Reducing variance with sample allocation based on expected response rates in stratified sample designs

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Abstract

This paper demonstrates that the sample allocation that takes the expected response rates (ERRs) into account has certain advantages over other approaches in terms of reducing the variances of the estimates. The performance of the ERR allocation is assessed within the framework of stratified sampling by comparing the resulting variances with those obtained using the classical procedure of proportional to stratum size (PS) allocation and then applying post-stratification. The main theoretical tool is asymptotic calculations using the δ-method, which are complemented with extensive finite sample evaluations using various combinations of specific population parameters. The main finding was that within a stratified sample design, ERR allocation leads to lower variances than PS allocation, not only when the response rates are correctly specified but also under a wide range of conditions where the response rates can only be approximately specified in advance.

Keywords — sample allocation, response rate, delta-method

1 Introduction

To achieve an optimum balance between data collection costs and estimation efficiency (variance reduction), complex selection methods are typically required for the sampling design of household and individual surveys. Samples that are representative according to previously appointed variables may be obtained via a precise allocation of the sample sizes within different strata, if the relevant information is available both for the entire population, e.g., from a census, and also for every individual in a sampling frame, e.g., in a register. Generally, the proportional-to-stratum size (PS) allocation method (Larsen 2008) is used. However, the realised (observed) sample sizes within the strata tend to differ from the planned (allocated) ones. The larger the reluctance to participate within a stratum, the larger the difference between the planned and realised sample sizes (Stoop 2004). Previous field experiences and the analysis of current survey meta-data indicate that the overall increase in survey nonresponse does not equally apply to different population subgroups (Meyer et al. 2015, Osier 2016). The resulting distortion of sample composition is usually dealt with using post-stratification (Groves et al. 2009). It has been found that single-person households, renters and individuals outside of the labour force are less likely to participate in surveys than members of other social groups (Abraham et al. 2006, Meyer et al. 2015). This suggests that giving a larger proportional allocation to these groups may improve the realised sample. To be able to determine the exact proportions during the allocation procedure, estimates from previous surveys are needed. In case of item-nonresponse the expected historical response rates are easy to determine using publicly available survey data. In the case of unit-nonresponse, the contact data (or survey meta-data) are typically not available publicly, but survey organizations can use their own historical data.
This paper demonstrates that if specific response rates are available for different strata, the sample allocation that takes these into account has certain advantages over the PS allocation methods, not only when the response rates are precisely known but also when they are approximated. In fact, an allocation that takes the expected response rates (ERRs) into account results in lower variance than when adopting PS allocation. The remainder of the paper is organised as follows. First, we briefly introduce the PS procedure with post-stratification (section 2.1) before section 2.2 formally presents the method of ERR allocation. The relative performance of ERR allocation is assessed by comparing the variances in the resulting estimates in section 3. The asymptotic variances are calculated using the δ-method in section 4.1 and are then initially compared by assuming correctly specified response rates in section 4.2. Here, the assumed response rates are subject to random fluctuations, which are then corrected using post-stratification. In section 4.3, variance comparison is performed in terms of misspecified response rates, and the results of an extensive assessment using various combinations of specific population parameters are presented.

2 Sample Allocation

Let N denote the population size and let N_h $(h = 1, 2, \ldots, H)$, be the sizes of the strata relevant to the sampling procedure, with $N = N_1 + ... + N_H$. In a stratified random sample, a simple random sample of n_h elements is taken from each stratum h $(h = 1, 2, ..., H)$, with a total sample size of n elements.

When the survey aims to collect m responses, the response rate which characterizes the population needs to be taken into account in deciding about the attempted sample size. Of course, such decisions should be made based on the true response rate, but it is rarely known. Thus, the ERR, say r, is used which is based on former experience. Then, a total of $n = m/r$ observations are allocated.

