous studies regarding the development and differences in mathematical problem-solving skills are debatable. Thus, the present study has been conducted to meet these gaps. This study investigated the development of mathematical problem-solving skills in students and the differences owing demographic backgrounds. Three aspects were considered: (1) student profiles of mathematical problem-solving skills, (2) development of their mathematical problem-solving skills across grades, and (3) significant differences in mathematical problem-solving skills based on gender and school location. The results of the present study will provide detailed information regarding the subsample that contributes to the development of mathematical problem-solving skills in students based on their demographic backgrounds. In addition, the description of the score is in the form of a logit scale from large-scale data providing consistent meaning and confident generalization. This study can be used to determine appropriate teaching and learning in the best period of students' development in mathematical problem-solving skills as well as policies to achieve educational equality.

2. Theoretical background

2.1. Mathematical problem-solving skills and their development

Solving mathematical problems is a complex cognitive ability that requires students to understand the problem as well as apply mathematical concepts to them [20]. Researchers have described the phases of solving a mathematical problem as understanding the problem, devising a plan, conducting out the plan, and looking back [20–24]. Because mathematical problems are complex, students may struggle with several phases, including applying mathematical knowledge, determining the concepts to use, and stating mathematical sentences (e.g., arithmetic) [20]. Studies have concluded that more students fail at later stages of the solution process [25,26]. In other words, fewer students fail in the phase of understanding a problem than during the plan implementation phase. Different studies have stated that students face difficulties in understanding the problem, determining what to assume, and investigating relevant information [27]. This makes them unable to translate the problem into a mathematical form.

Age or grade is viewed as one factor that influences mathematical problem-solving skills because the skills of the students improve over time as a result of the teaching and learning processes [28]. Neuroscience research has shown that older students have fewer problems with arithmetic than younger students; however, the hemispheric asymmetry is reduced [29]. In other words, older students are more proficient, but their flexibility to switch among different strategies is less. Ameer & Sigh [28] obtained similar results and found a considerable difference in mathematical achievement; specifically, older students performed better than younger students in number sense and computation using one-way analysis of variance (ANOVA) (F) of $F(2,411) = 4.82$, $p < 0.01$. Molnár et al. [3] found that the student grade affects domain-specific and complex problem-solving skills. They observed that the development of problem-solving skills was noticeable across grades in elementary school but stopped in secondary school. The fastest development of domain-specific problem-solving occurred in grades 7 and 8 [3], but the fastest development of complex problem-solving occurred in grades 5–7 [3]. No development was detected between grades 4 and 5 as well as grades 6 and 7 for domain-specific and complex problem-solving skills, respectively. Similarly, Greiff et al. [4] concluded that students developed problem-solving skills across grades 5–11 with older students being more skilled. However, the grade 9 students deviated from the development pattern, and their problem-solving skills dropped. The theories from Molnár et al. [3] and Greiff et al. [4] are the benchmark cases herein.

The above studies showed that problem-solving skills mostly developed during compulsory schooling and developed most quickly in specific grades. This indicates that specific development times can be targeted to enhance the problem-solving skills [3]. However, Jabor et al. [30] observed contradictory results showing statistically significant differences with small effects in mathematical performance between age groups: those under the age of 19 outperformed those over the age of 19 years old. Grissom [13] observed a negative correlation between age and school achievement that remained constant over time.

2.2. Effects of school location and gender on mathematical problem-solving skills

School location has been shown to affect mathematical achievement [9,14]. In 15 countries, students in rural schools performed considerably worse than students in urban schools in mathematics [9,10], science and reading [9]. In addition, Nepal [15] discovered that urban students significantly outperformed rural students in mathematical problem-solving skills ($t = -5.11$, $p < 0.001$) and achievement ($t = -4.45$, $p < 0.001$) using the results of an independent sample *t*-test (t). However, other countries have found that rural students outperformed urban students in mathematics [6,10]. These variations may be attributed to a lack of instructional resources (e.g., facilities, materials, and programs), professional training (e.g., poorly trained teachers), and progressive instruction [6]. The results of Williams's study [10] serve as the basis for the current study.

