2+1+1 GENERAL RELATIVISTIC HAMILTONIAN DYNAMICS AND GAUGE FIXING IN HORNDESKI GRAVITY

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Abstract. A novel 2+1+1 decomposition of space-time based on a nonorthogonal double foliation is worked out and applied for the Hamiltonian description of general relativity, recovering earlier results in the proper limit. The complexity of the formalism allows for an unambiguous gauge-fixing of spherically symmetric, static black hole perturbations in the effective field theory approach of scalar-tensor gravitational theories. This gauge choice is also the closest to the general relativistic Regge-Wheeler gauge.

Key words: Gravitation - Hamiltonian formalism - Black hole perturbation.

1. INTRODUCTION

The $3+1$ Arnowitt-Deser-Misner space + time decomposition has been widely exploited in the Hamiltonian treatment of general relativity (Arnowitt *et al.*, 1962), in dealing with the Cauchy-problem and in numerical evolutions (Lehner, 2001). It is also useful when considering cosmological perturbations (Gleyzes *et al.*, 2013, 2015), as cosmological symmetries single out comoving time as a preferred coordinate, also constant comoving time spatial hypersurfaces. Phase transitions along these hypersurfaces can be described by suitable junction conditions (Lanczos, 1922; Sen, 1924; Darmois, 1924; Israel, 1966) in terms of the induced metric and extrinsic curvature (Padilla and Sivanesan, 2012; Nishi *et al.*, 2014).

In generalisations of the second Randall-Sundrum brane-world scenario (Randall and Sundrum, 1999) another, temporal hypersurface (the brane) emerges as our observable universe embedded in a five-dimensional bulk. The existence of the very structure in our universe, all of its energy-momentum content in fact is consequence of similar junction conditions imposed on the temporal hypersurface, more precisely is due to a discontinuity of the extrinsic curvature of the brane (Maartens and Koyama, 2010; Shiromizu *et al.*, 2000; Gergely, 2003, 2008).

Combining the brane approach with the initial-value problem led to a $2+1+1$

Romanian Astron. J. , Vol. **30**, No. 1, p. 45–54, Bucharest, 2020

decomposition of space-time. The original version of this formalism (Gergely and Kovács, 2005; Kovács and Gergely, 2008) assumed orthogonal double foliations and has been worked out as an $s+1+1$ decomposition, hence it can be equally applied in general relativistic context. This is desirable, whenever the initial-value problem is discussed in space-times with a preferred family of surfaces, e.g. provided by spherical or cylindrical symmetry. Employing various $2+1+1$ decompositions, mostly in terms of kinematical quantities has a long history and has been applied for discussing perturbations of space-times with various symmetries (Clarkson and Barret, 2003; Clarkson, 2007).

The orthogonal double foliation formalism (Gergely and Kovács, 2005; Kovács and Gergely, 2008) is simpler, but proved its limits in the context of black hole perturbations (Kase *et al.*, 2014) in scalar-tensor gravitational theories. Such theories include both second order Horndeski theories (Horndeski, 1974; Deffayet *et al.*, 2011) and its beyond-Horndeski generalisations in which only the degrees of freedom propagate driven by second-order evolutions (Gleyzes *et al.*, 2013, 2015). Perpendicularity consumed one important gauge degree of freedom, leaving the even sector of the perturbations of spherically symmetric, static black holes ambiguous. Hence, only the odd sector of perturbations has been tackled in this formalism (Kase *et al.*, 2014).

In order to remedy this situation, we developed a new $2+1+1$ decomposition of the space-time $\mathcal{B}$, allowing for nonorthogonal double foliation (Gergely *et al.*, 2019), succintly presented in Section 2.

We applied this formalism for generalising for nonorthogonal double foliation the discussion i) of the orthogonally decomposed $2+1+1$ Hamiltonian evolution (Kovács and Gergely, 2008) in Section 3 and ii) of the gauge fixing for black hole perturbations in scalar-tensor gravitational theories in Section 4. In the final section we present our conclusions.

