Robust Fuzzy Control using the Type2 Distending Function

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***Abstract*—Here, we develop a novel robust type2 fuzzy controller (FC). It is based on a new type-2 membership function called the type2 Distending Function (T2DF). It can model the uncertainties associated with the measurement noise. The design process does not include the implication and the type reduction steps, hence it is computationally efficient. Also, aggregation is carried out using fuzzy arithmetic operations and T2DF is closed under linear combination. Because of these unique features, the proposed type2 FC performs better even in the presence of large measurement noise. The efficiency of the proposed FC is demonstrated using the altitude control of a quadcopter with a noisy feedback signal.**

***Index Terms*—Type2 fuzzy sets, type2 distending function, measurement noise, robust control.**

I. Introduction

Fuzzy theory exploits uncertainty and imprecise information obtained from the real world information in order to derive robust, tractable and low cost solutions. It is achieved by defining a type-1 membership function (MF) which has crisp grade values. However, sometimes it happens that these grade values become uncertain. The main reasons for these uncertainties are: 1) The measuring device is imprecise; 2) Sensor noise adds to the measured signal; 3) The actuators have nonlinear characteristics which are not known; 4) The experts have different opinions about the consequent values. Type2 fuzzy sets are defined to handle such uncertainties [1], and they have numerous proven practical applications [2–5].
The performance of the fuzzy logic system (FLS) depends on the measured data. If noise enters the measurement system then the data values become uncertain. It results in performance degradation or an unstable response by the FLS. A lot of research has been done to minimize the effect of noise and design robust FLSs. It has been shown that type2 fuzzy sets (T2FS) are better at handling the uncertainties introduced by noise than to type-1 fuzzy sets [6–8]. In one key article [9], a new type2 MF was introduced to minimize the uncertainty due to noise. Motivated by this approach, we present a novel type2 MF to model the uncertainty and we design a robust system against measurement noise.

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Designing a control system using a T2FS consists of five steps; 1) Fuzzification; 2) Rules strength calculation; 3) Calculation of the rules output using implication and aggregation; 4) Type-reduction; 5) Defuzzification. Type reduction is carried out using the Karnik Mendel (KM) algorithm [10]. This design process has however several drawbacks:

1) It is computationally expensive due to the implication, aggregation and the type reduction steps.
2) Various types of T2MF are used to model the uncertainty. There is no systematic connection between the choice of T2MF and the type of uncertainty.
3) The number of parameters associated with a T2MF is usually large. Therefore parameter optimization is not an easy task.

In this paper, we will define a new type2 MF called the Distending Function (DF) to handle the uncertainty introduced by measurement noise. Based on this type2 MF, a new technique for designing a type2 fuzzy controller will be presented. The unique features of our approach are:

1) The design technique does not include implication and type reduction steps. Aggregation is performed using fuzzy arithmetic operations. Hence it is computationally inexpensive.
2) The DF has a few parameters which can handle different types of uncertainties.
3) All the parameters of the DF except one (associated with the uncertainty) are kept fixed. This makes the optimization process easier and faster.

Because of the above advantages, the proposed technique implements a robust type2 fuzzy controller which can handle the uncertainties associated with measurement noise.
The rest of the paper is organized as follows. In Section II, we present the type2 DF and uncertainty representation. In Section III, we outline the aggregation of the type2 DF. In Section IV, we describe the proposed fuzzy controller. In Section V, we present the benchmark system, simulation results and a discussion. Lastly, in Section V, we draw some conclusions and make a suggestion for future research.

