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Research Reports

SUCCESS AND STRATEGIES IN 10 YEAR OLD STUDENTS' MENTAL THREE-DIGIT ADDITION

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In this study, 4th grade students' achievement and strategy use on three-digit addition tasks are presented. 78 students (40 boys, 38 girls, mean age 10 year 4 months) participated in the study. Students solved 8 tasks of various difficulties aiming to evoke the use of typical strategies revealed by previous research: stepwise, split, compensation, simplifying strategies, and indirect addition. The results show that students used the split strategy for the majority of tasks independently of how effectively that strategy could be used. There was no sign of using compensation, simplifying and indirect addition strategies. The results points to the potentials addition strategy trainings may have in developing students three-digit addition skills.

INTRODUCTION

The title of this paper paraphrases the title of Selter's (2001) work on success, methods and strategies of German elementary school children solving three-digit addition and subtraction. Success refers to students' achievement in terms of correct solution to mathematical, namely, addition problems. Methods of solution can take either written or oral computation forms. In the current study, only oral computation procedures are investigated. The term strategy remains implicit in the majority of recent articles on children's and adults' computation. However, a rather general definition given by Richard Mayer (2010, p. 164.) may serve well the purposes of the current study: "Strategies are general methods for planning and monitoring how to accomplish some task." In the case of mental arithmetical computations, strategies are therefore conscious planning and monitoring processes that can be used for solving a variety of different tasks.

The importance of research on elementary school children's success and strategies on mental computation can be supported not only by the widely recognized importance of mathematical skills (see e.g., Smith, 1999), but also by the challenges raised by research on adaptive strategy use. These two aspects are intertwined, and – from an educational point of view – there may be a bidirectional link between them. Developing expertise in mental computation may lead to a broad repertoire of calculation strategies, and at the same time enrichment of students' strategies may lead to better results both in correctness and the time needed for the solution. There is a growing body of evidence pointing to the importance of adaptive strategy use in mathematics (De Corte, Mason, Depaepe & Verschaffel, 2011).

Strategies in three-digit addition

Selter (2001) stated that there had been barely any research on addition and subtraction with three-digit numbers, except for a study by Fuson et al. (1997). In the past decade, some new findings have been reported, and besides investigating the achievement on and the strategies used for three-digit addition and subtraction, results of educational intervention programs have contributed to extend our knowledge of the topic. As for the categorization of strategies used for addition with three-digit numbers, there are different category systems using different labels for slightly different (or identical) strategies. The most recent one is provided by Heinze, Marschick and Lipowsky (2009) and is "denoted as an idealized because [it is] based on a mathematical systemization" (p. 592). There are five strategies listed by them:

- *stepwise strategy*: when the second addend is added in three steps. For example: 123+456=((123+400)+50)+6. This is called the "begin-with-one-number" method by Fuson et al. (1997).
- *split strategy*: adding first the hundreds, then the tens, and finally the ones. For example: 123+456=(100+400)+(20+40)+(3+6). This is called the "decompose hundreds-tens-and-ones" method by Fuson et al. (1997), and "htu (hundreds, tens, units)" strategy by Selter (2001).
- *compensation strategy*: one of the addend is rounded off to the nearest hundreds number. For example: 527+398=527+400-2. This is very similar to the *simplifying strategy* when both addends are changed by moving some from one of them to the other, e.g., 527+398=525+400. This latter strategy is called the "change-both-numbers" method by Fuson et al. (1997), and is labelled auxiliary or simplifying by Selter (2001).
- the strategy called *indirect addition* refers to a subtraction strategy when mental computation is executed like it was an addition task. For example: 701-698 is the number to be added to 698 in order to get 701.

All of the examples above were borrowed from Heinze, Marschick and Lipowsky's (2009) study. The tasks administered to students in the current investigation represent these four main bullet list categories. It means that although each three-digit addition task can be solved by any of the first three methods, and all three-digit subtraction tasks can be solved by means of "indirect addition", there are tasks that are especially suitable for effective use of the above-mentioned strategies.

Aims of the current study

The current paper presents results of a larger research project aiming at enriching students' mental computation strategy use. The research presented here can be considered as the pre-test phase of an intervention program. Due to the sample size and sample heterogeneity (in terms of SES-background and type of residence) the following research topics can yield generalizable data and results.

(1) Students' achievement on three-digit addition problems by means of mental computation, and in terms of correctness and the time needed for the solution.

(2) Students' errors during the mental computation process. These errors are often 'rational errors' (Ben Zeev, 1996), and may refer to a misused or inefficient strategy.

(3) Students' self-report of their strategy use.

(4) Inter-relations among achievement, errors and strategy use.