2. THE NONORTHOGONAL DOUBLE FOLIATION

Let a spatial hypersurface $\mathcal{S}_t$ being characterized by constant t, having the (time-like) normal n^a and a temporal hypersurface $\mathfrak{M}_\chi$ characterized by constant χ and (space-like) normal l^a (see Fig. 1). The intersection of the hypersurfaces generates the spatial surface $\Sigma_{t\chi}$. The tangent space of its codimension 2 space-time is spanned by any of the orthonormal bases (n^a, m^a) and (k^a, l^a).

The metric can also be $2+1+1$ decomposed in two equivalent fashions:

$$\tilde{g}_{ab} = -n_a n_b + m_a m_b + g_{ab} \,, \tag{1}$$

$$\tilde{g}_{ab} = -k_a k_b + l_a l_b + g_{ab} \,. \tag{2}$$

Here $\tilde{g}_{ab}$ and g_{ab} are the metrics on $\mathcal{B}$ and induced on $\Sigma_{t\chi}$, respectively.

The evolutions along these coordinate lines are decomposed in both bases as

$$\left(\frac{\partial}{\partial t}\right)^a = Nn^a + N^a + \mathcal{N}m^a = \frac{N}{\mathfrak{c}}k^a + N^a \ , \tag{3}$$

$$\left(\frac{\partial}{\partial \chi}\right)^a = Mm^a + M^a = M\left(-\mathfrak{s}k^a + \mathfrak{c}l^a\right) + M^a \ . \tag{4}$$

The coefficients N, M are the lapse functions of these evolutions, the 2-vectors N^a, M^a the shifts along $\Sigma_{t\chi}$, while $\mathcal{N}$ is the third component of the shift vector of $(\partial/\partial t)^a$ in the basis (n^a, m^a). Note that the evolutions proceed along $\mathfrak{M}_\chi$ and $\mathcal{S}_t$, respectively. This is why there is no third component of the shift vector of $(\partial/\partial\chi)^a$ in the basis (n^a, m^a), also there is no third component of the shift vector of $(\partial/\partial t)^a$ in the basis (k^a, l^a). The two bases are Lorentz-rotated with angle $\psi = \tanh^{-1}(\mathcal{N}/N)$ and $\mathfrak{s} = \sinh\psi, \ \mathfrak{c} = \cosh\psi$.

Nonorthogonality appears in the third component $\mathcal{N}$ of the shift vector, also generating the angle ψ of the Lorentzian rotation between the two bases adapted to the two sets of hypersurfaces. In the orthogonal limit $\mathcal{N}$ vanishes and the two bases coincide. The vorticity of the basis vectors k^a and m^a is also generated by $\mathcal{N}$, disappearing with it. Being hypersurface-orthogonal, the basis vector n^a and l^a are vorticity-free.

In the generic case the covariant derivatives of the basis vectors can be $2+1+1$ decomposed in their own basis each as follows:

$$\tilde{\nabla}_a n_b = K_{ab} + 2m_{(a}\mathcal{K}_{b)} + m_a m_b \mathcal{K} + n_a m_b \mathcal{L}^* - n_a \mathfrak{a}_b \ , \tag{5}$$

$$\tilde{\nabla}_a l_b = L_{ab} + 2k_{(a}\mathcal{L}_{b)} + k_a k_b \mathcal{L} + l_a k_b \mathcal{K}^* + l_a \mathfrak{b}_b \ , \tag{6}$$

$$\tilde{\nabla}_a k_b = K^*_{ab} + l_a \mathcal{K}^*_b + l_b \mathcal{L}_a + l_a l_b \mathcal{K}^* + k_a l_b \mathcal{L} - k_a \mathfrak{a}^*_b \ , \tag{7}$$

$$\tilde{\nabla}_a m_b = L^*_{ab} + n_a \mathcal{L}^*_b + n_b \mathcal{K}_a + n_a n_b \mathcal{L}^* + m_a n_b \mathcal{K} + m_a \mathfrak{b}^*_b \ . \tag{8}$$