Gender differences in mathematics have received attention because studies show that male students outperform female students on higher-level cognitive tasks [31]. This is a shift from a meta-analysis study that found gender differences in mathematics to be insignificant and favored female students [32]. At the college level, female students slightly outperform male students in computation while male students outperform female students in problem solving. However, no gender differences have been observed among elementary and middle school students. This result was strengthened by other meta-analysis studies [7,8], which concluded that there was no gender difference in mathematical performance and problem-solving skills [15,33–35]. Gender similarity in mathematics is achieved when equal learning opportunities and educational choices are provided and the curriculum is expanded to include the needs and interests of the students [16,18,31].

From a sociological perspective, gender similarity in mathematics makes sense. If there is a gender difference in mathematics, this has been attributed to science, technology, engineering, and mathematics (STEM) being stereotyped as a male domain [8]. Stereotypes influence beliefs and self-efficacy of students and perceptions of their own abilities [8,19]. This is the reason for the low interest of female students in advanced mathematics courses [18,19]. However, Halpern et al. [16] found that more female students are entering many occupations that require a high level of mathematical knowledge. Moreover, Anjum [36] found that female students outperformed male students in mathematics. This may be because female students prepared better than the male students before the test and were more thorough [36,37]. The study of Anjum [36] is one of the basis cases of the current study.

Differences in brain structure support the argument that there are gender differences in mathematical performance [16,17]. Females have less brain lateralization (i.e., symmetric left and right hemispheres), which helps them perform better verbally. Meanwhile, males have more brain lateralization, which is important for spatial tasks [17]. In addition, the male hormone testosterone slows the development of the left hemisphere [16], which improves the performance of right brain-dominant mathematical reasoning and spatial tasks.

3. Method

3.1. Instrumentation

In this study, a science-related mathematical problem-solving test was used. This is a mathematics essay test where the problems are in the form of scenarios related to environmental management. Problems are solved by using technology as a tool (e.g., calculator, grid paper). The test was developed in an interdisciplinary STEM framework, and it is targeted toward grades 7–9. There were six scenarios in total: some were given to multiple grades, and others were specific to a grade. They included ecofriendly packaging (grade 7), school park (grade 7), calorie vs. greenhouse gas emissions (grades 7–9), floodwater reservoir (grade 8), city park (grades 8–9), and infiltration well (grade 9). These scenarios cover topics such as number and measurement, ratio and proportion, geometry, and statistics. Every scenario had a challenge, and students were provided with eight metacognitive prompt items to help them explore their problem-solving skills.

The test was administered by using paper and pencils for a 3-h period with a break every hour. At the end of the test, students were asked to fill in their demographic information. Each prompt item had a maximum score of 5 points: a complete and correct answer (5 points), a complete answer with a minor error (4 points), an incomplete answer with a minor error (3 points), an incomplete answer with a major error (2 points), and a completely wrong and irrelevant answer (1 point). Each scenario had a maximum total score of 40 points.

The test was validated to determine whether it contained good and acceptable psychometric evidence. It had an acceptable content validity index (CVI >0.67), moderate intraclass correlation coefficient (ICC) ($r_{xx} = 0.63$), and acceptable Cronbach's alpha ($\alpha = 0.84$). The construct validity indicated all scenarios and prompt items were fit ($0.77 \leq$ weighted mean square ≤ 1.59) with an acceptable discrimination value ($0.48 \leq$ discrimination value ≤ 0.93), acceptable behavior of the rating score, and good reliability (scenario reliability = 0.86; prompt item reliability = 0.94).

3.2. Participants

The test was administered to grades 7–9 students in east Java, Indonesia (n = 1067). The students were selected from A-accreditation schools in urban and rural areas; random classes were selected for each grade. The majority of the students were Javanese (95.01%), with the remainder being Madurese (3.3%) and other ethnicities. Table 1 describes the demographics of the participants.

3.3. Data analysis

Data were collected between July and September 2022. Prior to data collection, ethical approval was sought from the institutional review board (IRB) of the Doctoral School of Education, University of Szeged and was granted with the ethical approval number of 7/2022. In addition, permission letters were sent to several schools to request permission and confirm their participation. The test answers of the students were scored by two raters – the first author of this study and a rater with master's degree in mathematics education – to ensure that the rating scale was consistently implemented. The results showed good consistency with an ICC of 0.992 and Cronbach's alpha of 0.996.