2.1 Allocation Proportional to Size

In the case of PS allocation, let n_h^{PS} $(h = 1, 2, ..., H)$ denote the subsample size within stratum h. The sampling fraction n_h^{PS}/N_h is specified to be the same for each stratum and thus

$$n_h^{PS} = \frac{1}{r}\frac{N_h}{N}m \qquad \textit{h=1,...,H,} \tag{1}$$

which implies that the overall sampling fraction n/N is the same as the fraction taken from each stratum. The total allocated sample size is then as follows:

$$n^{PS} = m\sum_{h=1}^{H}\frac{N_h}{N}\frac{1}{r} = \frac{m}{r} \tag{2}$$

2.2 Allocation Based on Different ERRs

In the case of ERR allocation, let n_h^{ERR} $(h = 1, 2, ..., H)$ denote the allocated subsample size within stratum h. Let r_h $(h = 1, 2, ..., H)$ denote the stratum-specific ERRs, which are also assumed to be population parameters. Clearly,

$$r = \sum_{h=1}^{H}\frac{r_h N_h}{N}.$$

In ERR allocation, the allocated sample size in each stratum n_h^{ERR} is specified using, instead of the population level ERR, the stratum-specific ERRs. The allocated sample size in each stratum is

$$n_h^{ERR} = \frac{1}{r_h}\frac{N_h}{N}m \qquad \textit{h=1,...,H.} \tag{3}$$

Consequently, the total allocated sample size is

$$n^{ERR} = m\sum_{h=1}^{H}\frac{N_h}{N}\frac{1}{r_h}. \tag{4}$$

3 Estimation Procedures

To assess the ERR and PS allocations, the variances of the estimates obtained will be compared in Section 4 using the δ-method. Here, we describe the estimating procedures.

The main aim is to estimate the proportion of respondents within a given population who would choose a fixed category, e.g., 'yes', of a given close-ended question based on observed samples in terms of both ERR and PS allocations. In both cases, post-stratification is applied prior to the estimation to appropriately reproduce the relative sizes of the strata in the population (Groves et al. 2009).

It is assumed that responding to the survey is probabilistic and occurs in stratum h with probability p_h and is independent from the true answer to the question of interest. It should be noted that the r_h response rates represent the expectation of the researcher based on previous knowledge and that p_h is the true probability of responding. The probability of nonresponse[1] is therefore $1 - p_h$ in each stratum h. Thus, the data are missing completely at random (Rubin 1976). The probability of a 'yes' response is assumed to be q_h in each stratum h.
Under the previous assumptions, the complete data for each stratum, would be the observation of a variable $\mathbf{Z}_h$ with the following four components:

1. Z_{h1} counts the number of cases when the selected respondent did answer and the answer was 'yes'.
2. Z_{h2} counts the number of cases when the selected respondent did answer and the answer was 'no';
3. Z_{h3} counts the number of cases when the selected respondent did not answer and the answer would have been 'yes';
4. Z_{h4} counts the number of cases when the selected respondent did not answer and the answer would have been 'no';

Within stratum h, $\mathbf{Z}_h$ has a multinomial distribution with parameters n_h and $\mathbf{q}_h$, where n_h is the allocated sample size for stratum h, which depends on the type of allocation, and under the assumed independence of the true response from whether or not the answer is received,

$$\mathbf{q}_h = (p_h q_h,\ p_h(1-q_h),\ (1-p_h)q_h,\ (1-p_h)(1-q_h)). \tag{5}$$

The observed sample size is $o_h = Z_{h1} + Z_{h2}$ in stratum h, and for each observation, a post-stratification weight of

$$\frac{\frac{N_h}{N}\sum_{i=1}^{H} o_i}{o_h} \qquad h=1,\ldots,H.$$

[1]For the present argument, it is irrelevant whether nonresponse applies to the entire survey because of no-contact or refusal or only to the current question.

is applied, which adjusts the fraction of the sample size in stratum h to be equal to the population fraction of stratum h but does not change the total observed sample size. After the weight is applied, Z_{h_j} is replaced by

$$\frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h_j}, \qquad j=1,2,3,4 \qquad h=1,\ldots,H.$$

As such, the natural estimator for the fraction of 'yes' responses in stratum h is

$$\hat{q}_h = \frac{\frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h1}}{\frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h2} + \frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h1}} = \frac{Z_{h1}}{Z_{h1}+Z_{h2}}, \tag{6}$$

which is the relative frequency of 'yes' responses among all responses observed in stratum h. It should be noted that as $\hat{q}_h$ refers to a single stratum, the post-stratification weights are cancelled out because they are identical within each stratum.