The quantities K_{ab}, L_{ab}, K^*_{ab} and L^*_{ab} represent the extrinsic curvatures (second fundamental forms) of $\Sigma_{t\chi}$ for the respective orthonormal basis vectors. The one-forms $\mathcal{K}_b$, $\mathcal{L}_b$ and the scalars $\mathcal{K}$, $\mathcal{L}$ represent normal fundamental forms and normal fundamental scalars for the hypersurface normals, respectively, while $\mathcal{K}^*_b$, $\mathcal{L}^*_b$, $\mathcal{K}^*$ and $\mathcal{L}^*$ are similarly defined quantities for the basis vectors with vorticities. The 2-dimensional nongravitational accelerations $\mathfrak{a}_b$, $\mathfrak{a}^*_b$ of the time-like vectors are complemented by similar quantities $\mathfrak{b}_b$, $\mathfrak{b}^*_b$ for the space-like basis vectors.

Note the symmetry of the first two expressions (5)-(6) and its lack in the last two decompositions (7)-(8), a feature related to the vorticity of the respective basis vectors. The definition of all these embedding variables, as arising from the respective projections of Eqs. (5)-(6) are given in Tables 2 and 2. These quantities are not independent (Gergely *et al.*, 2019), their complicated interrelations being summarized in Table 2.

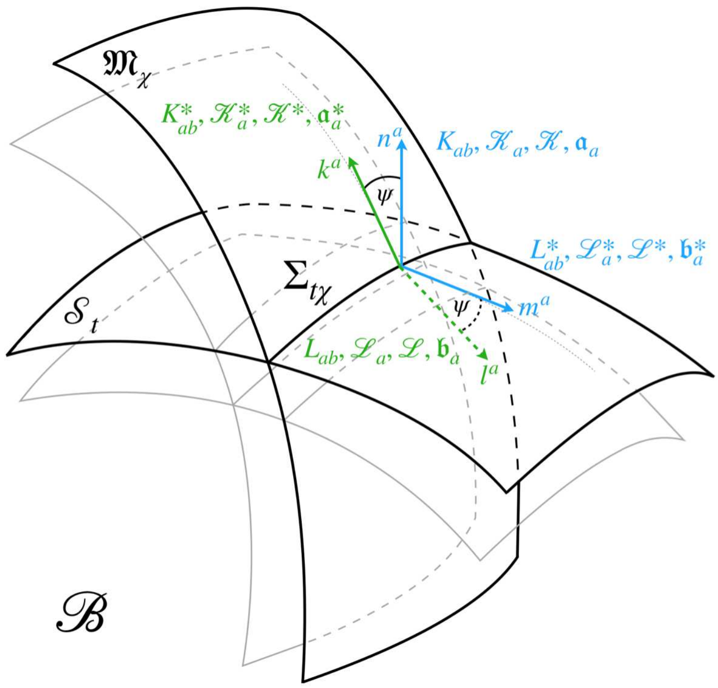

Fig. 1 – The $2+1+1$ decomposition of space-time represented by two nonorthogonal foliations, the two sets of bases related by a Lorentzian rotation and the embedding variables for each basis vector.

Table 1

The embedding variables for the normal basis vectors. The vanishing of their 3-dimensional vorticities $\hat{\omega}^{(n)}_{ab}$ and $\breve{\omega}^{(l)}_{ab}$ is also emphasized

$K_{ab} = g^c_a g^d_b \tilde{\nabla}_c n_d$	$L_{ab} = g^c_a g^d_b \tilde{\nabla}_c l_d$
$\mathcal{K}_a = g^c_a m^d \tilde{\nabla}_c n_d$	$\mathcal{L}_a = -g^c_a k^d \tilde{\nabla}_c l_d$
$\mathcal{K} = m^d m^c \tilde{\nabla}_c n_d$	$\mathcal{L} = k^d k^c \tilde{\nabla}_c l_d$
$\mathfrak{a}_a = g^d_a n^c \tilde{\nabla}_c n_d$	$\mathfrak{b}_a = g^d_a l^c \tilde{\nabla}_c l_d$
$\hat{\omega}^{(n)}_{ab} = 0$	$\breve{\omega}^{(l)}_{ab} = 0$

