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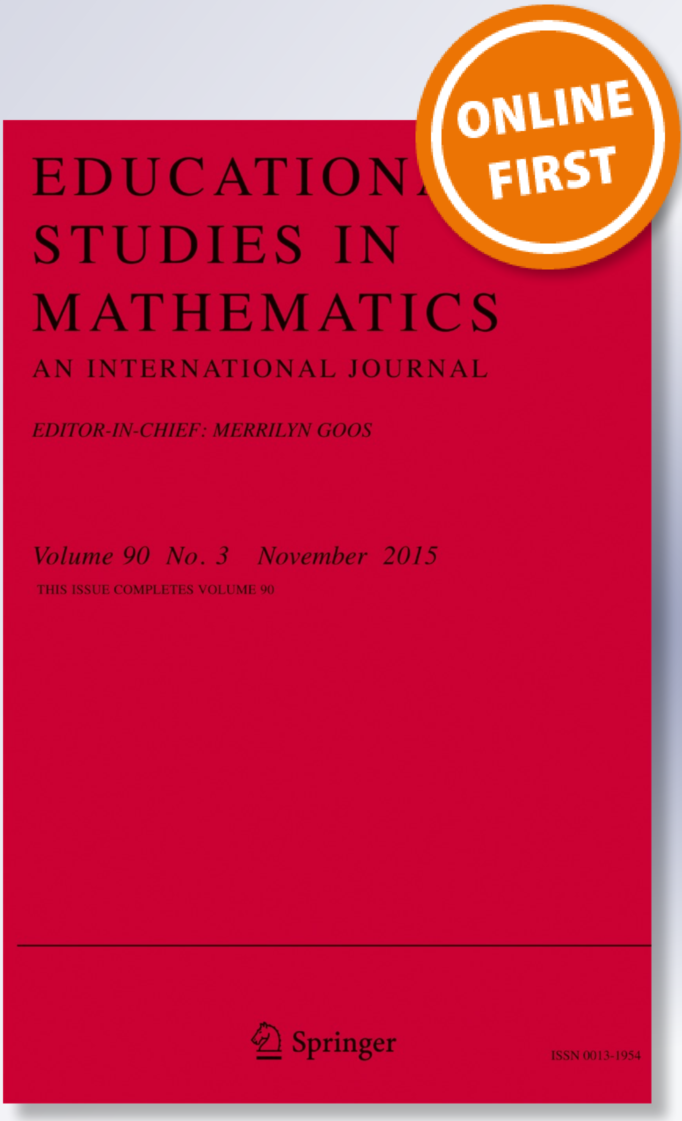

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# Strategies and performance in elementary students' three-digit mental addition

Csaba Csíkos [1]



**Abstract** The focus of this study is the relationship between students' performance in mental calculation and the strategies they use when solving three-digit mental addition problems. The sample comprises 78 4th grade students (40 boys and 38 girls). Their mean age was 10 years and 4 months. The main novelties of the current research include (1) exploration of the relationship between strategy use and response time, (2) revealing the uniformity of the strategies used throughout the series of tasks, and (3) pointing out between-school differences in strategy use, but not in success rate or response time. Although connections between strategy use and success have been demonstrated, about half of the students insisted on one given strategy throughout the series of eight tasks. The results indicate that teachers developed their students' mental claculation skills in a way such that some strategies became preferred and others ignored. In the discussion a comparison to previous research results and educational implications are provided.

**Keywords** Mental calculation · Arithmetic · Metacognition

## 1 Introduction

This study provides empirical results in a field that might be considered to be a well-defined segment of mathematical thinking and also discusses more general questions of instructional science. In recent years, there were reform mathematics education curricula introduced in the field of arithmetic, and some newly introduced principles and changes may be exemplary in other fields of school studies. Verschaffel, Luwel, Torbeyns, and Van Dooren (2009) list some initiatives to illustrate how the idea of adaptive or flexible strategy use appeared in some countries' curricula. "Less routine-based instructional approaches might be more appropriate, even for the younger and mathematically weaker children." (Verschaffel et al., p. 346–347). In

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Hungary, where the current research has been conducted, Julianna Szendrei's (2005) book offers new insights for teachers how self-regulation necessary for the afore-mentioned less routine-based instruction can be gradually introduced.

The focus of this study is the relationship between students' performance in mental calculation and the strategies they use when solving mental addition problems. In a rapidly changing socio-economic environment, it is increasingly important to foster students' strategic thinking, i.e., their potential to plan, monitor and evaluate their own thinking processes. Mental calculation (besides being useful in itself in everyday life, see Hope & Sherrill, 1987) is especially suitable for studying students' strategic and automatized thinking processes (Threlfall, 2002). Comparing to paper and pencil computations, mental calculation processes may and should be based on a rich repertory of calculation strategies that are adjusted to task and individual characteristics, and to contextual constraints. The usefulness and importance of mental calculation strategies have been emphasized by Lemaire, Lecacheur, and Farioli (2000), who refer to everyday situations where for instance the number of people in a crowd or the amount of spending can be calculated and at least estimated by means of using different mental calculation strategies.

The selection and usage of a mental addition strategy depends on several factors, three of which seem to be of primary importance (Verschaffel et al., 2009): the nature of the task (task variables), student's individual characteristics (subject variables), and contextual clues (contextual variables). An actual mental addition strategy can be efficient in a given context, for a given task and individual, while the same strategy may be time consuming and even increase the chances of errors under different circumstances. For instance, the accuracy and speed of the solution process may not be equally important in the classroom or in the supermarket when mentally adding three-digit numbers.

