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Detailed response to reviewer's comments after 3nd revision

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At this time again we would like to thank the reviewer's thorough work on our paper submitted to the International Journal of Production Economics that resulted in the fact that he / she strengthened that we had managed to built in all his / her comments received during the revision process.

Modeling and long-term forecasting demand in spare part logistics businesses

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Modeling and Long-term Forecasting Demand in Spare Parts Logistics Businesses

Abstract

In order to provide high service levels, companies competing in the electronics manufacturing sector need to ensure the availability of spare parts for repair and maintenance operations. This paper examines the purchase life-cycles of electronic spare parts and presents a new way of modeling and forecasting spare part demand for electronic commodities in the spare parts logistics services. The presented modeling methodology is founded on the assumption that the purchase life-cycles of spare parts can be described by a curve with short term fluctuations around it. For this purpose, a flexible Demand Model Function is introduced. The proposed forecasting method uses a knowledge discovery-based approach that is built upon the combined application of analytic and soft computational techniques and is able to indicate the turning points of the purchase life-cycle curve. The novelty lies in the fact that the model function has certain characteristics which support describing and interpreting the demand trend as a function of time. The application of our methodology is mainly advantageous in long-term forecasting, it can be especially useful in supporting purchase planning decisions in the ramp-up and declining phases of purchase life-cycles of product specific spare parts. A demonstrative example is used to illustrate the applicability of the proposed methodology. Its forecasting capability is compared to those of some widely

applied methods in business practice. From the results, the new method may be viewed as a viable alternative spare part demand forecasting technique in spare part logistics sector.

Keywords: spare part logistics; electronic aftermarket services; purchase life-cycle forecasting; knowledge discovery; clustering time series

1. Introduction

Customers have rising expectations concerning the quality and reliability of electronic products and associated services. As the in-warranty and out-of-warranty repairs play a dominant role, maintenance processes and the level of aftermarket services are significant factors of competitiveness in the electronic industry. In accordance with that, a well-established spare part management system is an effective way to enhance customer loyalty. Spare part demand forecasting is of crucial importance for maintenance systems, however, it is a complex issue due to the following reasons: in case of most electronic products the number of managed spare parts may often be high (Cohen and Agrawal, 2006), spare part demand patterns are usually lumpy or intermittent (Boylan and Syntetos, 2010), high responsiveness is required (Murphy et al., 2004), and there is a risk of spare part obsolescence (Solomon et al., 2000).

Spare parts can be characterized by their own life-cycles which is associated with the life-cycle of the final products that utilize them (Fortuin and Martin, 1999). Spare part life-cycle can be divided into three different phases with special characteristics for spare part demand (Fortuin, 1980). The purchase life-cycle (PLC) curve of an electronic spare part typically consists of three characteristic phases: an increasing first phase, a quasi-constant second phase and a declining third phase. Figure 1 depicts an example for the time series D_t that represents weekly demand for a spare part ($t = 1, 2, \ldots, 250$).

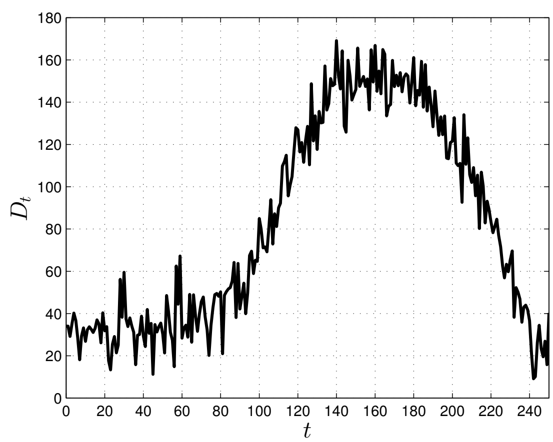

Figure 1: A typical demand time series of a spare part

In this paper a new way of modeling and forecasting electronic spare part demand is addressed and discussed. Stipan et al. (2000) points out that the commodity nature of modern electronic products dictates their operational life. Groen et al. (2004) emphasize that already available reliability data of different, yet similar products could be utilized for products under (re)desing by considering that these will typically have similar reliability characteristics. Therefore, reliability-related and technological data can be taken into account when estimating reliability features of new products. Following this, we may assume that the reliability characteristics of electronic spare parts of the same commodity are similar. The trend curves of time series representing the demands for spare parts can be considered as purchase life-cycles (PLC) of spare parts.