II. Uncertainty Representations using the Type2 Distending Function (T2DF)

The Distending Function (DF) is based on the Dombi operator and it belongs to the parametric family of membership functions [11]. The RHS and LHS of the DF (shown in Fig. 1) are given by

$$\Delta_R(x-c) = \frac{1}{1 + \frac{1-\nu}{\nu}\left|\frac{x-c}{\varepsilon}\right|^{\lambda} \frac{1}{1+e^{(-\lambda^{*}(x-c))}}}, \tag{1}$$

$$\Delta_L(x-c) = \frac{1}{1 + \frac{1-\nu}{\nu}\left|\frac{x-c}{\varepsilon}\right|^{\lambda} \frac{1}{1+e^{(\lambda^{*}(x-c))}}}. \tag{2}$$

These two sides can be combined using the Dombi conjunctive operator to form a symmetric DF (shown in Fig. 2). The symmetric DF is defined as

$$\Delta^{(\lambda)}_{\varepsilon,\nu}(x-c) = \frac{1}{1 + \frac{1-\nu}{\nu}\left|\frac{x-c}{\varepsilon}\right|^{\lambda}}. \tag{3}$$

It has four parameters, namely 1) The threshold ($\nu \in (0,1)$); 2) Tolerance ($\varepsilon > 0$); 3) Sharpness ($\lambda \in (1,+\infty)$); 4) Coordinate of the peak value ($c \in \mathbb{R}$). The DF can be translated to the right and left along the x-axis using the c parameter. At $x = c$, the value of DF is 1. The sharpness λ controls the steepness of the DF. The threshold ν and the tolerance ε have logical meanings and they control the width of the DF. If $x = \pm\varepsilon$, then $\Delta(x) = \nu$.
Any type of uncertainty present in the system can be modeled using the DF. Uncertainty may be associated with each of the four parameters of the DF and it results in the generation of different types of T2DF. In our previous study [12], the uncertainty in the c parameter leading to development of the uncertain peak value DF was discussed. Here, we will deal with the measurement noise and it can be modeled as an uncertainty in the boundary values of the DF. It can be realized by adding uncertainty in the ε parameter. It results in the generation of a set of various DFs. Among these DFs, the one with the highest grade value is called the upper membership function (UMF) and the one with the lowest grade values is called the lower membership function (LMF). The LMF, UMF and all the DFs in between can be combined to form a type2 DF (T2DF). Consider the LHS and RHS of the DF, as shown in Fig. 1. The uncertainty associated with the membership grade can be modeled by adding the uncertainty to the ε value. It will generate the LHS and RHS of T2DF Fig. 3. Applying the Dombi conjunctive operator will result in a T2DF with uncertain boundary values, as shown in Fig. 4. Next, we will show that the T2DF is closed under linear combination i.e. a linear combination of T2DFs is also a T2DF. We will use fuzzy arithmetic operations to prove this.

III. Aggregation of T2DFs

Fuzzy quantities can be aggregated using fuzzy arithmetic operations [1, 13]. Let $\Delta_2^1, \Delta_2^2, \ldots, \Delta_2^n$ be n T2DFs with

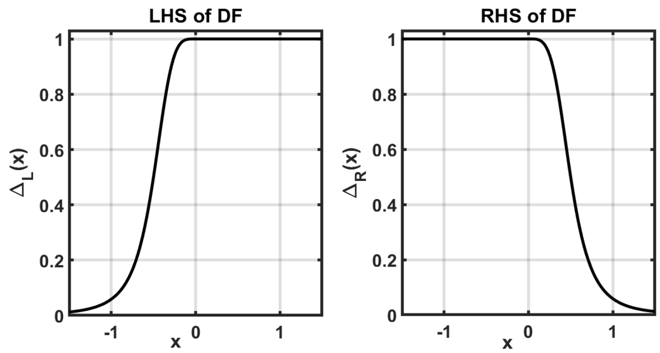


Fig. 1: LHS and RHS of DF

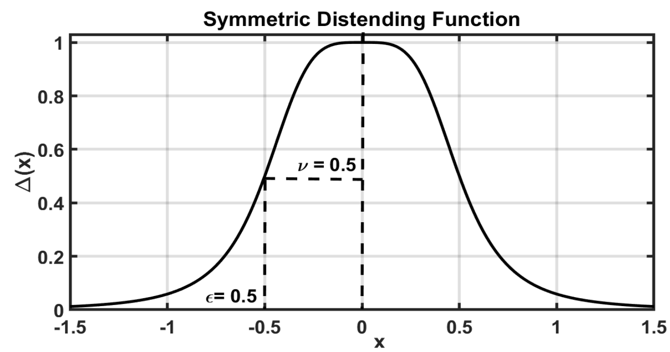


Fig. 2: Distending Function (here c=0)