The scores from one of the raters were converted to a logit scale by weighted likelihood estimation (WLE) using the ConQuest software. A logit scale provides a consistent value or meaning in the form of intervals. The logit scale represents the unit interval between locations on the person–item map. WLE was chosen rather than maximum likelihood estimation (MLE) because WLE is more central than MLE, which helps to correct for bias [38]. The WLE scale was represented by using descriptive statistics to profile the students' mathematical problem-solving skills in terms of the percentage, mean score (*M*) and standard deviation (*SD*) for each phase. The WLE scale was also used to describe common difficulties for each phase. The development of students' mathematical problem-solving skills across grades was presented by a pirate plot, which is used in R to visualize the relationship between 1 and 3 categorical independent variables and 1 continuous dependent variable. It was chosen because it displays raw data, descriptive statistics, and inferential statistics at the same time. The data analysis was performed using R studio version 4.1.3 software with the YaRrr package. A one-way ANOVA was performed to find significant differences across grades. An independent sample *t*-test was used to analyze significant differences based on gender and school location. The descriptive statistics, one-way ANOVA test, and independent sample *t*-test were performed using the IBM SPSS Statistics 25 software.

4. Results

4.1. Student profiles

The scores of students were converted to the WLE scale, where a score of zero represented a student with average ability, a positive score indicated above-average ability, and a negative score indicated below-average ability. A higher score indicated higher ability. The mean score represented a student with average mathematical problem-solving skills ($M = 0.001$, $SD = 0.39$). Overall, 52.1% of students had a score below zero. The distribution of scores among students was predominantly in the interval between −1 and 0. When the problem-solving process was analyzed by phase, the results showed that exploring and understanding were the most mastered problem-solving skills ($M = 0.24$, $SD = 0.51$). Only 27.9% of students had below-average scores for the exploring and understanding phases, which indicates that they mostly understood the given problem and recognized the important information. However, the problem-solving skills decreased with higher phases. The students had below-average abilities in the phases of representing and formulating ($M = -0.01$, $SD = 0.36$), planning and executing ($M = -0.15$, $SD = 0.41$), and monitoring and reflecting ($M = -0.16$, $SD = 0.36$). About 57.9% of the students had below-average scores for the representing and formulating phase, which indicates that they had problems making hypotheses regarding science phenomena, representing problems in mathematical form, and designing a prototype. The obvious reason for their difficulty with making hypotheses was that they did not understand simple concepts of science (e.g., CO_2 vs. O_2). In the planning and executing phase, 66.8% of the students failed to achieve a score greater than zero. This happened because they failed to apply mathematical concepts and procedures. Because they were unable to plan and execute a strategy, this affected the next phase of the problem-solving process. In the monitoring and reflecting phase, 68.0% of the students had a below-average score.

4.2. Development of mathematical problem-solving skills across grades

The development of the mathematical problem-solving skills of the students across grades was observed based on the increase in the mean score. The problem-solving skills developed from grade 7 to grade 8. The students of grade 7 had a mean score of −0.04 while grade 8 students had the highest mean score of 0.03. The students in grades 7 and 8 also showed more varied problem-solving skills than the grade 9 students did. In contrast, the grade 9 students showed a different pattern of development, and their mean score dropped to 0.01. Although the difference was not large, further analysis was needed to determine its significance.

Fig. 1 displays the development of the mathematical problem-solving skills of the students. The dots represent raw data or WLE

Table 1
Demographic characteristics of participants.

Demographic Characteristics		N	%
Gender	Male	452	42.4
	Female	615	57.6
Grade	7 (M age = 12.59, SD = 0.61)	380	35.61
	8 (M age = 13.42, SD = 0.59)	331	31.02
	9 (M age = 14.50, SD = 0.59)	356	33.36
School location	Rural	427	40.02
	Urban	640	59.89

scores. The middle line shows the mean score. The beans represent a smoothed density curve showing the full data distribution. The scores of the students in grades 7 and 9 were concentrated in the interval between −0.5 and 0. However, the scores of the grade 8 students were concentrated in the interval between 0 and 0.5. The scores of the students in grades 7 and 8 showed a wider distribution than those of the grade 9 students. The bands which overlap with the line representing the mean score, define the inference around the mean (i.e., 95% of the data are in this interval). The inference of the WLE score was close to the mean.

The one-way ANOVA results indicated a significant difference among the problem-solving skills of the students of grades 7–9 ($F(1,066) = 3.01$, $p = 0.046$). The students of grade 8 showed a significant difference in problem-solving skills and outperformed the other students. The students of grades 7 and 9 showed no significant difference in their mathematical problem-solving skills. Table 2 presents the one-way ANOVA results of the mathematical problem-solving skills across grades.