The introduced comparative forecasting methodology regarding lumpy spare part demand is based on knowledge discovery techniques. By combining analytic and fuzzy clustering techniques, spare part demand for new electronic products is forecasted based on the life-cycles of spare part demands of end-of-life (EOL) products. The paper investigates the life-cycles of spare parts that are supplied to the repair network by companies which provide the so-called spare part logistics (SPL) as a service. The method presented in this paper can be used to typify the PLC curves of EOL spare parts that belong to the same commodity category (e.g. motherboards, power supply units etc.). The presented modeling methodology is founded on the assumption that the purchase life-cycles of spare parts can be described by a curve with short-term fluctuations around it, and this model curve has the following characteristics:

(i) the curve is unimodal; that is, first it is increasing, then comes a plateau and finally it is decreasing

(ii) the curve can have maximum two inflexion points, one in the increasing and one in the decreasing phase

(iii) the curve can be zero or positive, both at the start and at the end

(iv) transformations allow this curve to fit required heights and locations.

The remaining of the paper is organized as follows. In Section 2, we report some related works encountered in the literature. Section 3 describes the methodology framework. Section 4 presents the modeling methodology of demand time series. Section 5 presents and industrial application and the goodness of the forecasting methodology is evaluated. In Section 6, main findings are discussed and some conclusive remarks and limitations of the study are pointed out.

2. Literature review

Due to the rapid progress in the electronic industry, new electronic products are constantly being launched to the market, the time of which can be measured in weeks and months rather than years, which results in shortening product life-cycles and delivery times. As a result, a typical consumer electronic product may go through all of its life-cycle stages within a year or less. Accordingly, the final order is now typically placed within a year after production kick-off (Pourakbar et al., 2012; Teunter and Fortuin, 1999). Shortening innovation cycles result in shortening production periods which means that in case of an increasing number of durable products original equipment manufacturers (OEMs) must provide spare parts for legal and service reasons. This end-of-life service period may last for many years (Teunter and Fortuin, 1999). Therefore, trends in technological lifetimes, particularly that of electronic parts are important to OEMs that must perform long support life applications (Sandborn et al., 2011). Forecasting the expected demand for a certain period of time with one or more spare parts is a relevant target in an organization dealing with spare part logistics.

The above mentioned phenomena require the management of demand and inventory also for parts for which historical demand or failure data are not available (Boylan and Syntetos, 2010). Spare part classification, spare part management and spare part demand forecasting is a hot issue in the relevant literature (Gajpal et al., 1994; Huiskonen, 2001; Boone et al., 2008; Boylan and Syntetos, 2010; Kennedy et al., 2002; Bacchetti and Saccani, 2012). Many studies focusing on spare part demand have resulted in specific methods in the last decades.

Time series demand forecasting methods have been widely applied to spare parts. Traditional time series methods are usually highly dependent on historical data, which can be incomplete, imprecise and ambiguous. These uncertainties are likely to hinder forecasting accuracy, thus limiting the applicability of these methods. Traditional forecasting techniques can deal with many forecasting cases, but cannot solve forecasting problems in which historical data are given in linguistic values (Hwang et al., 1998). Fuzzy forecasting approaches are capable of dealing with vague and incomplete time series data under uncertain circumstances (Song and Chissom, 1993, 1994; Chen, 1996; Chen and Chung, 2006; Chen and Chang, 2010; Egrioglu et al., 2011; Chen and Chen, 2011; Wang et al., 2013, 2014; Lu et al., 2014).