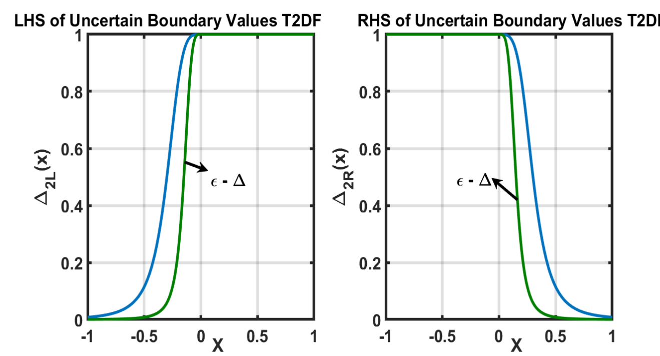


Fig. 3: LHS and RHS of uncertain boundary values T2DF

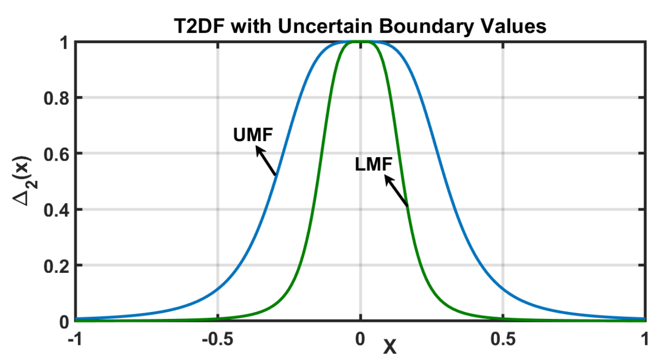


Fig. 4: Uncertain boundary values T2DF

tolerance values $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$, let $\varsigma_1, \varsigma_2, \ldots, \varsigma_n$ be the uncertainties in the tolerance values and $c_1, c_2, \ldots, c_n$ be the peak value coordinates. We define

$$\overline{\varepsilon}_a = \sum_{j=1}^{n} \overline{v}_j \overline{\varepsilon}_j, \quad \underline{\varepsilon}_a = \sum_{j=1}^{n} \underline{v}_j \underline{\varepsilon}_j, \quad \overline{c}_a = \sum_{j=1}^{n} \overline{v}_j \overline{c}_j,$$
$$\underline{c}_a = \sum_{j=1}^{n} \underline{v}_j \underline{c}_j, \quad \underline{\varsigma}_a = \sum_{j=1}^{n} \underline{v}_j \underline{\varsigma}_j, \quad (4)$$

where $\overline{c}_j,\ \underline{c}_j$ are the peak value coordinates, $\overline{\varepsilon}_j,\ \underline{\varepsilon}_j$ are the tolerance values and $\overline{v}_j,\ \underline{v}_j$ are the weights corresponding to the UMF and LMF of the jth T2DF, respectively. These n T2DFs can be aggregated using arithmetic operations and it results in

$$\underline{\Delta}_a(x) = \frac{1}{1 + 2\left(\frac{1-\nu}{\nu}\right)\left|\frac{x-\underline{c}_a}{\underline{\varepsilon}_a - \underline{\varsigma}_a}\right|^{\lambda}}, \quad (5)$$

$$\overline{\Delta}_a(x) = \frac{1}{1 + \frac{1-\nu}{\nu}\left|\frac{x-\overline{c}_a}{\overline{\varepsilon}_a}\right|^{\lambda}}. \quad (6)$$

Here $\underline{\Delta}_a(x)$ and $\overline{\Delta}^a(x)$ are the LMF and the UMF of the aggregated membership function. As $\underline{\Delta}_a(x)$ and $\overline{\Delta}^a(x)$ are DFs, this proves that the T2DF is closed under linear combination (a linear combination of T2DFs is also a T2DF).