METHODS

Sample

The students involved were recruited from two different schools: one school is situated in a county seat town and the other in a village of Hungary. Both schools have two 4th grade classes, and the students come from rather diverse socio-economic background families. The sample comprised 78 students (40 boys and 38 girls). Their mean age was 123.92 months (10 years and 3.92 months).

Test and procedure

Eight tasks were developed for this investigation. There were six three-digit addition tasks and two three-digit subtraction tasks. The first task was considered as a warm-up one. Students had to compute the following operations:

(1) 342 + 235 = 577	(2) 143 + 426 = 569	(3) 702 + 105 = 807
(4) 284 + 202 = 486	(5) 527 + 398 = 925	(6) 498 + 256 = 754
(7) 701 – 694 = 7	(8) 646 – 583 = 63	

The first four tasks could be effectively solved either by the stepwise or the split strategies. The 5th and 6th ones were planned to evoke the compensation or simplifying strategies, while the last ones gave the opportunity for using the indirect addition strategy.

All tasks were printed on a separate A4 sheet of paper, and were handed over to the students. At the moment of handover, timing was started. Students saw the operation to be computed in a form like e.g., "342 + 235 =", and they were not allowed to write down anything to the paper.

The interviewers noted all erroneous answers (if any) to their answer sheet, and at the moment of hearing the right answer, they stopped the watch, and wrote down the time, then proceeded to the next task. The maximum time allowance for a task was 60 seconds.

After having completed all the eight task, they turned on the dictaphone, and asked the students to tell how each task was solved. The students could saw again the tasks while talking about their solution strategy. The key encouragement question in case of silence was: "What partial results did you have?"

Students were tested individually in a quiet, separated room of the school. Data collection was managed by three university students who were previously trained and then paid for their contribution. Data collection took place in the form of an interview, the protocol of which had been previously rehearsed during the training session with the interviewers.

RESULTS

Achievement in three-digit addition

The rate of correct solutions within the 60 second time limit is shown in Table 1, along with the average time needed for the correct solution. Please note that the first task can be considered a warming-up one.

Task	Rate of correct solutions (%)	Mean time (SD in parentheses)
342 + 235 = 577	94.9	13.35 (10.36)
143 + 426 = 569	97.4	10.95 (9.57)
702 + 105 = 807	98.7	5.53 (5.65)
284 + 202 = 486	100.0	8.39 (8.90)
527 + 398 = 925	70.5	24.14 (17.90)
498 + 256 = 754	69.2	22.02 (15.02)
701 – 694 = 7	52.6	24.37 (16.82)
646 – 583 = 63	50.0	28.28 (14.75)

Table 1: The rate of correct solutions yielded within 60 seconds, and mean response time (SD in parentheses) N = 78

The results suggest that the first four tasks were solved by almost everyone within a rather short time. However, the fifth and sixth tasks that would have been easily solved by the so-called compensation or simplifying strategies required much longer solution time, and about one third of the students failed to solve them. The two subtraction tasks proved to be even more difficult.

"Rational errors"

Students' erroneous answers were noted down. In some cases, there were several erroneous answers provided; in Table 2 only each student's first non-correct solution is considered (if there were any). Please note that only the incorrect answers given by at least 3 students (3.8%) are shown. Table 2 includes incorrect answers of those who later (within 60 seconds) gave the correct answer as well.

Task	The most frequent incorrect answers (relative frequency in parentheses)
342 + 235 = 577	5707 (5.1%); 5777 (3.8%); 587 (3.8%)
143 + 426 = 569	579 (3.8%); 590 (3.8%)
702 + 105 = 807	
284 + 202 = 486	
527 + 398 = 925	915 (6.4%); 625 (5.1%)
498 + 256 = 754	654 (10.3%)
701 – 694 = 7	193 (15.4%); 5 (7.7%); 16 (5.1%); 13 (3.8%); 93 (3.8%)
646 – 583 = 63	43 (9.0%); 143 (9.0%); 163 (6.4%); 57 (3.8%); 67 (3.8%); 137 (3.8%)

Table 2: The most frequent incorrect answers.