### 1.1 Strategic components of mental arithmetic

Any description of how mental arithmetic processes are accomplished requires multi-level interdisciplinary approaches. From neuroscience research (see Piazza & Dehaene, 2004) through cognitive psychological accounts (e.g., Fürst & Hitch, 2000) to school text-book content analysis (e.g., Heinze, Marschick, & Lipowsky, 2009), a number of perspectives have appeared in the past decades. From an educational point of view, the processes of mental arithmetic that can be improved by means of appropriate instructional interventions are of special importance. In order to be able to add three-digit numbers mentally, student must detach from the artefacts they used in the first grades of the primary school (Bartolini Bussi, 2011). In Hungarian schools, the most widely used artefacts in learning to add and subtract are the Cuisenaire-type colorful rods (for a description see Walter & Gerson, 2007), but these – of course – cannot be effectively used when introducing three-digit addition. Nevertheless, in some subtraction tasks, a visualization of the process of indirect addition (see later) can be realized by means of using these tools.

In different cultures and countries there are different traditions and constraints about the teaching and learning of adding numbers mentally. The importance of linguistic factors is well known from the literature. Miller, Kelly, and Zhou (2005) describe the differences between the Chinese and English number names with a view to the transparency of the base ten structure. It seems that linguistic differences have an effect on the development of arithmetic skills, but they should be regarded as "stumbling blocks" (p. 173) that can be overcome. The Hungarian composition of number words between 10 and 1000 is rather similar to the Chinese way of naming two- and three-digit numbers, and Cankaya, LeFevre, and Sowinski (2012) call this system "regular".

In the classroom (and during everyday activities) children learn to mentally add and subtract numbers of one or more digits. It is usually reflected in curricular targets or taken for granted by teachers and even by lay people that by the age of ten children can mentally add and subtract three-digit figures. For a quantitative analysis of whether and how they can attain this goal, two important considerations should be observed. First, in order to measure children's performance, both the rate of success and the time within which the child can solve a problem can be used. Another important aim is to identify strategies children use thus making it possible to link their correct or erroneous answer patterns with the strategy they used.

Measuring students' performance in mental addition or subtraction is usually accomplished by means of developing a set of tasks of various types. The construction of this set may depend on the number of genuinely different types of tasks that can be identified. The identification of task types may be based on purely mathematical considerations, or also on psychological considerations. The latter approach means that the repertory of strategies assumed to be used by the students is taken into consideration in deciding on the number of task types the experimenter should include.

In the following sections, first we present a general discussion of what 'strategy' or 'strategic' means in an educational context, then a short overview will be given of the strategies of mental three-digit addition and subtraction identified in the literature so far, and in the next section the consequences for research on adaptive strategy use are discussed.

Defining what 'strategy' in human thinking means is still a challenge in the scientific literature. A very general approach was provided by Flavell (1987) which was claimed to be 'speculation' by the author himself: strategy variables are metacognitive components. They both refer to knowledge about cognitive procedures that are used to achieve various goals. While cognitive strategies are about attaining a goal, the purpose of metacognitive strategies is to be confident about reaching that goal. In various cognitive domains, the term 'strategy' is often used without any precise definition. Thinking processes and artificial intelligence levels in chess and in other 'strategy' games are often divided into two parts: Strategy refers to the planning processes where actions and subgoals are connected, and tactics refers to a lower level of considerations focusing on immediate inferential chains (see Cannice, 2013; Wiener, 1960/1999).

In education theory and practice, strategies have been first metaphorically (Almasi, 2003) and then analytically defined in the field of reading. Afflerbach, Pearson, and Paris (2008) started out from the definition found in The Literacy Dictionary. In an educational sense strategy is a systematic plan that is consciously adapted and monitored in order to improve performance in learning. The appearance of the term 'conscious' raises further questions, however. Further clarification is provided by Hacker (1998) who reserves the term metacognitive for potentially reportable thoughts. Pondering on how to proceed from reading-related definitions to an arithmetic skill-related one, it is useful to cite Gourgey's (1998, p. 86) ideas. The author explicitly juxtaposes the automated and strategic processes of reading and mathematics:

> Metacognition in mathematics is, in principle, the same as metacognition in reading. That is, once students have acquired the basics (computation in mathematics as compared with decoding in reading), their ability to think in the domain is based on

clarifying goals, understanding important concepts, monitoring understanding, clarifying confusion, predicting appropriate directions, and choosing appropriate actions.

There is one more concept in this strategy definition that requires further examination: In the above understanding of strategic thinking, there seems to be a sequence of components, 'basics' coming first, and strategic components next (for an analysis of the sequence of learning phases, see Baroody, 2003). However, automated and strategic components develop together – even attaining the 'basics' requires metacognitive strategic considerations (for an example, see Siegler, 1987). Further support for this claim comes from the "overlapping waves" developmental model (Siegler, Adolph, & Lemaire, 1996; Siegler & Lin, 2010).