Fig. 2 shows the development of the mathematical problem-solving skills of the students across grades based on school location and gender. The problem-solving skills of the urban students increased from a mean score of 0.07 in grade 7 to 0.14 in grade 8. However, the mean score of urban students in grade 9 dropped. In contrast, the mean scores of the rural students increased continuously with grade. The improvements were significant for both the rural ($F(426) = 10.10, p < 0.001$) and urban ($F(639) = 6.10, p < 0.01$) students. For the rural students, grade 9 students showed a significant difference in problem-solving skills. In contrast, urban students in grades 8 and 9 showed significant differences in problem-solving skills but not in grade 7.

When divided by gender, both female and male students showed improvements in their problem-solving skills from grades 7 and 8. However, female students in grade 9 showed a stable score while the scores of male students in grade 9 declined. Only male students in grade 7 showed a significant difference in the mean score. In urban schools, the scores of male and female students increased and decreased, respectively, from grade 7 to grade 8. Male students in rural schools showed an increase in score from grade 7 to grade 9. However, the scores of female students in rural schools decreased from grade 7 to grade 8. Table 3 presents the one-way ANOVA results for the mathematical problem-solving skills of the students considering gender and school location.

Fig. 2 shows that the distributions of the male and female scores of students were similar for every grade except rural grade 9 students. The scores of the rural female students were concentrated in the interval between 0 and 0.5 while the scores of the rural male students were mostly below 0. The scores of rural students in grade 7 and urban students in grade 9 (both male and female) were concentrated in the interval between −0.5 and 0. The scores of urban students in grades 7 and 8 were concentrated in the interval between −0.5 and 0.5.

Fig. 3 shows a detailed analysis of the development of mathematical problem-solving skills across grades for each phase of the problem-solving process. Similar patterns were observed in the exploring and understanding and the representing and formulating phases: the mean score increased from grade 7 to grade 8 but decreased from grade 8 to grade 9. Grade 8 students had the highest mean score and differed significantly from the scores of students in other grades.

The scores of the students for the planning and executing phase increased with grade. However, the difference was only significant at grade 9. Grades 7 and 8 students showed an increase in score, but the improvement was not significant. There was no pattern detected in the monitoring and reflecting phase. The score was stable for grades 7 and 8 students but improved for grade 9 students. The mean score for each phase and the one-way ANOVA results are presented in Table 4.

Fig. 3 shows that the distributions of the problem-solving skills of the students were similar across grades and phases. However, the distributions were different for grade 9 students in the representing and formulating, planning and executing, and monitoring and reflecting phases, where 95% of the data were in the interval between −0.5 and 0.5.

4.3. *Effects of demographic background*

4.3.1. *School location*

The mathematical problem-solving skills of the students differed significantly based on school location. Urban students scored higher than rural students. The results of the *t*-test for mathematical problem-solving skills based on school location are presented in Table 5.

The effects of the school's location on the performances of male and female students were analyzed. The results showed that the

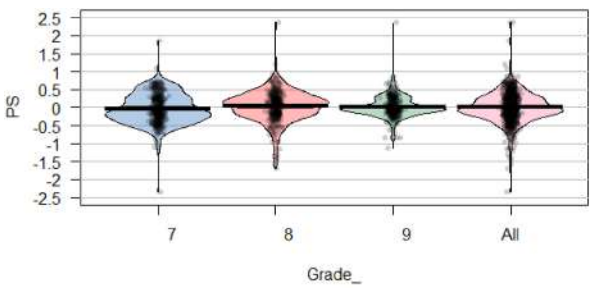
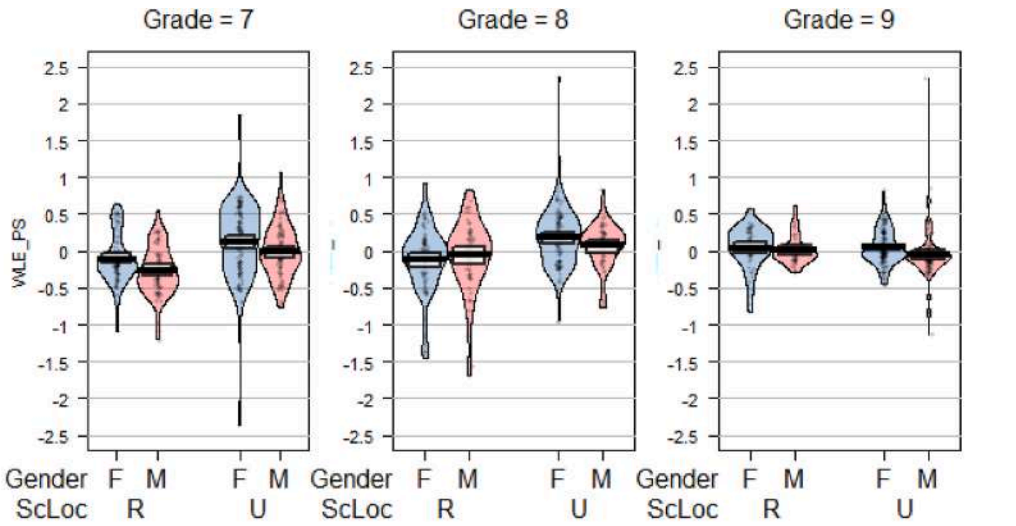