IV. Robust Fuzzy Controller Design

The design presented here is based on our previous articles [11, 12]. A multi-input multi-output system (MIMO) having n inputs and m output is described by the following expert rules:

$$\text{If } x_1 \text{ is } F_1^j \text{ and ... and } x_n \text{ is } F_n^j$$
$$\text{then } y_1 \text{ is } O_1^j \text{ ; ... ; } y_m \text{ is } O_m^j\ , \quad (7)$$

where $j = 1, ..., l$ are the number of fuzzy rules and $x_1, x_2, ..., x_n$ are the input linguistic variables which take the values from the input fuzzy sets $F_1, F_2, ..., F_n$. The output linguistic variables $y_1, y_2, ..., y_m$ take the values from the output fuzzy sets $O_1, O_2, ..., O_m$. If the output variables are independent of each other, then each rule given by Eq. (7) can be written as m multi input single output (MISO) rules of the form:

$$\text{If } x_1 \text{ is } F_1^j \text{ and ... and } x_n \text{ is } F_n^j \text{ then } y \text{ is } O^j\ . \quad (8)$$

If any source of uncertainty is present in the system, then it can be handled (modeled and minimized) via the T2DF. The shape of each type2 DF will depend on the magnitude of the uncertainty associated with the tolerance parameter ε of the DF. The effect of this uncertainty on the system output is minimized by defining a proper inference mechanism. This inference mechanism will utilize the T2DFs to generate a crisp output by mapping the input-output space. The part of the fuzzy rule between *If* and *then* is called the antecedent and the remaining part after *then* is called the consequent. We will consider these two parts separately in our design.

1) The antecedent: The antecedent of the jth fuzzy rule is

$$\ell(\Delta_1(x_1)^j, \Delta_2(x_2)^j, \ldots, \Delta_n(x_n)^j) = \hat{v}_j(x). \quad (9)$$

This ℓ represents a fuzzy logical expression comprising of conjunction, disjunction and negation operators. The rule applicability interval $\hat{v}_j(x)$ contains two values. These values corresponds to the lower and upper membership grade values of the T2DF. We can use a general parametric operator [14] to evaluate Eq. (9). It has the form

$$D_\gamma(x) = \frac{1}{1 + \left(\frac{1}{\gamma}\left(\prod_{i=1}^{n}\left(1 + \gamma\left(\frac{1-\Delta(x_i)}{\Delta(x_i)}\right)^{\alpha}\right) - 1\right)\right)^{\frac{1}{\alpha}}}. \quad (10)$$

This operator covers the Hamacher, Dombi, Einstein, product, min/max and the drastic operators. For specific input values $\underline{x}^*$, Eq. (9) can be evaluated and this results in a set of two numeric values $\underline{\hat{v}}_j(\underline{x}^*)$ and $\hat{\overline{v}}_j(\underline{x}^*)$, representing an interval

$$\ell(\underline{\Delta}^j_{1(x_1^*)}, \underline{\Delta}^j_{2(x_2^*)}, \ldots, \underline{\Delta}^j_{n(x_n^*)}) = \underline{\hat{v}}_j(\underline{x}^*),$$
$$\ell(\overline{\Delta}^j_{1(x_1^*)}, \overline{\Delta}^j_{2(x_2^*)}, \ldots, \overline{\Delta}^j_{n(x_n^*)}) = \hat{\overline{v}}_i(\underline{x}^*),$$

where $\hat{\overline{v}}_j(\underline{x}^*)$ and $\underline{\hat{v}}_i(\underline{x}^*)$ are the respective lower strength and upper strength of the jth rule. Also, $\underline{\Delta}_n$ and $\overline{\Delta}_n$ are the LMF and the UMF of the nth T2DF, respectively. The rule strengths are normalized to obtain the firing strengths ($\overline{v}_j(x^*)$, $\underline{v}_j(x^*)$). The upper firing strength (UFS) and lower firing strength (LFS) of the jth rule are given by