In accordance with the overlapping waves model, in the case of addition and subtraction, a variety of strategies may suffice for a given task. In the simplest case, one may decide to use a written computation algorithm instead of mental computation. Threlfall (1998) doubts whether in teachers' understanding and in the current convenient use of the term "strategy" a priori decisions before mental calculation are always involved. For some children, mental computation strategies seem to be easier to complete first, and for others it is the written algorithm that provides the smartest solution. From the various types of mental addition strategies, some children may first prefer the split or hundreds-tens-units strategy, and later start using the stepwise strategy[1] or *vice versa*. There might be rather different mental processes even when the result of the computation is the same. For the purposes of educational interventions, a list of possible mental addition strategies is needed, and then the links between performance and strategy use can eventually be identified.

Searching for a description of the most widely known and used mental addition strategies, we have found three studies in which a system of mental three-digit addition strategies was constructed. Fuson et al. (1997), Heinze et al. (2009), and Selter (2001) described a system of these strategies with no substantial differences between their taxonomies. The studies do differ, however, in the names given to the strategies. Table 1 summarizes the structurally identical systems that can be inferred from these three studies, and makes the taxonomical differences comparable.

The compensation/simplifying strategies might be somewhat different according to Selter (2001) and Heinze et al. (2009), but from a research methodological point of view we can unite them in a single strategy cluster. In this study we will use Heinze, Marschick and Lipowsky's taxonomy, using the term 'simplifying' for the compensation/simplifying strategy cluster. Another feature of Table 1 is the presence of the strategy of indirect addition. From a purely mathematical point of view, the inclusion of indirect addition as a mental addition strategy in connection with an operation that is obviously subtraction in mathematics requires further consideration. Subtraction problems may be solved by different strategies, including mental indirect addition (see Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). What is more, indirect addition as a mental strategy can be more generalizable to other place value systems.

Another important question about the taxonomy of the strategic components of mental three-digit addition is whether the system is complete in the sense that no further important strategies can be added to the list – at least for the elementary school student age group. For example, it may be possible for someone to follow the algorithm of the written computation,

[1] For the same task, e.g., 526+233, using the split or hundreds-tens-units strategy means transforming the original task mentally to 500+200 plus 20+30 plus 6+3, while the stepwise strategy results in a mental procedure that can be observed (and usually verbally reported by the child) as 526+200+30+3.

**Table 1** Taxonomies of three-digit mental addition strategies in three key publications

| Example | Fuson et al. (1997) | Selter (2001) | Heinze et al. (2009) |
|---|---|---|---|
| 123+456=123+400+50+6 | "begin-with-one-number" | stepwise | stepwise |
| 123+456=(100+400)+(20+50)+(3+6) | decompose hundreds-tens-and-ones | htu (hundreds, tens, units) | split |
| 527+398=527+400−2 | change-both-numbers | auxiliary (simplifying) | compensation (simplifying) |
| 701−698=the number which must be added to 698 in order to get 701 | unknown addend | adding up | indirect addition |

imagining the addends (or subtrahends) written one above the other, and following the steps of the algorithm in his or her mind. If this strategy is indeed used, it can be considered a combination of the stepwise and split strategies with the important difference that students may explicitly report their use of the written computation algorithm. This strategy thus deserves its own label: "paper-and-pencil mental analogue" (term borrowed from Hope & Sherrill, 1987, who used it for mental multiplication) is the label used in the current study.

### 1.2 Connections between mental addition strategies and performance

Hope and Sherrill's (1987) research on mental multiplication provided evidence on the connection between strategy use and performance in the case of mental multiplication. In their study, upper secondary students were divided into two groups: the best and worst performers, and there was strikingly marked difference in their strategy use indicating that unskilled students overwhelmingly used some kind of paper-and-pencil-like mental strategy without much success. On the contrary, skilled students used a more sophisticated distribution strategy when one or both of the factors to be multiplied are transformed to sums or differences. Although that study focused on mental multiplication, the main results on the connection between strategy use and performance may be analogous with and may serve as starting hypotheses for other mental arithmetic operations as well.

Torbeyns, et al. (2009a) revealed that young adults can adaptively use the indirect addition strategy in the case of subtraction problems with three-digit numbers, whereas only 15 % of the fourth grade students used spontaneously indirect addition in the case of subtraction problems with two-digit numbers. According to their results, the developmental route from childhood to adulthood encompasses not only higher frequency in using indirect addition when the difference between the minuend and the subtrahend are relatively small (i.e., it is very simple to determine how much should be added to the subtrahend), but the frequency of strategy use depends also on individual characteristic (subjective preference or objective efficiency).

Further studies have revealed connections between strategy choice in mental calculation and performance in various tasks. Torbeyns, Verschaffel, and Ghesquière (2006) investigated second grade pupils' mental two-digit addition and subtraction strategies, and it was only the high achievers who fitted their strategy choice to task characteristics. In another study of the Leuven group, Torbeyns, Ghesquière, and Verschaffel (2009), again young adults' mental direct subtraction and indirect addition strategy choice proved to be connected to their performance on three-digit subtraction tasks. The most immediate antecedents of our current investigation was the study by Heinze et al. (2009) which provided results on the accuracy (performance) of third grade students on eight addition and subtraction problems, and on their strategy choice. They deliberately chose the second half of Grade 3, i.e., the time when the formal writing computation was just about to be taught.