Fig. 1. Differences in students' mathematical problem-solving skills across grades.
Note: PS: Problem-Solving Skills of Students.

Table 2
One-way ANOVA results of the mathematical problem-solving across grades.

Grades	N	*M*	*SD*	*F*(1,066)	*P*	Significant difference between the sub-samples
7	380	−0.04	0.42	3.01	.046	{7, 9} < {8}
8	331	0.03	0.44			
9	356	0.01	0.29			

Note. Post hoc test: Dunnett's T3. 7, 8, and 9: subsample grade. <: direction of significant difference ($p < 0.05$).



Note: WLE_PS: The students' problem-solving skills in WLE scale; F: Female; M: Male; ScLoc: School location; R: Rural; U: Urban.

(a) Differences in students grade 7 of mathematical problem-solving skills across grades and different demographic backgrounds

(b) Differences in students grade 8 of mathematical problem-solving skills across grades and different demographic backgrounds

(c) Differences in students grade 9 of mathematical problem-solving skills across grades and different demographic backgrounds

Fig. 2. Differences in students' mathematical problem-solving skills across grades and different demographic backgrounds.
(a) Differences in students grade 7 of mathematical problem-solving skills across grades and different demographic backgrounds
(b) Differences in students grade 8 of mathematical problem-solving skills across grades and different demographic backgrounds
(c) Differences in students grade 9 of mathematical problem-solving skills across grades and different demographic backgrounds
Note: WLE_PS: The students' problem-solving skills in WLE scale; F: Female; M: Male; ScLoc: School location; R: Rural; U: Urban.

scores of the female students differed significantly based on school location ($t(613) = -6.09$, $p < 0.001$). Female students in urban schools ($M = 0.18$, $SD = 0.39$) outperformed female students in rural schools ($M = -0.08$, $SD = 0.37$). Similar results were observed for male students with urban students ($M = -0.01$, $SD = 0.35$) outperforming rural students ($M = -0.12$, $SD = 0.39$) by a significant margin ($t(382.764) = -3.25$, $p < 0.01$).

When analyzed by grade, grades 7 and 8 students contributed to the difference based on school location with $t(377.952) = -6.34$, $p < 0.001$ and $t(300.070) = -5.04$, $p < 0.001$, respectively. Urban students in grades 7 and 8 performed significantly better than their rural counterparts did. However, there was no significant difference between rural and urban students in grade 9 ($t(354) = 0.71$, $p = 0.447$).

4.3.2. Gender

Male and female students showed a significant difference in their mathematical problem-solving skills. Overall, female students outperformed male students. The detailed results of the independent sample *t*-test for mathematical problem-solving skills based on gender are presented in Table 6.

The results were analyzed to determine whether the school location contributed to the gender difference. The gender difference was

Table 3
One-way ANOVA results for mathematical problem-solving skills across grades and different demographic backgrounds.