For the entire sample, the $\mathbf{Z}_h$ variables have a product multinomial distribution. The estimator for the fraction of 'yes' responses in the total sample is

$$\begin{aligned}\hat{q} &= \frac{\sum_{h=1}^{H} \frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h1}}{\sum_{h=1}^{H}(\frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h2} + \frac{N_h}{N} \cdot \frac{\sum_{i=1}^{H}(Z_{i1}+Z_{i2})}{Z_{h1}+Z_{h2}} Z_{h1})} \\ &= \frac{1}{N}\sum_{h=1}^{H} N_h \frac{Z_{h1}}{Z_{h1}+Z_{h2}} \end{aligned} \tag{7}$$

which is the weighted fraction of 'yes' responses among all responses observed in the total sample. Here, post-stratification has the effect of weighting the stratum-specific estimates in terms of their population weights.

4 Variance Comparison

In this section we compare the variances of the estimates derived from the ERR and PS allocations using the δ-method.

4.1 The δ-Method

Theorem 4.1 (Multidimensional δ-method)**.** *Let $X_n, n = 1, 2, \ldots$ be a sequence of k-dimensional vector-valued random variables such that,*

$$\sqrt{n}(X_n - a) \xrightarrow{d} Y, \tag{8}$$

where $a \in \mathbb{R}^k$ and $Y \sim N(0, \Sigma)$. If a function $f : \mathbb{R}^k \to \mathbb{R}^l$ is differentiable at $a \in \mathbb{R}^k$, and D is its $l \times k$ matrix of partial derivatives with $d_{ij} = \frac{\partial f_i(a)}{\partial x_j}$, then

$$\sqrt{n}(f(X_n) - f(a)) \xrightarrow{d} Z, Z \sim N(0, D\Sigma D^T). \tag{9}$$

The proof of the Theorem can be found in Bishop et al. (2007) or Lehmann & Romano (2005).

As condition (8) holds for the multinomial distribution, theorem 4.1 may be applied within each stratum. The estimator for the proportion of 'yes' responses in the total population (7) in section 3, is the weighted fraction of 'yes' responses among all responses observed across all strata.

In one strata, when omitting the index h, the estimation function is $f(Z) = \frac{Z_1}{Z_1+Z_2}$ and the partial derivatives are as follows:

$$\frac{df}{dZ_1} = \frac{Z_2}{(Z_1+Z_2)^2} \qquad \frac{df}{dZ_2} = -\frac{Z_1}{(Z_1+Z_2)^2}$$
$$\frac{df}{dZ_3} = 0 \qquad \frac{df}{dZ_4} = 0$$

The partial derivative vector D with the components evaluated above at the expectations $E(Z_1) = np_1$ and $E(Z_2) = np_2$, is

$$D = \begin{bmatrix} -\frac{np_1}{(np_1+np_2)^2} \\ \frac{np_2}{(np_1+np_2)^2} \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

As $\boldsymbol{Z}$ has a multinomial distribution with the probability vector given in (5), its covariance matrix is

$$\Sigma = \begin{bmatrix} np_1(1-p_1) & -np_1p_2 & -np_1p_3 & -np_1p_4 \\ -np_2p_1 & np_2(1-p_2) & -np_2p_3 & -np_2p_4 \\ -np_3p_1 & -np_3p_2 & np_3(1-p_3) & -np_3p_4 \\ -np_4p_1 & -np_4p_2 & -np_4p_3 & np_4(1-p_4) \end{bmatrix} \tag{11}$$

Then, one has the following results for the asymptotic variances.