$$\overline{v}_j(\underline{x}^*) = \frac{\hat{\overline{v}}_j(\underline{x}^*)}{\sum_{i=1}^{l}\hat{\overline{v}}_j(\underline{x}^*)}, \qquad \text{where} \quad \sum_{j=1}^{l}\overline{v}_i(\underline{x}^*) = 1. \quad (11)$$

$$\underline{v}_j(\underline{x}^*) = \frac{\underline{\hat{v}}_j(\underline{x}^*)}{\sum_{i=1}^{l}\underline{\hat{v}}_j(\underline{x}^*)}, \qquad \text{where} \quad \sum_{j=1}^{l}\underline{v}_i(\underline{x}^*) = 1. \quad (12)$$

2) The consequent: The consequent in each rule is represented using a single T2DF. Multiply the LMF and UMF of this T2DF by LFS and UFS of the rule. This will generate the fuzzy output of the rule (it is a T2DF). Then aggregate the output of all the rules to get the fuzzy output of the rule-based system. This aggregated output ($\Delta_a(x)$) will also be a T2DF. The UMF and the LMF of the $\Delta_a(x)$ are given by

$$\underline{\Delta}_a(x) = \sum_{j=1}^{l}\underline{v}_i(\underline{x}^*)\underline{\Delta}_{jo}(x). \quad (13)$$

$$\overline{\Delta}_a(x) = \sum_{j=1}^{l}\overline{v}_i(\overline{x}^*)\overline{\Delta}_{jo}(x), \quad (14)$$

where $\underline{\Delta}_{jo}(x)$ and $\overline{\Delta}_{jo}(x)$ are the LMF and the UMF of the jth consequent T2DF. The UMF ($\overline{\Delta}_a(x)$) and LMF ($\underline{\Delta}_a(x)$)

Algorithm 1: Robust fuzzy control using T2DF
Step 1: Construct the T2DFs for the outputs and normalized inputs. **Step 2:** Normalize the input data and fuzzify using the T2DFs. **Step 3:** Calculate the UFS and LFS of each rule using Eq. (11) and Eq. (12). **Step 4:** Construct the UMF and LMF of the aggregated output T2DF using Eq. (13) and Eq. (14). **Step 5:** Generate the crisp output control u_c using Eq. (15), Eq. (16) and Eq. (17)

have the form given by Eq. (5) and Eq. (6), respectively. The parameters in these equations can be calculated using

$$\overline{c}_a = \sum_{j=1}^{l} \overline{v}_j(\overline{x}^*)\overline{c}_j, \qquad \overline{\varepsilon}_a = \sum_{j=1}^{l} \overline{v}_i(\overline{x}^*)\overline{\varepsilon}_j, \tag{15}$$

$$\underline{c}_a = \sum_{j=1}^{l} \underline{v}_j(\underline{x}^*)\underline{c}_j, \qquad \underline{\varepsilon}_a = \sum_{j=1}^{l} \underline{v}_j(\underline{x}^*)\underline{\varepsilon}_j. \tag{16}$$

The COG values of $\overline{\Delta}_a(x)$ and $\underline{\Delta}_a(x)$ can be used to generate the crisp output control u_c. Here,

$$u_c = \frac{\overline{c}_a + \underline{c}_a}{2} \tag{17}$$

Algorithm 1 briefly describes the procedure for designing the robust type2 FC.

V. Benchmark System, Simulation Results and Discussion

Now, we will demonstrate the effectiveness of the proposed procedure. The altitude of a quadcopter will be controlled using a robust type2 FC in the presence of noisy measurement data.