Foxman and Beishuizen (2002) reanalyzed 11-year-old students' results on a nationwide survey. One of the items was a simple mental addition task with two three-digit numbers (238+143=?). A detailed analysis of the strategy choice was presented (81 % of the students got the right answer), both for the whole sample and for three attainment-category subsamples (top, middle, bottom achievers in the whole test). As for the correlation between strategy choice and performance, the top and middle achievers proved to be more successful in that item, and these two groups used more frequently the "paper-and-pencil mental analogue" strategy (in the authors' terminology: "algorithm"), whereas the third group tried to use the split strategy more frequently. There are two major points to consider here. First, although the split strategy perfectly "fits" the ominous task, the higher achievers tended to rely on the mental execution of the written computation algorithm. Second, children above the age of the school introduction of written addition algorithm do use this strategy (51 % of them) instead of any evenly smart strategies.

### 1.3 Research questions

In accordance with the literature review, several research questions have been addressed.

(Q1) What solution patterns do 10 year old students have in terms of success rate and response time on three-digit mental addition problems?
(Q2) What type of incorrect answers do they provide during the mental computation process?
(Q3) What is the frequency of different mental computation strategies among 10 year old students?
(Q4) Is there any connection between students' strategy use and their performance?
(Q5) Are there between-class and between-school differences either in strategy use, performance, or incorrect answer patterns? If so, can these differences be attributed to the text-book used?

The first two questions have previously been examined and the results have been presented (Csíkos, 2012). The current paper focuses on students' strategy use (Q3), and its correlations with the observed performance (Q4). Following Heinze et al.'s hypothesis about the role different instructional practices and text-books may play in shaping students' mental computation, differences between schools and between classes are also examined (Q5).

## 2 Methods

### 2.1 Sample

The student participants were recruited from two different schools: One school is situated in a county seat town and the other in a village in Hungary. From both schools, two 4th grade classes participated, i.e., all students belonging to that age group in the schools. Students come from diverse socio-economic family backgrounds, thus assuring generalizability of the results. The sample comprises 78 4th grade students (40 boys and 38 girls). Their mean age was 10 years and 4 months (with an age range of 9 years and 6 months to 11 years and 9 months).

## 2.2 Measures

A task system consisting of eight tasks was developed for the purpose of the study. In accordance with the research questions, this task system may facilitate the use of diverse mental computing strategies, and all tasks are expected to be correctly solved by 4th grade students:

| | | |
|---|---|---|
| (1) 342+235 = | (2) 143+426 = | (3) 702+105 = |
| (4) 284+202 = | (5) 527+398 = | (6) 498+256 = |
| (7) 701−694 = | (8) 646−583 = | |

The first four tasks were designed to elicit either the stepwise or the split strategy of mental addition. Tasks 5 and 6 were expected to encourage the use of the simplifying strategy, whereas the last two tasks were expected to elicit the use of the indirect addition strategy.

The system of these eight tasks as a whole aimed to represent the four main strategy cluster shown in Table 1. The task system has a Cronbach-a reliability of 0.70, computed from 7 items only, since Task 4 was solved by every students. This reliability coefficient indicates appropriate reliability for the purpose of the current study.

## 2.3 Procedures

Students were tested individually in three separate rooms of the school they attended. Three research assistants were trained and the data collection procedure was previously rehearsed. The tasks were printed on A4 size sheets (one task per sheet) with 20 pt Times New Roman characters, center-aligned, on the top of the page. The research assistants handed over the sheets to the students, and the timer was started. For each task, a maximum of 60 s were given. This time limitation is reasonable not only for methodological purposes, but it is in line with Frank and Barner's (2012) findings who claim that when adding mentally two-digit numbers, after several trials children are usually capable to solve the tasks within 15 s. Within this time frame, the assistants wrote down any incorrect answer given by the students on the answer sheet. When hearing the right answer, they stopped the timer, noted the time needed for the correct solution, and proceeded to the next task.

Having completed the eight tasks, the assistants asked permission from the students to start the dictaphone, and asked them to explain how they had solved each task. The students could see the task sheets again while talking about their solution process. In case of a relatively longer pause, the assistants encouraged the students by asking: "What partial results did you have?"

## 2.4 Analysis

As the quantitative measures of students' performance, success rate, erroneous answers and response time were recorded. The dichotomous variable was 1 in case the student gave the right answer within 60 s, and was 0 in case of lack of the right answer, independently of the erroneous answer(s) the student gave.

To identify the strategy used in a given task, a post-hoc judging procedure was established. Two experts were asked to listen to the recorded audio-files, and to judge independently which strategy was used for a given task by the student. They had to choose either from the strategies described in Table 1, or an additional strategy of "paper-and-pencil mental analogue" as described above. As an example for "paper-and-pencil mental analogue" strategy, in the task 527+398 a student may start adding 7 and 8, and virtually writes down 5 in the place of units, and brings 1 to the second column, for the tens etc. Due to technical errors, 3 students' recorded answers were lost, therefore we have a 75 student sample on the assigned strategies (Table 2).

As it will be shown in Table 3, the rate of answers judged to be differently by the two raters fell into the interval of 1 to 13 % which is comparable to the reported 10 % of unclassified strategies in Foxman and Beishuizen's (2002) analysis of a three-digit mental addition task. It is important to note that in their analysis in the bottom third of the sample the rate of unclassified answers was 21 %, drawing attention to the possible limitations in children's accurate report of their thinking processes. The validity or veridicality of children's self-report depend on several things: on one hand, in this age-group (and even in adulthood) the mental calculation processes needed for the tasks involved in our study are far from being automatic, consequently the information did enter into working memory (Kirk & Ashcraft, 2001), therefore it might be validly reported. On the other hand, by means of the uniform protocol we tried to avoid any bias towards reporting any strategies children did not use. Nevertheless, their vocabulary sometimes may lack technical terms like hundreds, tens, units, borrowing etc., the two independent experts who were in-service mathematics teachers, could well identify children's strategies as reflected in the low rate of unclassified strategies.