Table 2

The embedding variables for the complementary basis vectors

$L^*_{ab} = g^c_a g^d_b \tilde{\nabla}_c m_d$	$K^*_{ab} = g^c_a g^d_b \tilde{\nabla}_c k_d$
$\mathcal{L}^*_a = -g^d_a n^c \tilde{\nabla}_c m_d$	$\mathcal{K}^*_a = g^d_a l^c \tilde{\nabla}_c k_d$
$\mathcal{L}^* = n^d n^c \tilde{\nabla}_c m_d$	$\mathcal{K}^* = l^d l^c \tilde{\nabla}_c k_d$
$\mathfrak{b}^*_a = g^d_a m^c \tilde{\nabla}_c m_d$	$\mathfrak{a}^*_a = g^d_a k^c \tilde{\nabla}_c k_d$

Table 3

The embedding variables for the complementary basis vectors m^a and k^a expressed as functions of the embedding variables of the normals. The nonvanishing components of the 3-dimensional vorticities $\hat{\omega}_{ab}^{(k)}$ and $\breve{\omega}_{ab}^{(m)}$ are also given.

$K_{ab}^{*}=\frac{1}{\mathfrak{c}}\left(K_{ab}+\mathfrak{s}L_{ab}\right)$	$L_{ab}^{*}=\frac{1}{\mathfrak{c}}\left(L_{ab}-\mathfrak{s}K_{ab}\right)$
$\mathcal{K}_{a}^{*}=\mathcal{K}_{a}+\frac{\mathfrak{s}}{\mathfrak{c}}\left(\mathfrak{a}_{a}+\mathfrak{b}_{a}\right)$	$\mathcal{L}_{a}^{*}=\mathcal{L}_{a}+\frac{\mathfrak{s}}{\mathfrak{c}}\left(\mathfrak{a}_{a}+\mathfrak{b}_{a}\right)=\mathcal{K}_{a}^{*}+D_{a}\psi$
$\mathcal{K}^{*}=\frac{1}{\mathfrak{c}}\left(\mathcal{K}-\mathfrak{s}\mathcal{L}\right)+\frac{1}{\mathfrak{c}^{2}}\left(l^{a}-\mathfrak{s}n^{a}\right)\tilde{\nabla}_{a}\psi$	$\mathcal{L}^{*}=\frac{1}{\mathfrak{c}}\left(\mathfrak{s}\mathcal{K}+\mathcal{L}\right)+\frac{1}{\mathfrak{c}^{2}}\left(\mathfrak{s}l^{a}+n^{a}\right)\tilde{\nabla}_{a}\psi$
$\mathfrak{a}_{a}^{*}=\mathfrak{a}_{a}+\frac{\mathfrak{s}}{\mathfrak{c}}\left(\mathcal{K}_{a}-\mathcal{L}_{a}\right)=\mathfrak{a}_{a}-\frac{\mathfrak{s}}{\mathfrak{c}}D_{a}\psi$	$\mathfrak{b}_{a}^{*}=\mathfrak{b}_{a}+\frac{\mathfrak{s}}{\mathfrak{c}}\left(\mathcal{L}_{a}-\mathcal{K}_{a}\right)=\mathfrak{b}_{a}+\frac{\mathfrak{s}}{\mathfrak{c}}D_{a}\psi$
$\hat{\omega}_{ab}^{(k)}g_{c}^{b}=0$	$\breve{\omega}_{ab}^{(m)}g_{c}^{b}=0$
$\hat{\omega}_{ab}^{(k)}l^{b}=\frac{1}{2}D_{a}\psi-\frac{\mathfrak{s}}{2\mathfrak{c}}\left(\mathfrak{a}_{a}+\mathfrak{b}_{a}\right)$	$\breve{\omega}_{ab}^{(m)}n^{b}=\frac{1}{2}D_{a}\psi+\frac{\mathfrak{s}}{2\mathfrak{c}}\left(\mathfrak{a}_{a}+\mathfrak{b}_{a}\right)$