A. Mambo quadcopter

The Parrot mini-drone Mambo is usually used in academic research. The Matlab Simulink provides a flight simulation model for this quadcopter [14]. It consists of: 1) A flight controller; 2) A sensor model; 3) An airframe model; 4) An environment model . The sensor model consists of sonar, accelerometers and gyroscopes. The airframe model (shown in Fig. 5) describes the structure of the mini drone. It consists of the translational components (x, y, z), rotational components (ϕ, θ, ψ), motor angular speeds ω, torques τ and upward forces f. The environment model consists of external noise acting on the system. The model of the quadcopter is given by:

$$\dot{X} = f(X, u) + N, \tag{18}$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}; \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \omega_r \end{bmatrix}; \quad N = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}.$$

Here X is the state vector, u is the input and N is the additive white noise that affects the system states. Also,

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \omega_r \end{bmatrix} = \begin{bmatrix} b\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right) \\ b\left(-\omega_2^2 + \omega_4^2\right) \\ b\left(\omega_1^2 - \omega_3^2\right) \\ d\left(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2\right) \\ -\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2 \end{bmatrix}.$$

Here b and d are the thrust and drag coefficients. u_1, u_2, u_3, u_4 controls the altitude, roll, pitch and yaw movements of the quadcopter. ω_r is the residual angular speed and u_1 is the thrust. The state equations of the system are:

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \\ \dot{z} \\ \ddot{z} \\ \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{u_1}{m} \\ \dot{y} \\ (\cos\phi\sin\theta\sin\psi + \sin\phi\cos\psi)\frac{u_1}{m} \\ \dot{z} \\ g - (\cos\phi\cos\theta)\frac{u_1}{m} \\ \dot{\phi} \\ \dot{\theta}\dot{\psi}\frac{I_{yy}-I_{zz}}{I_{xx}} + \dot{\theta}\frac{J_r}{I_{xx}}\Omega_r + \frac{L_a}{I_{xx}}u_2 \\ \dot{\theta} \\ \dot{\phi}\dot{\psi}\frac{I_{zz}-I_{xx}}{I_{yy}} - \dot{\phi}\frac{J_r}{I_{yy}}\Omega_r + \frac{L_a}{I_{yy}}u_3 \\ \dot{\psi} \\ \dot{\theta}\dot{\phi}\frac{I_{xx}-I_{yy}}{I_{zz}} + \frac{1}{I_{zz}}u_4 \end{bmatrix} + \begin{bmatrix} 0 \\ n_1 \\ 0 \\ n_2 \\ 0 \\ n_3 \\ 0 \\ n_4 \\ 0 \\ n_5 \\ 0 \\ n_6 \end{bmatrix}.$$

The altitude z is measured by a sonar device. Here, we design a T2DF-based fuzzy controller to generate appropriate thrust u_1 to control the altitude z of the quadcopter in the presence of external noise n_3.

B. Altitude control in the presence of measurement noise

Now we will consider a situation where the altitude measured by the sonar is corrupted by an additive white measurement noise. The quadcopter is ordered to take off and reach an altitude of 1m, maintain this height for 20 seconds and then increase the altitude to 2m. A fuzzy controller based on T2DFs is designed to generate the required thrust signal u_1 during this entire operation. The altitude error and rate of change of altitude are the two inputs of the controller and u_1 is the output. The controller knowledge base is shown in Table. I, and the T2DFs of the antecedent and consequent are shown in

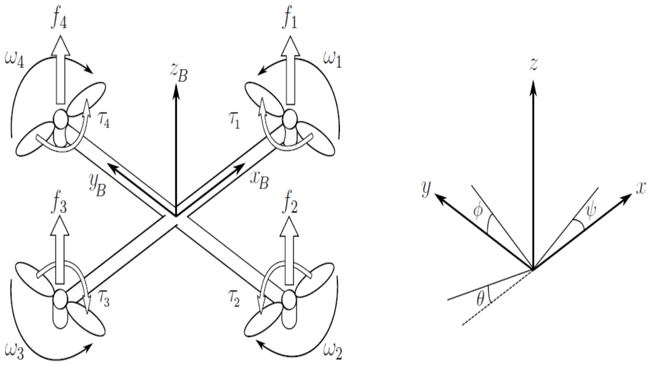


Fig. 5: Body and inertial frames of the Quadcopter structure [15]