## 3 Results

The results will be presented in the order of the research questions. As was mentioned above, Q1 and Q2 have already been detailed elsewhere, and only a condensed summary of those results is given here.

### 3.1 Performance and typical incorrect answers

Table 2 shows the performance and the incorrect answer patterns for each task. The success rate was satisfactory for the first four tasks, but dissatisfactory for the next four items. Mean

**Table 2** The rate of correct solutions, the time needed for the solution, and the most frequently observed incorrect answers (Csíkos, 2012, pp. 182–183)

| Task | Success rate (%) | Mean RT (SD in parentheses) | The most frequent incorrect answers (relative frequency in parentheses in %) |
|---|---|---|---|
| 342+235=577 | 94.9 | 13.35 (10.36) | 5707 (5.1); 5777 (3.8); 587 (3.8) |
| 143+426=569 | 97.4 | 10.95 (9.57) | 579 (3.8); 590 (3.8) |
| 702+105=807 | 98.7 | 5.53 (5.65) | – |
| 284+202=486 | 100.0 | 8.39 (8.90) | – |
| 527+398=925 | 70.5 | 24.14 (17.90) | 915 (6.4); 625 (5.1) |
| 498+256=754 | 69.2 | 22.02 (15.02) | 654 (10.3) |
| 701−694=7 | 52.6 | 24.37 (16.82) | 193 (15.4); 5 (7.7); 16 (5.1); 13 (3.8); 93 (3.8) |
| 646−583=63 | 50.0 | 28.28 (14.75) | 43 (9.0); 143 (9.0); 163 (6.4); 57 (3.8); 67 (3.8); 137 (3.8) |

Incorrect answers with fewer than 3 occurrences are suppressed

**Table 3** Frequency of observed computation strategies (%)

| Task | Stepwise | Split | Simplifying | Indirect addition | Written algorithm | Indeterminate |
|---|---|---|---|---|---|---|
| 342+235=577 | 26.7 | 44.0 | – | – | 25.3 | 4.0 |
| 143+426=569 | 29.3 | 42.7 | – | – | 25.3 | 2.7 |
| 702+105=807 | 26.7 | 50.7 | – | – | 21.3 | 1.3 |
| 284+202=486 | 28.0 | 45.3 | – | – | 25.3 | 1.3 |
| 527+398=925 | 32.0 | 37.3 | – | – | 26.7 | 4.0 |
| 498+256=754 | 28.0 | 36.0 | 2.7 | – | 28.0 | 5.3 |
| 701−694=7 | 36.0 | 17.3 | – | 9.3 | 24.0 | 13.3 |
| 646−583=63 | 38.7 | 21.3 | – | 6.7 | 26.7 | 6.7 |

*N*=75

response times are shown in the second column, and the most frequent incorrect answers (three of more occurrences) are shown in the third.

Table 2 shows that the last four tasks proved to be more difficult both in terms of success rate and in response time. The patterns of the most frequent incorrect answers show some obvious examples of 'rational errors' (Ben-Zeev, 1996) indicating the use of a rational but inappropriate strategy, e.g., in Task 7 the result 193 can be explained by subtracting the smaller from the larger value for each place: 7−6=1; 9−0=9, and 4−1=3, which gave 193 for almost one sixth of the students.

### 3.2 Strategies in mental addition

In order to identify students' strategies two independent experts listened to the audio taped interviews, and made their decisions about the category the strategy fell in. They could select any of the categories listed in Table 1, or they could select "paper-and-pencil mental analogue." As this fifth category is actually a subset of the split (or hundreds-tens-units) strategy category with an extra reference to the use of written computation algorithm, in the cases where one of the experts selected the split strategy and the other selected the written computation algorithm, the broader category was accepted. If there was any other disagreement between the experts, a sixth category, 'indeterminate' was applied. Table 3 shows the frequencies of the different strategies.

As Table 3 shows, the frequency of the indeterminate strategy is low (1.3 % means one student), i.e., there was little disagreement between the experts. For the first six tasks, three strategies were observed with high frequencies: split, stepwise and written computation algorithm strategies. What is more, the distribution of these three most frequent strategies shows uniform distribution for the first six tasks (Kolmogorov-Smirnov's Z values were 1.03 ($p$=.24); 0.60 ($p$=.86); 1.05 ($p$=.22), respectively. At the same time this suggests that students overwhelmingly insisted on one given strategy throughout the first six tasks. This claim is supported by the fact that in School 1, 78 % of the students, and in School 2, 67 % of students used the same strategy for the first six tasks. The difference between schools in this respect is non-significant.

Forty seven percent of the students applied the same and only strategy throughout the eight tasks. This insistence on a given strategy may be explained in part by individual characteristics (knowledge, beliefs and attitudes), but there might be group-level differences (see later, in the section on Q6). In general, there may be a tendency to reiterate strategy use throughout a series of tasks, and Schillemans, Luwel, Bulté, Onghena, and Verschaffel (2009) suggest that switching strategies has a cost and thus the repeated application of a given strategy may be preferred.