3. HAMILTONIAN EVOLUTION

The embedding variables can be expressed in terms of coordinate derivatives of the metric components, either directly from their definition or by exploring identities involving the algebra of the basis vectors (Gergely *et al.*, 2019). As the set of the embedding variables pertinent to the basis (n^a, m^a) contains fewer elements containing time derivatives than the other set, in particular L_{ab}^* and $\mathcal{L}^*$ are expressible with spatial derivatives alone, it is more convenient to employ them in a dynamical analysis. The respective $2+1+1$ decomposition of the curvature scalar and metric determinant allows for the decomposition (Gergely *et al.*, 2018) of the Einstein-Hilbert action

$$\begin{aligned} S_{EH} &= \int dt \int d\chi \int_{\Sigma_{t\chi}} d^2x \mathcal{L}^{EH} \,, \\ \mathcal{L}^{EH} &= \sqrt{-\tilde{g}}\tilde{R} \,. \end{aligned} \tag{9}$$

As expected, the lapse and shift components $\{N, N^a, \mathcal{N}\}$ turn out to be nondynamical, while the momenta (Kovács and Gergely, 2008)

$$\pi^{ab} = \frac{\partial \mathcal{L}^{EH}}{\partial \dot{g}^{ab}} = \sqrt{g}M\left[K^{ab} - g^{ab}\left(K + \mathcal{K}\right)\right] \,, \tag{10}$$

$$p_a = \frac{\partial \mathcal{L}^{EH}}{\partial \dot{M}^a} = 2\sqrt{g}\mathcal{K}_a \,, \tag{11}$$

$$p = \frac{\partial \mathcal{L}^{EH}}{\partial \dot{M}} = -2\sqrt{g}K \tag{12}$$

can be employed in order to carry out the Legendre transformation, resulting in the Liuville form, boundary terms and the gravitational Hamiltonian density

$$\mathcal{H}^G = N\mathcal{H}_{\perp}^G + N^a\mathcal{H}_a^G + \mathcal{N}\mathcal{H}_{\mathcal{N}}^G \,, \tag{13}$$

with the Hamiltonian constraint $\mathcal{H}_{\perp}^G$ and diffeomorphism constraints $\mathcal{H}_a^G$, $\mathcal{H}_{\mathcal{N}}^G$ expressed solely in terms of the canonical pairs $\left\{\left(g_{ab}, \pi^{ab}\right), \left(M^a, p_a\right), \left(M, p\right)\right\}$, as given by Eqs. (61)-(63) of Ref. Gergely *et al.* (2019).

Denoting the canonical coordinates by $g^A \equiv \{g_{ab}, M^a, M\}$ and canonical momenta by $\pi_A \equiv \left\{\pi^{ab}, p_a, p\right\}$, also by $y = \left\{y^1, y^2\right\}$ the coordinates adapted to $\Sigma_{t\chi}$, the Poisson bracket of any two arbitrary functions $f(\chi,y) \equiv f\left(\chi,y;g^A(\chi,y),\pi_B(\chi,y)\right)$ and $h(\chi,y) \equiv h\left(\chi,y;g^A(\chi,y),\pi_B(\chi,y)\right)$ is defined as:

$$\begin{aligned}\left\{f(\chi,y), h\left(\chi',y'\right)\right\} = \int d\chi'' \int dy'' \left(\frac{\delta f(\chi,y)}{\delta g^C(\chi'',y'')} \frac{\delta h(\chi',y')}{\delta \pi_C(\chi'',y'')}\right. \\ \left. - \frac{\delta f(\chi,y)}{\delta \pi_C(\chi'',y'')} \frac{\delta h(\chi',y')}{\delta g^C(\chi'',y'')}\right) . \qquad (14)\end{aligned}$$

The canonical pairs obey

$$\begin{aligned}\left\{g^A(\chi,y), g^B\left(\chi',y'\right)\right\} &= 0 , \\ \left\{\pi_A(\chi,y), \pi_B\left(\chi',y'\right)\right\} &= 0 , \\ \left\{g^A(\chi,y), \pi_B\left(\chi',y'\right)\right\} &= \delta^A_B \delta\left(\chi-\chi'\right)\delta\left(y-y'\right) . \qquad (15)\end{aligned}$$