Fig. 9. The upper and lower control surfaces generated using the T2DFs are shown in Fig. 6. The control surface used to generate the thrust u_1 is shown in Fig. 7. The altitude of the quadcopter and the command signal sent during the simulation have been plotted in Fig. 8. For comparison purposes, the altitude response generated using a Mamdani (type-1) fuzzy controller has also been plotted in Fig. 8. The top part of Fig. 8 shows the altitude response of both controllers in the absence of external noise. The bottom part of Fig. 8 shows the response in the presence of a large measurement noise (15 dB SNR).

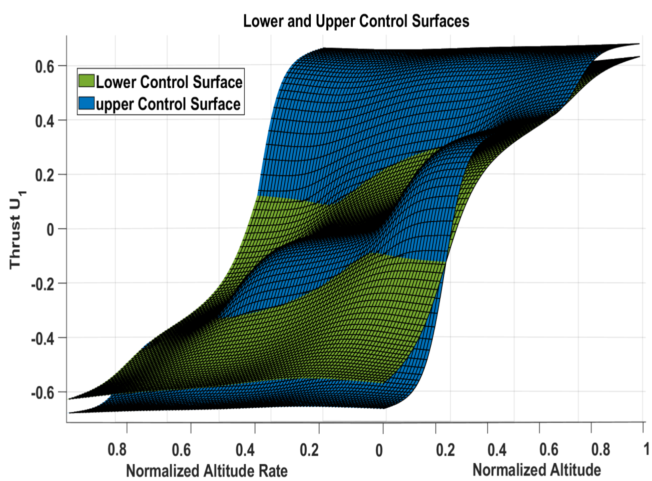


Fig. 6: Upper (blue) and lower (green) control surfaces

C. Discussion

A T2DF-based controller is able to control the altitude of the quadcopter using a few rules. The control surface used to generate the thrust u_1 has smooth transitions for different input values. In the absence of external noise, the response of the proposed controller is comparable with the conventional Mamdani controller. However in the presence of large measurement noise (15 dB SNR), the Mamdani controller produces small oscillations in the quadcopter altitude.