### 3.3 Connections between strategies and performance

The effects of strategy choice on students' performance can be investigated by group comparisons. Performance has two indicators (success rate and response time), and strategy choice can also be characterized by at least two measures (the strategy used in a given task, and the recurrence of strategies throughout the series of tasks). The first strategy indicator can be applied for a given task, while the second can be used as a general strategy indicator for the series.

In our first analysis, the relationship between strategy use and response time is shown in the form of eta-squared coefficients. Eta-squared coefficients are useful indicators of the explained variance, i.e., to what extent strategy use determines the response time.

Following Cohen (1969), the volume of the eta-squares presented in Table 4 can be interpreted as follows: 1 % means small, 5 % indicates medium, and 16 % large effect size. Consequently, in general the strategy used in a given task has medium or large effect on the response time achieved. The response time values are, of course, computed only for successful task solution.

The highest eta-squared values were observed for the tasks that were to evoke the indirect addition strategy. Task 7 proved to be especially divisive with respect to the role of strategies in response time. To have a closer look at how a given strategy affects response time, let us consider the response time averages for Task 7 in Table 5.

Table 5 convincingly demonstrates that in this task the indirect addition strategy proved to be the most efficient (in accordance with our previous expectations). It has also been revealed that the members of the second best performing group did not report their mental computation processes clearly enough, since the experts could not unambiguously decide which strategy they used.

As for the connection between the uniformity of strategy use and response time, the students were divided into two groups (those who used the same strategy for all eight tasks (47 % of students) and those who switched strategies at least once) and their response times were compared. The results are shown in Table 6

**Table 4** Eta-squared coefficients between strategy use and response time for each task

| Task | Eta-squared (%) |
|---|---|
| 1 | 6.5 |
| 2 | 7.4 |
| 3 | 7.2 |
| 4 | 7.1 |
| 5 | 5.1 |
| 6 | 10.1 |
| 7 | 26.7 |
| 8 | 16.3 |

**Table 5** Mean response times for different mental computation strategies in Task 7

| Strategy | N | Mean response time |
|---|---|---|
| stepwise | 15 | 28.35 |
| split | 5 | 28.02 |
| compensation (simplifying) | – | – |
| indirect addition | 6 | 11.77 |
| written algorithm | 6 | 38.65 |
| indeterminate | 7 | 16.97 |

ANOVA $F$=3.09, $p$=.03

Results in Table 6 indicate that in itself the uniformity of strategy use has not proved to be advantageous nor disadvantageous, since the differences between the mean response times were not significant, with the exception of Task 1. In Tasks 1, Task 3 and Task 7, the tendency (which manifests itself as significant difference in the case of Task 1) is that those who insisted on using the same mental calculation strategy throughout the tasks were quicker in solving these tasks.

The second performance indicator is the success rate. How the success rate in a given task depends on strategy use, was analyzed by cross-tabulation with Cramér's V (see Table 7).

According to Table 7, there was no significant connection between success on a given task and the mental calculation strategy used revealed.

In order to analyze the relationship between success rate and strategy use at the whole task series level, mean number of successfully solved tasks were 3.17 for the 'uniform strategy' group, and 2.70 for the other group. The difference is not significant ($t$=1.25; $p$=.22). The impact of uniform strategy use on success in each task was analyzed cross-tabulation. $\chi^2$-

**Table 6** Mean response times for two groups: those who used the same strategy throughout the series of the tasks, and those who changed the strategy use at least once

| Task | Uniform strategy use | Mean response time (SD in parentheses) | \|t\| | p |
|---|---|---|---|---|
| 1 | Uniform ($N$=37) | 10.24 (8.02) | 2.82 | .01 |
| | Not uniform ($N$=34) | 17.05 (11.81) | | |
| 2 | Uniform ($N$=38) | 10.74 (10.76) | 0.11 | .91 |
| | Not uniform ($N$=35) | 10.99 (8.12) | | |
| 3 | Uniform ($N$=39) | 4.33 (4.69) | 1.93 | .06 |
| | Not uniform ($N$=35) | 6.87 (6.53) | | |
| 4 | Uniform ($N$=40) | 8.72 (11.06) | 0.19 | .85 |
| | Not uniform ($N$=35) | 8.33 (6.13) | | |
| 5 | Uniform ($N$=31) | 24.17 (17.58) | 0.03 | .98 |
| | Not uniform (N=21) | 24.02 (17.78) | | |
| 6 | Uniform ($N$=28) | 20.98 (15.07) | 0.42 | .68 |
| | Not uniform ($N$=23) | 22.76 (15.39) | | |
| 7 | Uniform ($N$=24) | 22.29 (16.46) | 1.44 | .16 |
| | Not uniform ($N$=15) | 30.11 (16.52) | | |
| 8 | Uniform ($N$=24) | 27.17 (15.34) | 0.50 | .62 |
| | Not uniform ($N$=13) | 29.78 (14.98) | | |

In the case of Task 1, the Welch-test could be used ($df$=57.41)

**Table 7** Cramér's V coefficients between strategy use and success in a given task ($N$=75)

| Task | Cramér's V | p |
|---|---|---|
| 1 | 0.28 | .12 |
| 2 | 0.28 | .11 |
| 3 | 0.12 | .80 |
| 4 | – | – |
| 5 | 0.18 | .51 |
| 6 | 0.26 | .30 |
| 7 | 0.33 | .09 |
| 8 | 0.25 | .32 |

Since Task 4 was solved by everyone, no connection could be computed

statistics revealed that in the case of Task 8 a significant connection exists between the two variables ($\chi^2$=3.90, $p$=0.048).