In order to derive the Hamiltonian equations of motion for the canonical variables we introduce the smeared Hamiltonian density

$$\begin{aligned}\mathcal{H}^G[N,N^a,\mathcal{N}] &= \mathcal{H}^G_\perp[N] + \mathcal{H}^G_a[N^a] + \mathcal{H}^G_\mathcal{N}[\mathcal{N}] , \\ \mathcal{H}^G_\perp[N] &= \int d\chi \int dy N(\chi,y)\mathcal{H}^G_\perp(\chi,y) , \\ \mathcal{H}^G_a[N^a] &= \int d\chi \int dy N^a(\chi,y)\mathcal{H}^G_a(\chi,y) , \\ \mathcal{H}^G_\mathcal{N}[\mathcal{N}] &= \int d\chi \int dy \mathcal{N}(\chi,y)\mathcal{H}^G_\mathcal{N}(\chi,y) . \qquad (16)\end{aligned}$$

Then each canonical variable evolves as given by the Poisson bracket of the canonically conjugate variable with the smeared Hamiltonian density

$$\begin{aligned}\dot{g}^A &\equiv \left\{g^A(\chi,y), \mathcal{H}^G\right\} = \frac{\delta \mathcal{H}^G[N,N^a,\mathcal{N}]}{\delta \pi_A(\chi,y)} , \\ \dot{\pi}_A &\equiv \left\{\pi_A(\chi,y), \mathcal{H}^G\right\} = -\frac{\delta \mathcal{H}^G[N,N^a,\mathcal{N}]}{\delta g^A(\chi,y)} . \qquad (17)\end{aligned}$$

Their detailed form was presented in Eqs. (65) and (69)-(71) of Ref. Gergely *et al.* (2019). Previously derived results (Kovács and Gergely, 2008) reemerge in the orthogonal double foliation limit.

4. GAUGE FIXING IN SCALAR-TENSOR THEORIES

In general relativity a convenient gauge fixing of spherically symmetric, static black hole perturbations is achieved through the Regge-Wheeler gauge (Regge and Wheeler, 1957). The even and odd perturbations conveniently decouple. In scalar-tensor theories this feature is conserved. Spherically symetric, static black hole perturbations in Horndeski theories were discussed both for the odd (Kobayashi *et al.*, 2012) and even sectors (Kobayashi *et al.*, 2014).

In an effective field theory approach, which includes both second order Horndeski theories (Horndeski, 1974; Deffayet *et al.*, 2011) and its beyond-Horndeski generalisations in which only the degrees of freedom propagate driven by second-order evolutions (Gleyzes *et al.*, 2013, 2015) a similar analysis has provided the stability analysis of the perturbations of the odd sector (Kase *et al.*, 2014). With the new formalism based on the nonorthogonal double foliation the discussion of the even sector also becomes possible, as follows.

In the Helmholtz-type decompositions of perturbations of the 2-vectorial and 2-tensorial metric variables (Kase *et al.*, 2014):

$$\delta N_a = \bar{D}_a P + E^b{}_a \bar{D}_b Q\,, \tag{18}$$

$$\delta M_a = \bar{D}_a V + E^b{}_a \bar{D}_b W\,, \tag{19}$$

$$\delta g_{ab} = \bar{g}_{ab} A + \bar{D}_a \bar{D}_b B + \frac{1}{2}\left(E^c{}_a \bar{D}_c \bar{D}_b + E^c{}_b \bar{D}_c \bar{D}_a \right) C \tag{20}$$

the odd sector variables are C, Q, W. All the above mentioned approaches, including ours achieve by a suitable gauge fixing $\widehat{C} = 0$, leaving the physical odd degrees of freedom in $\widehat{Q}$, $\widehat{W}$ (an overbar and a wide overhat representing the respective quantity on the background and after the gauge transformation and fixing, respectively).

The even sector includes the above defined P, V, A, B, the perturbations of the scalar metric variables $\delta N, \delta\mathcal{N}, \delta M$ together with the scalar field perturbation. The latter is obviously absent in general relativity, where the Regge-Wheeler gauge gives, exploring the three remaining gauge degrees of freedom $\widehat{B} = \widehat{P} = \widehat{V} = 0$, with the physical even degrees of freedom $\widehat{\delta N}$, $\widehat{\delta\mathcal{N}}$, $\widehat{\delta M}$, $\widehat{A}$.