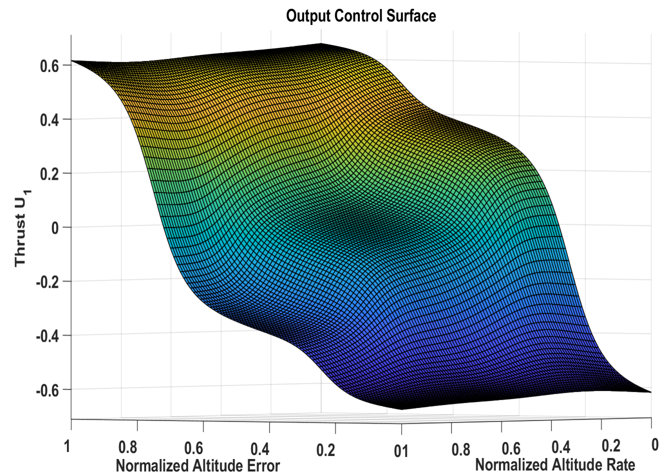


Fig. 7: The control surface used to generate the controller output

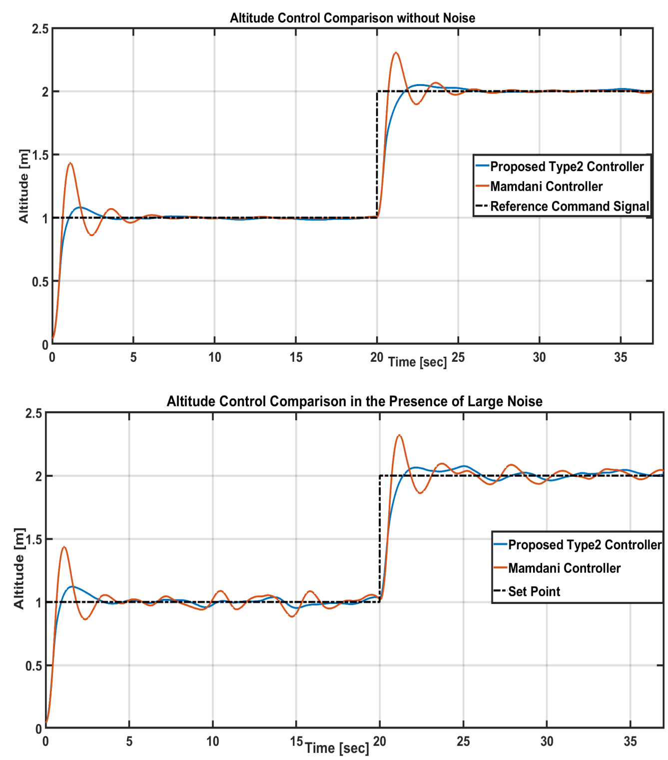
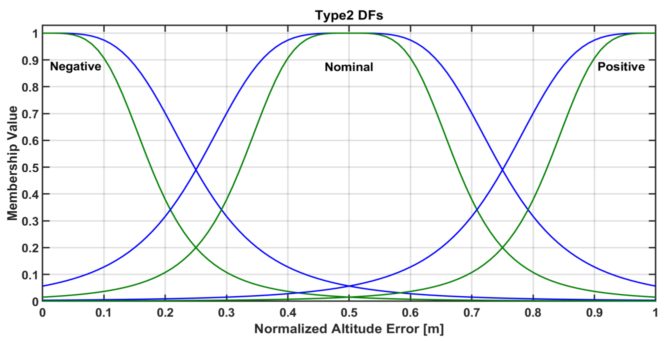


Fig. 8: Altitude control comparison of proposed Type2 and Mamdani controllers. a) Without noise (top). b) Large measurement noise (bottom)

In contrast, the proposed controller is more robust to this measurement noise and it produces a smooth altitude response compared to Mamdani controller. So, the proposed T2DF-based controller is better able to overcome the problem of external noise signals.

Rule No.	Altitude Error	Rate of change of Altitude	Thrust
1	Positive	-	Positive
2	Negative	-	Negative
3	Nominal	-	Minor
4	Nominal	Positive	Negative Small
5	Nominal	Negative	Positive Small

TABLE I: The rule base for the T2DF-based controller



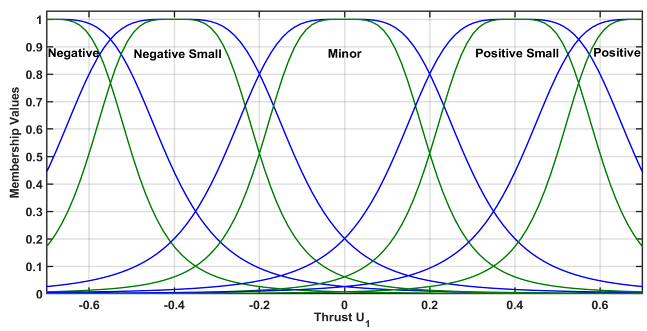


Fig. 9: T2DFs for Altitude Error and Thrust u_1

VI. Conclusion and Future Work

Here, a new parametric type2 MF called T2DF is proposed. It can handle the uncertainties introduced by measurement noise. Based on this T2DF, a novel technique for a robust type2 fuzzy controller is presented. The controller is robust against measurement noise, is computationally efficient and fast. The efficiency of the proposed fuzzy controller was demonstrated using the altitude control of a quadcopter. The results indicate that the performance of the proposed controller is better than that of the traditional Mamdani fuzzy controller in the presence of large measurement noise. This also reinforce the claim that the noise reduction property of type2 fuzzy controllers is better than that of type-1 fuzzy controllers. Future study will include the development of a data-driven-based type2 FC based on T2DFs.

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