### 3.4 Between-class differences

Every analysis presented until this point can be re-analyzed from the perspective of between-school and between-class differences. As for the first and second research questions (Q1 and Q2), no between-school differences have been found in either success rate ($t$=0.25, $p$=.81) or response time (p values for the independent-samples t-tests for the various tasks were between 0.25 and 0.99). Within the schools there were slightly larger between-class differences marked by a significant difference for Task 1 ($t$=2.10; $p$=.04) in the case of School 1, but there were no other significant between-class differences in performance indicators.

Looking at between-school differences in strategy use (Q3), we find that for School 1 the stepwise strategy was used the most frequently, around 50 % of the time in the first six tasks, while in School 2, the split strategy proved to be the most popular. According to the two-sample Kolmogorov-Smirnov-test, there were significant differences between the schools in strategy use for all but Task 7. Interestingly, there were no significant between-class differences found within the schools.

The question whether there are between-school differences in the nature and magnitude of correlations between strategy use and response time (Q4) was examined also by computing eta-squared coefficients (see Table 8).

**Table 8** Between-school differences in eta-squared coefficients for the connection between strategy use and response time for each task

| Task | Eta-squared (%) | |
|---|---|---|
| | School 1 | School 2 |
| 1 | 4.3 | 7.1 |
| 2 | 13.8 | 16.1 |
| 3 | 6.9 | 6.6 |
| 4 | 8.6 | 19.3 |
| 5 | 1.0 | 6.9 |
| 6 | 7.0 | 20.2 |
| 7 | 23.7 | 29.3 |
| 8 | 25.8 | 55.3 |

Table 8 suggests that there are relevant differences between the two participating schools with respect to the strength of correlations between strategy use and response time. For School 2 there are noticeable eta-squared values for Tasks 4, 6, 7, and 8. The latter two tasks also had notable explained variances in School 1. These values indicate large effect sizes, i.e. strategy selection and use in these cases explain response time differences to a large extent. Between-school differences indicate the different efficiency of the most frequently used strategies (stepwise strategy in School 1 and split strategy in School 2).

## 4 Discussion and implications

The main purpose of this study was to investigate the relationships between mental computation strategy use and mental addition performance. Three-digit addition and subtraction tasks were developed for the purposes of the study inducing the use of different strategies known from the literature (Fuson et al., 1997; Heinze et al., 2009; Selter, 2001). Besides these "real" mental calculation strategies, a further one imitating the written computation algorithm has been documented.

The main novelties of the current research include (1) exploration of the relationship between strategy use and response time, (2) revealing the constancy/uniformity of the strategies used throughout the series of tasks, and (3) pointing out between-school differences in strategy use, but not in success rate or response time.

The limitations of our study come from at least three sources. First, the length of the task series may have been enough to detect mental calculation strategies, but might have been insufficient to make reliable estimation on students' overall mental calculation skills. This possible constraint cautions about the interpretation of results concerning the sum of correct answers. Second, the time when children learn the formal written algorithm for addition (and subtraction) may influence the rate of occurrence of the "paper-and-pencil mental analogue" strategy. In Hungary, the formal written addition algorithm is usually taught in the first half of Grade 3. Third, since there were only two schools involved, the between-school difference found can be considered as one relevant case, but its generalizability cannot be evaluated.

The performance on tasks and the frequent error types have already been presented and interpreted (Csíkos, 2012). Students' performance can be characterized in a way that they could easily solve the first four tasks; two thirds of them succeeded in the tasks that were designed to evoke the simplifying strategy, and only half of them could solve the subtraction tasks. Their error patterns showed that an early over-automatization of the written computation strategy led to "rational errors" (e.g., subtracting always the smaller digit from the bigger one).

Research questions 3, 4 and 5 are discussed here. Question 3 concerned the observed strategies during mental calculation. The analysis of the frequency of mental addition strategies brought results about a population that has already been taught written computation algorithm. Thus, the relative frequency of the strategy "paper-and-pencil mental analogue" is of special interest. Comparing to Foxman and Beishuizen's (2002) results on the three-digit addition task (where 51 % of the 11-year-old students used the written algorithm strategy), in our sample less students (around 25 %, independently of the actual tasks) used that strategy. Another intriguing query was whether Hungarian 10–11 year old students use the indirect addition strategy in case of the two subtraction items. In Torbeyns et al.'s (2009b, Study 2) investigation, only 10 % of the second and third grade students, and 15 % of the fourth grade students used spontaneously the indirect addition mental strategy. In our study, for both subtraction tasks, less than 10 % of our fourth grade students used that strategy.

The uniformity of the strategy used throughout of the series of the tasks can be elucidated by two things. On one hand, students may in fact have a very limited repertory of mental addition strategies. It is possible, however, that students did not well recall the strategies they used, and a strategy they reported in a task may have been attributed to all other tasks. This debate could have been resolved by asking students about their strategy use after each task, but in that way the strategy use just being explicated may influence the strategy used in the next task. On the other hand, even if they had a rich repertory, the cost of strategy switching (Schillemans et al., 2009) may hinder them from using that repertory. The difference between the two participating schools suggests that whatever the explanation is, the uniformity of the strategy use is essentially independent of the actual strategy used throughout the tasks.