	Grade	*N*	*M*	*SD*	*F*	*P*	Significant difference between the sub-samples
Rural	7	168	−0.18	0.34	*F(426) = 10.10*	*<.001*	{7, 8} < {9}
	8	155	−0.10	0.46			
	9	104	0.03	0.25			
Urban	7	212	0.07	0.44	F(639) = 6.10	.001	{7, 9} < {8}
	8	176	0.14	0.38			
	9	252	0.05	0.30			
Female	7	219	0.03	0.44	F(614) = 0.18	.84	–
	8	204	0.05	0.45			
	9	192	0.05	0.26			
Male	7	161	−0.13	0.36	F(451) = 5.28	.005	{7} < {8, 9}
	8	127	0.001	0.42			
	9	164	−0.03	0.32			

Note. Post hoc test: Dunnett's T3. 7, 8, and 9: subsample grade. <: direction of significant difference ($p < 0.05$).

most significant among urban students ($t(596.796) = -4.36$, $p < 0.001$). Female students from urban schools ($M = 0.12$, $SD = 0.39$) outperformed male students from urban schools ($M = -0.01$, $SD = 0.35$). There was no significant difference between female and male students from rural schools ($t(425) = -1.31$, $p = 0.191$).

Grades 7 and 9 students contributed to the gender difference with $t(372.996) = -3.90$, $p < 0.001$ and $t(354) = -2.73$, $p < 0.01$, respectively. Female students in grades 7 and 9 outperformed their male counterparts. However, there was no significant gender difference among grade 8 students ($t(329) = -0.10$, $p = 0.323$).

5. Discussion

The mathematical problem-solving skills of the students were categorized as average. In addition, the difficulties of students increased in line with the problem-solving phase. Fewer students failed the exploring and understanding phase than the subsequent phases. This confirms the results of previous studies indicating that more students failed further along the problem-solving process [25,26]. Because the problem-solving process is sequential, students who have difficulty understanding a problem will fail the subsequent phases [27].

The development of mathematical problem-solving skills was evaluated according to the mean WLE score. The mathematical problem-solving skills of the students developed from grade 7 to grade 8 based on the increase in their mean scores. However, the development dropped in grade 9. This agrees with previous results that concluded that higher grades had the highest problem-solving skills, but the fastest skill development took place in grades 7–8 after which it dropped [3,4]. These results indicate that the mathematical problem-solving skills of the students should improve and be strengthened in grades 7–8, which will help them perform better in grade 9.

In this study, the effects of the demographic background of the students were analyzed in detail, which is an aspect missing from previous studies. The results showed that the mathematical problem-solving skills of urban students increased from grade 7 to grade 8 but decreased in grade 9. The same pattern was found among male and female students. However, a different pattern was observed for rural students, where the skills of grade 9 students continued to increase. The different patterns may be attributed to a structural reorganization of cognitive processes at a particular age [3]. However, more research is needed on the effects of the demographic backgrounds of students on mathematical problem-solving skills. These results were different from previous results because the previous studies only analyzed the development in general, without focusing on their demographic background. Hence, different patterns of development were observed when it was thoroughly examined.

Because solving problems is a cognitive process, the development of problem-solving skills for particular phases and processes needed to be analyzed. The students showed the same pattern for knowledge acquisition (i.e., exploring and understanding, and representing and formulating phases), with an increase in skill from grade 7 to grade 8 but a decrease in grade 9. However, the students showed increasing skill in knowledge application (i.e., planning and executing, as well as monitoring and reflecting phases) across grades. This means that the difference between the mean scores in grade 9 was not significant across phases. Grade 9 students had lower scores than grade 8 students for the knowledge acquisition phase but higher scores for the knowledge application phase. In contrast, the gap between the mean scores of grades 7 and 8 was large across phases.

These results proved that there is a significant difference in the mathematical problem-solving skills of students based on their demographic backgrounds. The urban students outperformed rural students, which confirms the results of previous studies [9,10,15]. The difference can be attributed to the availability of facilities, teacher quality, and interactive teaching and learning instruction [6]. In Indonesia, the policies for the public educational system for middle schools are set at the municipal level. This means that each city has its own policies for teacher training, teacher recruitment, teaching and learning processes, facilities, etc. Urban schools mostly have stricter policies as well as various programs to help students improve their knowledge and skills. In addition, they have supportive facilities for teaching and learning. This unequal environment is the strongest reason for the difference in mathematical problem-solving skills.