Students' self-report of mental computation strategies

Having completed all eight tasks, students reported of their strategy use task by task. In the simplest cases, the split (or decompose hundreds-tens-and ones) strategy was the most commonly used. The majority of them continued to use this strategy for the fifth and sixths tasks (albeit the compensation or simplifying strategies would have easily worked). For example in the case of Rozália (code number #106), the following self-report was received:

Rozália: 498 plus 256. I added in a way that 400 plus 200 is 600. 9 plus 5 is 14. This is 900… 600 and twelve. And 8 plus 2, no plus 6 is…

Finally she gave 625 as an answer which is not correct. Her self-report clearly indicates the insistence on using the split strategy. However, with these addends, the split strategy requires rather heavy memory load and fair computational skills. Another student (code number #125) tried to use the stepwise strategy in this task:

Boglárka: 498 plus 200 makes 698, plus 50 [pause], is 748, plus 6 [pause], is 713…

Neither Rozália nor Boglárka gave the correct answer in the first phase of the investigation. Rozália gave the same incorrect answer, Boglárka had 915 as her first erroneous answer. A third student (code number #126) had the correct answer before without any incorrect solution attempts, and he described his strategy in the following way:

Bendegúz: 498 plus 256. 498 plus 200 is 698; plus 50 is 748, plus 6 is 754.…

In this case, the stepwise strategy was correctly used. A final example is given for the sixth task showing a "pseudo" mental calculation strategy. This student (code number #221) solved all the previous tasks; too, in a way as they were written computational tasks.

Tamás: This was done in the same way, that is 8 plus 6 is 14. The remainder is 1, this is added to 9 to get 15…

Interviewer: You mean 9 + 1 + 5 = 15.

Tamás: Yes, and then again the remainder is 1, and then it will be 7.

This student solved the tasks in a way that he mentally put the addends one under another, and followed the algorithm learnt for written computations.

There was no sign of the compensation or simplifying strategy use in the case of the fifth and sixth tasks. Similarly, the last two tasks may have evoked the indirect addition method, but students (please note that half of them failed to give the correct answer within 60 seconds) used the split or stepwise strategies.

DISCUSSION

The results can be discussed along three lines. *Students' achievement* (success) on different types of three-digit addition tasks show that in the case of simpler tasks where there are less then ten tens, and less then ten ones in the addends, the solution is straightforward. In the tasks where the compensation or simplifying strategies might have given an easy solution, about one third of the students failed to give the correct answer. In the subtraction tasks, only half of them succeeded.

An *analysis of incorrect answers* shows that in some cases computational errors made while otherwise using an appropriate strategy led to incorrect answers.

In several cases, typical rational errors described in the literature can be observed. For example, 701 – 694 = 193 indicate that those students who had this solution, subtracted always the smaller digit from the bigger one: 7 – 6 = 1 for the hundreds, 9 – 0 = 9 for the tens, and 4 – 1 = 3 for the ones. This obviously erroneous strategy might reflect an early over-automatization of a wrong written subtraction algorithm.

Students' self-reports of their strategy use may point to two relevant phenomena. First, they are well aware of what they are doing when adding two numbers, at least in terms of the mathematical description of the process. They use the terms hundreds, tens, ones, remainder etc. Second, there are a rather limited variety of strategies used, at least the lack of the compensation and simplifying strategies, and the absence of indirect addition have been revealed. The narrow range of strategies used can be in part due to the perseverance effect known from the literature (Schillemans, Luwel, Bulté, Depaepe & Verschaffel, 2009).

According to Peters, De Smedt, Torbeyns, Ghesquière and Verschaffel (2010), adults tend to use the indirect addition for subtraction problems in rather reasonable cases, when the subtrahend was larger than the difference. Consequently, the indirect addition method can be labeled as a relatively late developmental stage in computational strategy use for subtractions.

Nevertheless, a kind of re-orchestration of the written computation algorithm for mental computation has been demonstrated. Therefore, this strategy might be considered as a real archetypical mental strategy.

IMPLICATIONS

There is an agreement in the literature on the need for greater flexibility in computations (Beishuizen & Anghileri, 1998). How it can be achieved raises several questions. One debate is about how teachers can become capable of fostering students' addition strategies. In Carpenter, Franke, Jacobs, Fennema and Empson's (1998) study, teachers themselves took part in a 3-year training program before the experiment. There are successful intervention studies with less demanding prerequisite resources, like that of Hiebert and Wearne's (1996) experiment. The second big issue is whether (and how) explicit addition strategies are taught. In Hiebert and Wearne's experiment "students were encouraged to develop their own procedures and to explain them to their peers" (p. 258). The debate on whether addition strategies should actively be taught to students or they can be left for spontaneous development is analyzed by Murphy (2004).

Our suggestion is – and this is in line with the results of the current investigation – that students should be actively taught to use a wide repertoire of addition strategies. Adaptive strategy use, i.e. when strategy choice is made according to task, individual and context variables, requires a range of possibly available strategies. While learning this strategy repertoire, students can constructively develop new strategies they have been never taught. Keeping in mind the educational goals of developing mathematical skills, fostering students' active and conscious strategy use in mental computation may well support the development of adaptive expertise.

Additional information

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