The apparition of the scalar field and its perturbation restricts the possibilities in the choice of the metric functions. A way to fix the latter is $\widehat{B} = \widehat{P} = \widehat{A} = 0$, leaving $\widehat{\delta N}$, $\widehat{\delta\mathcal{N}}$, $\widehat{\delta M}$, $\widehat{V}$, $\widehat{\delta\phi}$ as the even physical degrees of freedom (Kobayashi *et al.*, 2014).

Alternatively, similarly to the unitary gauge of cosmology, on a spherically symmetric and static background a radial unitary gauge $\widehat{\delta\phi} = 0$ can be chosen, such that the scalar stays unaffected by the perturbation. In this case one metric perturbation can be switched off as $\widehat{B} = 0$, however the use of the orthogonal double foliation formalism demanded to also fix $\widehat{\delta\mathcal{N}} = 0$ by wasting the last gauge degree of freedom

(Kase *et al.*, 2014). This resulted in the metric perturbation $\widehat{P}$ containing an arbitrary time function, hampering the physical interpretation of the even sector perturbations.

Instead, by relaxing the orthogonality of the double foliation we could use the last gauge degree of freedom for imposing $\widehat{P}=0$, leaving the physical even degrees of freedom as $\widehat{\delta N}$, $\widehat{\delta \mathcal{N}}$, $\widehat{\delta M}$, $\widehat{A}$, $\widehat{V}$ (Gergely *et al.*, 2019). The advantages are obvious: 1) the ambiguity is removed, allowing for a physical interpretation of perturbations, and 2) the conformal factor $\widehat{A}$ of the two-dimensional metric is kept among the variables, while the scalar field variation removed. Hence this becomes the gauge choice closest to the general relativistic Regge-Wheeler gauge. The scalar degree of freedom survives in $\widehat{V}$.

With this we have layed the foundations for the stability analysis of the even sector perturbations of spherically symmetric, static black hole in generic scalar-tensor theories in an effective field theory approach, which includes both second order Horndeski theories and its generalisations in which only the degrees of freedom propagate driven by second-order evolutions. For this purpose the action has to be written in terms of the scalars formed from the metric and embedding variables and variation carried out to the second order.

The first order variation give the equations of motion for the background. They can be investigated in order to derive new spherically symmetric, static solutions of the afore-mentioned generic scalar-tensor theories.

Morover, the second order variation of the action generates the dynamics of the perturbations, which can be investigated even without the knowledge of the background solution. Separating into even and odd parts, fixing the gauge as described above and expanding into spherical harmonics yields the required evolutions of the even sector perturbations. These lengthy calculations will be addressed elsewhere.

5. CONCLUSION

We developed a new $2+1+1$ decomposition of space-time based on a double foliation, the leaves of the two sets being nonorthogonal. Nonorthogonality doubled the set of embedding variables, however one set could be selected as more suitable for dynamical analysis, containing a lower number of time derivatives. The main applications of the new formalism up-to-date are A) a generalisation of the Hamiltonian analysis of general relativity developed earlier for orthogonal double foliation, and B) a gauge fixing suitable for dealing with perturbations of spherically symmetric, static black holes in scalar-tensor gravity theories in an effective field theory approach. These have already been worked out in detail (Gergely *et al.*, 2019).

The new formalism opens up the possibility to discuss in a unified way both the even and odd perturbations of spherically symmetric, static black holes in the effective field theory approach of scalar-tensor gravitational theories in the radial unitary gauge, in the closest possible way to the successful Regge-Wheeler gauge of general relativity.

Acknowledgements.
L. Á. G. acknowledges the generous support of the organisers of the Recent Developments in Astronomy, Astrophysics, Space and Planetary Sciences conference. This work was supported by the Hungarian National Research Development and Innovation Office (NKFIH) in the form of the Grant No. 123996 and has been carried out in the framework of COST actions CA15117 (CANTATA), CA16104 (GWverse) and CA18108 (QG-MM) supported by COST (European Cooperation in Science and Technology). C.G. was supported by the UNKP-19-3 New National Excellence Program of the Ministry of Human Capacities of Hungary. Z.K. was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences and by the UNKP-19-4 New National Excellence Program of the Ministry of Human Capacities of Hungary.

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Received on 26 November 2019