Question 4 concerned the connection between strategy use and performance. This research question has multiple facets, since both the actual strategy and the uniformity of strategy use, and the two types of performance indicators, i.e., success and reaction time may be intercorrelated. This results in four types of correlations. (1) The connection between the actual strategy used in a task and the reaction time was analyzed by means of nominal-by interval correlations, i.e. eta-squared coefficients. These coefficients show effect sizes ranging from middle to large sizes. (2) The overall uniformity of strategy use defined two groups that could be compared on their reaction times. In general, students' insistence on using the same strategy throughout of the eight tasks (47 % of them did so) does not play a relevant role in the average time during which the right calculation was done. (3) The connection between the actual strategy used in a task and success in that task was analyzed by means of cross-tabulation and Cramér's statistics. The results indicate that success (giving a 60 s time frame) does not depend on the actual strategy, or, more positively expressed, each mental calculation strategy observed may be eligible for use in three-digit mental calculation. (4) The overall uniformity of strategy use indicated a significant impact on whether the students could solve Task 8. Those who used the same strategy throughout the task series were less successful on Task 8. This obviously means that those who changed the strategy to for instance indirect addition had much better chance to provide the right answer.

Question 5 addressed the between group differences both in strategy use and in performance. Interestingly there was no significant difference between the two participating schools regarding performance. But there were conspicuous differences in strategy use between the two schools, but not between the two classes within a school. This surely points to a school-wide didactical practice or tradition in teaching mental calculation strategies. The relevant difference between the two schools suggests that the teachers developed their students' mental computation skills in a way such that some strategies became preferred and others ignored. On the other hand, it would be important to maintain the flexibility or adaptivity of mental addition strategy use. In a study on written computation algorithms, Laupa and Becker (2004) demonstrated the power of introducing a new computation algorithm based on teachers' authority. The new and erroneous algorithm was accepted by the majority of children when it was supported by teachers' authority, and it was ignored otherwise. Therefore, teachers really have the power to introduce new computing algorithms, but they have the responsibility to adapt those strategies to task and student characteristics, and to contextual constraints (Verschaffel et al., 2009).

The possible role textbooks may play in the development of and the emerging differences in mental addition strategy use is still not quite well known in Hungary. Differently from Heinze et al.'s (2009) study where a detailed analysis of the occurrence of different mental addition strategies in German textbooks was provided, in Hungary there has been no similar analysis published from two reasons. First, our textbook covers the topic of mental calculation marginally, and the situation is the same as in half of the German textbooks: half a page dealing with "computation tricks or 'advantageous computations'" (Heinze et al., p. 594).

Specifically, in our studies the two schools used two different textbooks. The two textbooks treated the topic of mental addition very similarly. The Grade 4 textbooks contained exactly one lesson (suggested time frame: 45 min) for mental additions and subtractions, and provided some examples of "smart" calculations, which means that they showed the use of the simplifying strategy where appropriate. However, in the previous year, in Grade 3, the textbooks contained several lessons on mental computations, and in one of the textbooks, one lesson was about different possible strategies. To compute the sum of 270+140, the textbook provides three possible solutions (Balassa, Csekné Szabó, & Szilas, 2009) which represent three different strategies: stepwise, split and simplifying strategies. The teachers' manual attached to this textbook emphasized that teachers should present several solution methods enhancing the chance that students will find the most appropriate strategy according to their preferences and the task characteristics.

It is worth emphasizing that the phenomenon of adaptive strategy use is apparent in the mathematics textbook the students used, albeit the time frame dedicated to the introduction and practice of different strategies is very narrow. The teachers participating in our study follow the textbooks they have chosen, i.e., they follow the sequence of lessons with the time frames suggested by the teachers' manuals. Consequently, and according to our results, it might be not mainly the textbooks but the teachers' beliefs and experiences that shape students' mental calculation. The lack of between-class differences within a school indicates an existing school-level coordinated effort in teachers' practices in the schools involved in the study. This school-level coordination of didactical issues is institutionalized in Hungary by means of teachers' "special interest groups" within schools. These groups have their role – among many other things – in choosing the textbooks and selecting continuous professional development courses.

The educational implications of the current study may concern both in-service teachers' practice and the training of elementary school teachers. Since there is no "optimal" mental calculation strategy even for a well-defined task, one possible conclusion is that it does not matter which strategy is taught in schools. Another equally plausible conclusion may be that several different strategies ought to be taught in order to offer a repertory from which children can choose. The teaching methods for building such a strategy repertory assume that the teacher plays an active role not in directly "providing [children] a rule for deciding when to use" (Torbeyns, et al., 2009a, p. 87.) but playing an active, although indirect role in giving "students the leeway to devise their own strategies" (Baroody, Cibulskis, Lai, & Li, 2004, p. 252).

Starting out from the current situation, it seems that two steps should be taken simultaneously: making the importance of flexible mental addition strategy use explicit for teachers, and letting students develop and use their own mental calculation strategies. According to the results presented in this study, students do possess a mental addition strategy they prefer to use, and that was probably taught and encouraged by their teacher. However, since more difficult tasks often require the use of different strategies in their different phases of the solution process, Harskamp and Suhre (2007) claim that in general students should possess a broad strategy repertory. Since besides task characteristics, also individual preferences might make a difference regarding which mental calculation strategy is the most effective in a given task context, any effort that aims at building a rich strategy repertory is in itself commendable.

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