Theorem 4.2 (Variance of the estimates)**.** *Let the population size be N, and let the population be divided into H strata of respective sizes of N_h, $(h = 1, .., H)$. Let m be the intended total sample size, r the ERR in the entire population and r_h the respective ERRs in the strata. The true population proportion of those possessing the characteristics of interest is denoted by q_h, which is the parameter to be estimated in each stratum h. Finally, let p_h be the true response rate in stratum h. Then, the asymptotic variances of the estimates obtained from samples based on PS and ERR allocations, with post-stratification applied, are as follows.*

$$V^{PS}(\hat{q}) = \frac{1}{Nm}\sum_{h=1}^{H} N_h q_h(1-q_h)\frac{r}{p_h} \tag{12}$$

$$V^{ERR}(\hat{q}) = \frac{1}{Nm}\sum_{h=1}^{H} N_h q_h(1-q_h)\frac{r_h}{p_h} \tag{13}$$

Proof. As stratified sampling leads to a product multinomial distribution (see, e.g., Rudas, 2018), theorem 4.1 is applied for each stratum. Then, the asymptotic variance is obtained as follows:

$$D^T\Sigma D = \frac{(p_h q_h)(p_h(1-q_h))^2 + (p_h q_h)^2(p_h(1-q_h))}{n_h((p_h(1-q_h)+p_h q_h)^4}$$

$$= \frac{p_h^3 q_h - p_h^3 q_h^2}{n_h p_h^4} = \frac{p_h^3 q_h(1-q_h)}{n_h p_h^4} = \frac{q_h(1-q_h)}{n_h p_h}$$

As the allocated stratum-specific sample sizes n_h are different in the PS and ERR allocations, different asymptotic variances will be obtained.
In the case of PS allocation, using (1),

$$V_h^{PS}(\hat{q}) = \frac{q_h(1-q_h)}{n_h^{PS} p_h} = \frac{q_h(1-q_h)}{\left(\frac{1}{r}\frac{N_h}{N}m\right)p_h},$$

whereas for the total sample, the following is obtained:

$$V^{PS}(\hat{q}) = \frac{1}{N^2}\sum_{h=1}^{H} N_h^2 \hat{V}_h(\hat{q}) = \frac{1}{N^2}\sum_{h=1}^{H} N_h^2 \frac{q_h(1-q_h)}{\left(\frac{1}{r}\frac{N_h}{N}m\right)p_h}$$

$$= \frac{1}{Nm}\sum_{h=1}^{H} N_h q_h(1-q_h)\frac{r}{p_h}.$$

The asymptotic variance in stratum h in case of the ERR allocation with (3) is

$$V_h^{ERR}(\hat{q}) = \frac{q_h(1-q_h)}{n_h^{ERR} p_h} = \frac{q_h(1-q_h)}{\left(\frac{1}{r_h}\frac{N_h}{N}m\right)p_h}$$

whereas for the total sample, the following is obtained:

$$V^{ERR}(\hat{q}) = \frac{1}{N^2}\sum_{h=1}^{H} N_h^2 \hat{V}_h(\hat{q}) = \frac{1}{N^2}\sum_{h=1}^{H} N_h^2 \frac{q_h(1-q_h)}{\left(\frac{1}{r_h}\frac{N_h}{N}m\right)p_h}$$

$$= \frac{1}{Nm}\sum_{h=1}^{H} N_h q_h(1-q_h)\frac{r_h}{p_h}$$

□

In terms of a general comparison of the variances obtained above, the difference of the variances for the ERR and PS allocations, disregarding a positive constant multiplier, may be written as a weighted sum of the quantities

$$\frac{r_h - r}{p_h}, \tag{14}$$

with weights equal to

$$N_h q_h (1 - q_h). \tag{15}$$

Large negative values and small positive values of (14) point to a better performance of the ERR allocation than of the PS allocation. The value of (14) is sometimes negative and sometimes positive, as r is a weighted average of the r_h values. Negative values of (14) are obtained when r_h is smaller than average and they will be made larger if p_h is small. Positive values of (14) are obtained when r_h is greater than average and will be made smaller if p_h is large. Thus, (14) may be viewed as a measure of how well the ERRs r_h approximate the true response rates p_h, with large negative and small positive values meaning better approximation.
Consequently, $V^{ERR}(\hat{q}) - V^{PS}(\hat{q})$ may be seen as a weighted average of how well r_h approximates p_h, as measured by (14), where the weights are the total variances of the strata, given in (15). The better the approximation, in particular in the strata with large total variances, the better the ERR allocation performs relative to the PS allocation.
In the next two subsections, more detailed comparisons are given.