The results were analyzed in detail to observe which groups in the rural and urban schools contributed to the difference. Both male and female students in urban schools performed better than their counterparts in rural schools did. In addition, urban students in

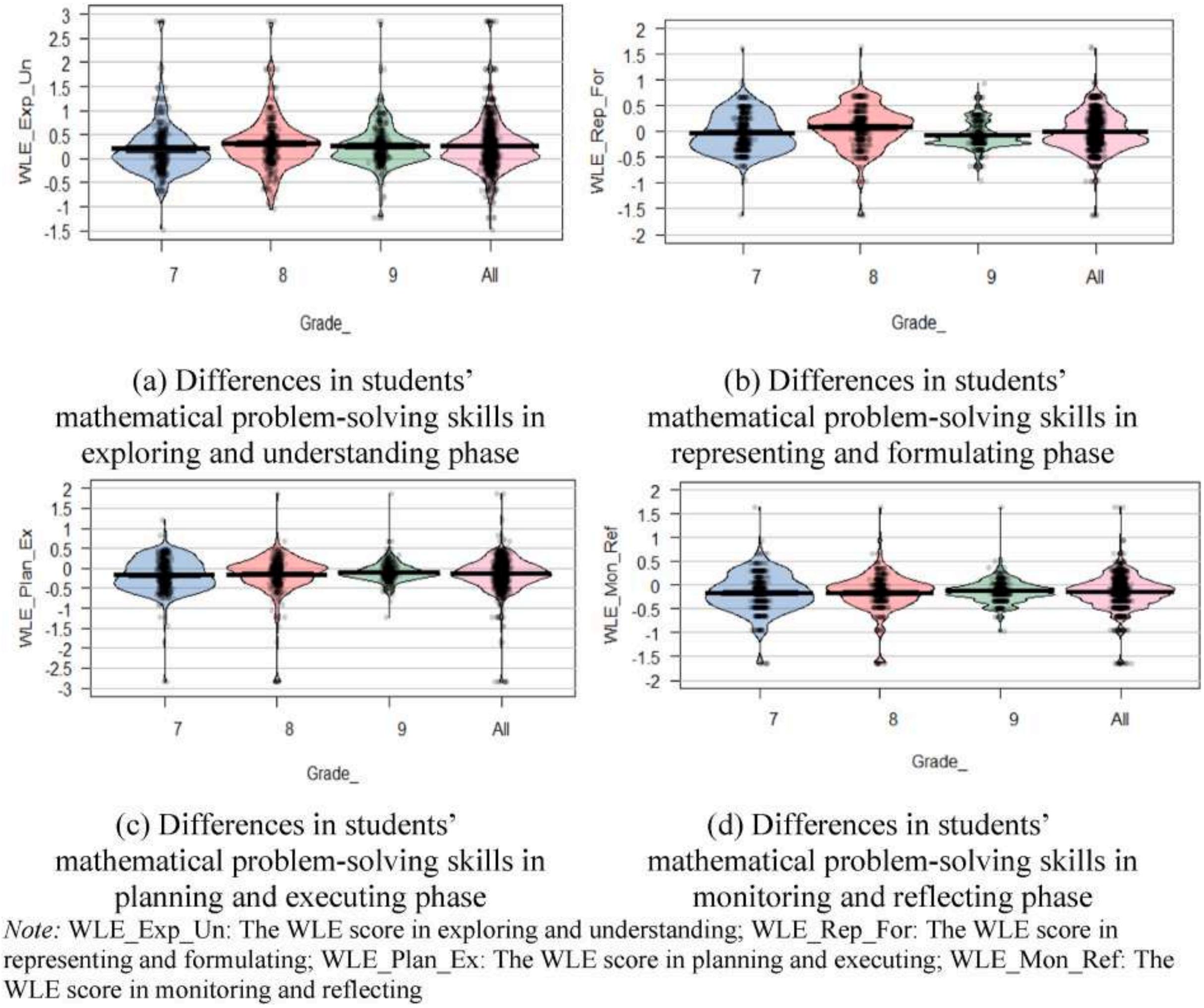


Fig. 3. Differences in students' mathematical problem-solving skills in every phase across grades: (1) Exploring & understanding, (2) Representing & formulating, (3) Planning & executing, (4) Monitoring & reflecting.
(a) Differences in students' mathematical problem-solving skills in exploring and understanding phase
(b) Differences in students' mathematical problem-solving skills in representing and formulating phase
(c) Differences in students' mathematical problem-solving skills in planning and executing phase
(d) Differences in students' mathematical problem-solving skills in monitoring and reflecting phase
Note: WLE_Exp_Un: The WLE score in exploring and understanding; WLE_Rep_For: The WLE score in representing and formulating; WLE_Plan_Ex: The WLE score in planning and executing; WLE_Mon_Ref: The WLE score in monitoring and reflecting.

grades 7 and 8 outperformed their rural counterparts. There was no significant difference between urban and rural students in grade 9. This may be because grade 9 is the last grade in middle school, so students have to prepare for high school entrance requirements, including exam and/or grade point average scores. Hence, both rural and urban schools focus much effort on the teaching and learning process in this grade.

In this study, the female students surprisingly had better mathematical problem-solving skills than the male students did. This confirmed the results of the meta-analysis by Hyde et al. [32] and study by Anjum [36], which found that female students slightly outperformed male students in mathematics. This difference may be because of motivation and attitude [39,40]. Female Indonesian students are typically more diligent, thorough, responsible, persistent, and serious with their tasks.

A detailed analysis was performed to evaluate which group of students contributed to the gender differences. The results showed that female students outperformed male students in urban schools. This may be because male students at urban schools typically display an unserious attitude toward low-stake tests. In addition, female students outperformed their male counterparts in grades 7 and 9. The reason for this difference requires further analysis.

Table 4
One-way ANOVA results for every phase of problem-solving across grades.

Phases	Grade	*M*	*SD*	*F*(1,066)	*P*	
Exploring & Understanding	7	0.19	0.54	3.98	.019	{7, 9} < {8}
	8	0.30	0.54			
	9	0.25	0.44			
Representing & Formulating	7	−0.03	0.37	16.20	<.001	{7, 9} < {8}
	8	0.07	0.42			
	9	−0.08	0.26			
Planning & Executing	7	−0.18	0.42	3.20	.041	{9} < {7, 8}
	8	−0.17	0.51			
	9	−0.11	0.24			
Monitoring & Reflecting	7	−0.17	0.41	1.48	.228	–
	8	−0.17	0.41			
	9	−0.13	0.23			

Note. Post hoc test: Dunnett's T3. 7, 8, and 9: subsample grade. <: direction of significant difference ($p < 0.05$).

Table 5
T-test results for mathematical problem-solving skills based on school location.

Location	*N*	*M*	*SD*	*F*	*P*	*t*(1065)	*P*
Rural	427	−0.10	0.38	0.18	.674	−6.90	<.001
Urban	640	0.07	0.38				

Table 6
T-test results for mathematical problem-solving skills based on gender.

Gender	*N*	*M*	*SD*	*F*	*P*	*t*(1006,013)	*P*
Female	615	0.04	0.39	5.20	.023	−4.312	<.001
Male	452	−0.06	0.38				

6. Conclusion

Studying the problem-solving skills of students is crucial to facilitating their development. In this study, the conclusions are presented as follows:

- The mathematical problem-solving skills of the students were categorized as average. More students failed at higher phases of the problem-solving process.
- Students showed development of their mathematical problem-solving skills from grade 7 to grade 8 but a decline in grade 9. The same pattern was detected across grades for urban students, both female and male. However, the problem-solving skills of rural students increased with the grade.
- A similar development was observed for the individual problem-solving phases. In the knowledge acquisition phase, the problem-solving skills of the students developed from grade 7 to grade 8 but decreased in grade 9. However, problem-solving skills increased across grades in the knowledge application phase.
- The school location was shown to have a significant effect on the mathematical problem-solving skills of the students. Urban students generally outperform students in rural schools. However, gender and grade contributed to differences in mathematical problem-solving skills based on school location. Female and male urban students in grades 7 and 8 outperformed their rural counterparts.
- In general, female students outperformed male students in mathematical problem-solving skills, particularly those from urban schools and in grades 7 and 9.

The sampling method and the number of demographic backgrounds limited the scope of this study. Only students from A-accreditation schools were selected because higher-order problem-solving skills were considered assets. Moreover, the study only included three demographic factors: grade, gender, and school location. More demographic information, such as school type, can be added (public or private schools). Hence, future studies will need to broaden the sample size and consider more demographic factors. Despite these limitations, this study can help teachers determine the best period for enhancing the development of mathematical problem-solving skills. Moreover, the differences in mathematical problem-solving skills due to demographic background can be used as a basis for educational policymakers and teachers to provide equal opportunity and equitable education to students.

Author contribution statement

Ijtihadi Kamilia Amalina: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Tibor Vidákovich: Conceived and designed the experiments; Contributed reagents, materials, analysis tools or data.

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Data availability statement

Data will be made available on request.

Additional information

No additional information is available for this paper.

Declaration of competing interest

No potential conflict of interest was reported by the authors.

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