4.2 Comparison Under Correctly Specified Response Rates

In this section, we prove that in the case of correctly specified response rates ($r_h = p_h$), the variance of the estimate based on the ERR allocation is less than or equal to that derived from the PS allocation:

Theorem 4.3 (Relationships among the variances)**.** *Let $\hat{V}^{PS}(\hat{q})$ be the total variance of the estimates based on a sample drawn via the PS allocation given in (12), and let $\hat{V}^{ERR}(\hat{q})$ be the total variance of the estimates based on a sample drawn by the allocation based on different ERRs, as given in (13). If the observed response rates are equal to the ERRs, then,*

$$V_{h_{ERR}}(\hat{q}) \leq V_{h_{PS}}(\hat{q}) \tag{16}$$

Proof. If $r_h = p_h$, the response rates are correctly specified, and then r is also the average ERR among all strata. Because N, N_h and q_h are population parameters, and m is a fixed constant, it is enough to see that

$$\sum_{h=1}^{H} N_h q_h(1-q_h) \leq \sum_{h=1}^{H} N_h q_h(1-q_h)\frac{\frac{1}{H}\sum_{j=1}^{H} p_j}{p_h}$$

or

$$\frac{1}{\sum_{h=1}^{H} w_h}\sum_{h=1}^{H} w_h \frac{1}{\frac{1}{H}\sum_{j=1}^{H} p_j} \leq \frac{1}{\sum_{h=1}^{H} w_h}\sum_{h=1}^{H} w_h \frac{1}{p_h}.$$

As the left hand side is the weighted harmonic mean of the values $\frac{1}{p_1}, \ldots, \frac{1}{p_H}$, and the right-hand side is the weighted arithmetic mean of the same numbers, by inequality between these means (Bullen 2003) demonstrates that the claim of the theorem is true.

□

4.3 Comparison Under Misspecified Response Rates

In this section, we compare the ERR and PS allocation methods under misspecification that is, when the true response rates differ from the ERRs used in the sample allocation ($p_h \neq r_h$). The variances were compared for all combinations of parameter values with a fixed number of strata, $H = 3$. Specifically, all possible combinations of the following parameter values were considered: all possible combinations of the values $\{0.1, 0.3, 0.5, 0.7, 0.9\}$ for the true response rates $\{p_1, p_2, p_3\}$ and for the ERRs $\{r_1, r_2, r_3\}$. The parameter to be estimated in every stratum h ($h = 1, 2, 3$) was given values between 0 and 1, with an increment of 0.05. The size of the population $N = 10^7$,the sizes of the strata $N_1 = 2*10^6$, $N_2 = 3*10^6$, $N_3 = 5*10^6$, and the desired total sample size $m = 1000$ were fixed. With the different choices, a total of 15.625.000 different sets of parameters were defined. The calculations were conducted using the R statistical environment.

Figure 1 shows the comparison of the variances of the estimates obtained using ERR and PS allocations. The comparison is given in terms of the total absolute misspecification of the response rates, $\sum_{h=1}^{H}|r_h - p_h|$ *(x-axis)* and of the total absolute distance of the ERRs $\{r_1, r_2, r_3\}$ from their weighted average, $\sum_{h=1}^{H}|r_h - r|$ *(y-axis)*.

The magnitude of the misspecification of the response rates appeared to have a greater impact on the relative performances of the two allocation procedures. When the total absolute misspecfication was less than 0.3, the ERR allocation almost always performed better. Meanwhile, the total absolute distance of the ERRs from their weighted average appears to have had a small and non-systematic effect.

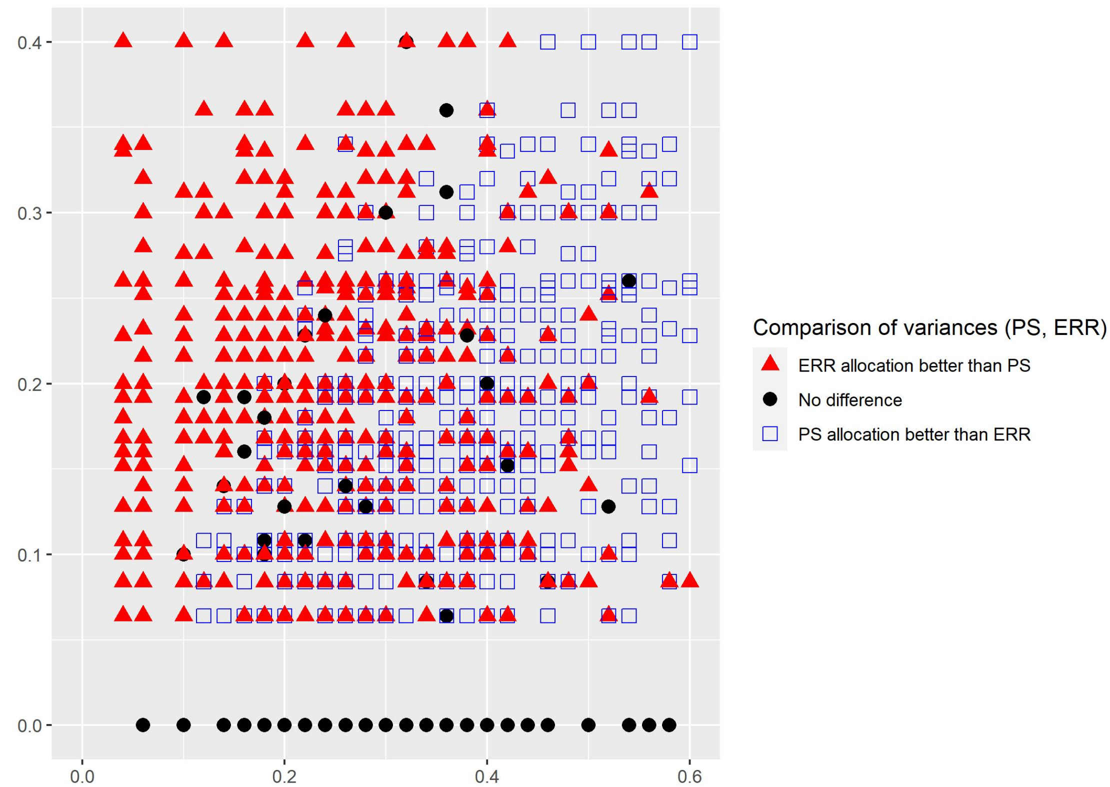

Figure 1: Comparison of the variances in the estimates obtained using ERR and PS allocations, in terms of the total absolute misspecification of the response rates *(x-axis:* $\sum_{h=1}^{H} |r_h - p_h|$*)* and the total absolute distance of the ERRs from one weighted average *(y-axis:* $\sum_{h=1}^{H} |r_h - r|$*)*.

Figure 2 shows the comparison of the variances of the estimates obtained using ERR and PS allocations in terms of the total absolute misspecification of the response rates, $\sum_{h=1}^{H} |r_h - p_h|$ *(x-axis)* and the difference in the absolute deviances of the response rates from their respective weighted averages, $\sum_{h=1}^{H}(|r_h - r| - |p_h - p|)$ *(y-axis)*.

When the total absolute misspecification of the response rates was lower than 0.3, the ERR allocation yielded mostly smaller variances. Meanwhile, in the range of $0.3 - 0.4$, the two allocations performed equally well. Most notably, an equal precision can be expected in the extreme areas of the plot.

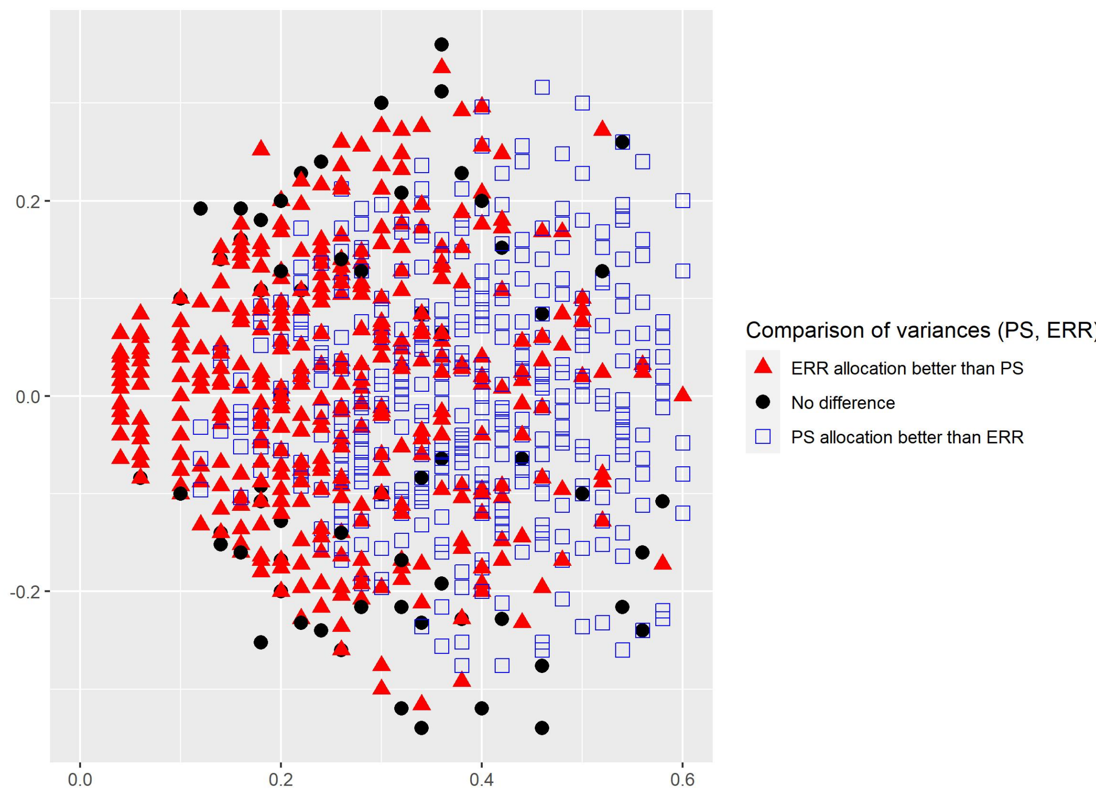

Figure 2: Comparison of the varianc in the estimates obtained using ERR and PS allocations, in terms of the total absolute misspecification of the response rates *(x-axis:* $\sum_{h=1}^{H} |r_h - p_h|$*)* and the difference in the absolute deviations of the response rates from their respective weighted averages *(y-axis:* $\sum_{h=1}^{H}(|r_h - r| - |p_h - p|)$*)*.

Figure 3 shows the comparison of the variances of the estimates obtained using the ERR and PS allocations in terms of the total absolute distance of the response rates from their weighted average $\sum_{h=1}^{H} |r_h - r|$ *(x-axis)* and the difference in the absolute deviations of the response rates from their respective weighted averages $\sum_{h=1}^{H}(|r_h - r| - |p_h - p|)$ *(y-axis)*.
Here, the total absolute misspecification shown on the x-axes of Figures 1 and 2 was disregarded but was clearly more influential than the characteristics shown in Figure 3. When the difference between the total absolute deviations of the expected rates and the ERRs was less than approximately half of the latter, the ERR allocation always performed better, irrespective of whether or not the individual response rates were correctly predicted.

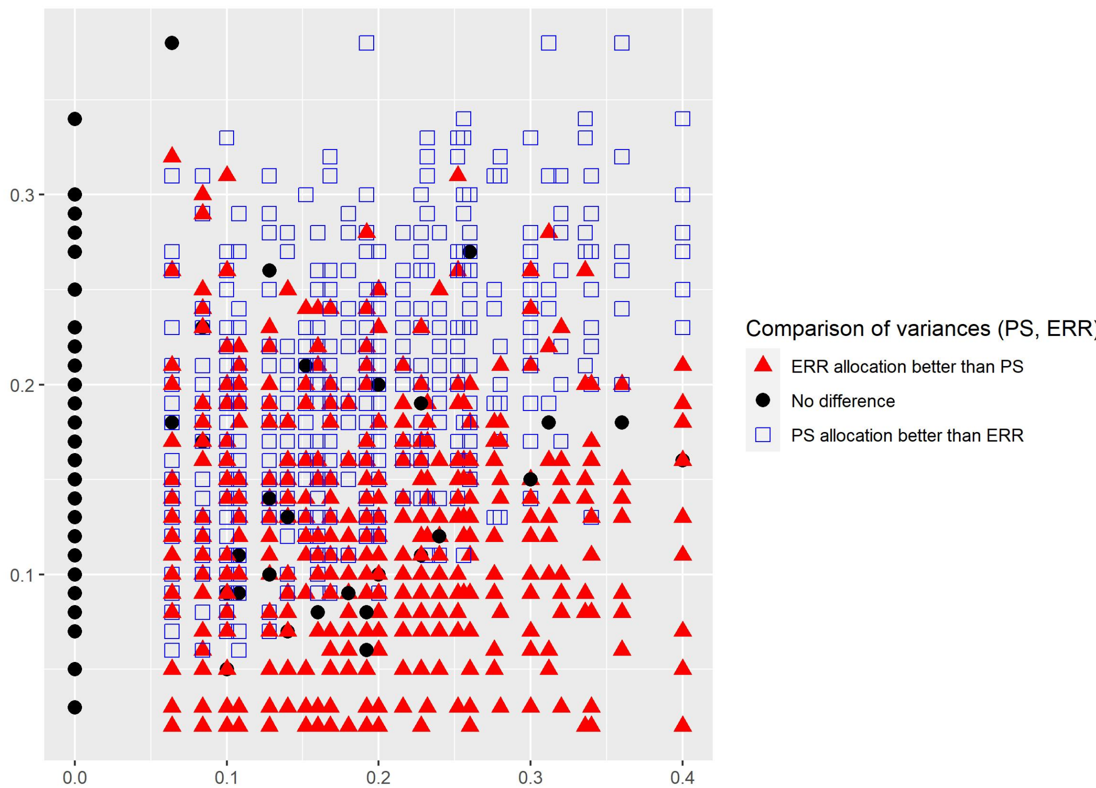

Figure 3: Comparison of the variances of the estimates obtained using the ERR and the PS allocations in terms of the total absolute distance of the response rates from their weighted average *(x-axis:* $\sum_{h=1}^{H} |r_h - r|$*)* and the difference in the absolute deviations of the response rates from their respective weighted averages *(y-axis:* $\sum_{h=1}^{H}(|r_h - r| - |p_h - p|)$*)*.

5 Conclusion

In this paper, we demonstrated how ERRs can be utilised in the sample allocation procedure. In the process, we introduced an ERR allocation procedure where the stratum-specific ERRs were used to determine the allocated sample sizes within each stratum. We assessed the method by comparing it with a standard proportional allocation method (PS) where stratum-specific response rates are not used. The assessment of the sample allocation procedures used a comparison of the resulting asymptotic variances based on the δ-method when assuming the expected responses were equal to the true responses. In the case of misspecified response rates, extensive enumeration was used. The first finding of the paper is that if the stratum-specific response rates are correctly specified, ERR allocation performs better than PS allocation in terms of the variances of the estimates. In practice, however, it may be difficult to precisely estimate the stratum-specific response rates prior to sampling. In such cases, approximate response rates based on experience need to be used. On the basis of the numerical results obtained:

(a) ERR allocation outperforms PS allocation if the total absolute distance of the ERRs from the true response rates is moderate,

(b) The total absolute distance of the ERRs from their weighted average and the total absolute distance of the true response rates from their weighted average do not appear to affect the aforementioned finding,

(c) When the difference between the total absolute deviations of the expected and of the true response rates is less than approximately half of the latter, ERR allocation always performs better, irrespective of whether or not the individual response rates were correctly predicted.

In this paper, statistics other than proportions were not investigated, because of the problematic nature of the distributional assumptions including the homogeneity of variances which would have to be made. However, given that the proportions were obtained as averages of specific indicators, we expect similar results to hold in more general cases. The allocation method described in this paper may be applied to other sampling designs which include separate allocation steps for subsamples. These designs include multistage sampling where sample allocations taking into consideration the different ERRs in the different primary sampling units may be applied.

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