Neural networks have also emerged as an alternative tool for modeling and forecasting due to their ability to capture the non-linearity in the data (Chen et al., 2010a,b; Kourentzes, 2013). Recent research activities in this area and successful forecasting applications suggest that neural networks can also be an important alternative for time series forecasting and are able to compete with linear models of time series (Dong and Pedrycz, 2008; Mukhopadhyah et al., 2012; Gutierrez et al., 2008; Hua and Zhang, 2006). Shortcoming of neural network models, however, is the large amount of training data.

The combined application of neural networks and fuzzy systems could provide better results than classic regression models and neural networks in time series prediction (Armano et al., 2005; Huarng and Yu, 2006; Chen and Wang, 2012). A distinct property of the neuro-fuzzy approaches is that they are able to indicate the turning points of the purchase life-cycle curve, while traditional statistical techniques lack this property.

In the field of spare part demand forecasting there is still no consensus on which is the best forecasting method for spare parts (Bacchetti and Saccani, 2012). Only very few studies propose criteria to differentiate the forecasting methods for different items and only a few papers deal with the practical applicability of methods to real cases for spare part management (Boylan and Syntetos, 2008).

Most of the above mentioned fuzzy time series methods provide reasonable accuracy over short periods of time, but the accuracy of time series forecasting diminishes sharply as the length of forecasting increases (Li et al., 2010). Nevertheless, there is an increasing need for long-term forecasting Simon et al. (2005), which is difficult to achieve because information is unavailable for the unknown future time steps. Li et al. (2010) propose a new method called deterministic vector long-term forecasting (DVL). Wang et al. (2015) propose a forecasting model combining the modified fuzzy c-means and information granulation for solving the problem of long-term prediction with time series. Kaushik and Singh (2013) apply long-term forecasting with fuzzy time series and neural networks.

The installed base of a product, that is, the number of products still in use can also be utilized to obtain forecasts (van der Heijden and Iskandar, 2013; Jalil et al., 2011; Dekker et al., 2013) An interesting installed-based approach to spare part demand modeling was provided by Kim et al. (2017) with the ability to capture the turning point of the purchase life-cycle curve.

The primary aim of this paper is to develop a time series method that is applicable to long-term forecasting and takes into account the uncertainty present in the time series under investigation. Taking the PLC curves of spare parts into consideration the trends of time series and the prediction of the turning points between the successive characteristic phases of the purchase life-cycle are of crucial importance in the long run.

3. The methodology framework

In this section, our method consisting of two main phases, the knowledge discovery and the knowledge application phase is introduced. In the knowledge discovery phase, a parametric *demand model function (DMF)* is fitted to each full historical demand times series of end-of-life spare parts. The *demand models (DMs)* of purchase life-cycle curves of end-of-life spare parts are the fitted demand model functions. The demand models are transformed to *standardized demand models (SDMs)* that may be viewed as primitives which represent the entire set of the studied purchase life-cycle curves. Since each parameter of a primitive has a geometric interpretation, that is, parameters of a primitive determine the shape of its curve. In the next step, the primitives are clustered based on their parameters. Clustering results in *cluster characteristic SDMs* that represent the typical standardized demand models. The cluster characteristic SDMs may be viewed as a knowledge base discovered from historical demand times series of EOL spare parts. Once the typical SDMs have been identified, the knowledge that they represent may be used to predict demand for active spare parts. The active spare parts are the ones for which there is a current demand. In the knowledge application phase, we describe how the cluster characteristic SDMs can be used for forecasting purposes.

The main steps of the knowledge discovery and knowledge application phases are described in the following subsections.

3.1. Knowledge discovery phase

Inputs. We assume that we have the time series $D_{i,t_1}, D_{i,t_2}, \ldots, D_{i,t_{n_i}}$, each of them represents the full purchase life-cycle of an end-of-life spare part, that is, each time series contains the full historical demand for an EOL spare part $(i = 1, 2, \ldots, m)$. Based on practical considerations, which we will discuss in our demonstrative example, the demand values $D_{i,t_1}, D_{i,t_2}, \ldots, D_{i,t_{n_i}}$ for the ith spare part are taken on weekly basis. It allows us to use the simplified $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$ notation for the time series $D_{i,t_1}, D_{i,t_2}, \ldots, D_{i,t_{n_i}}$.

Step 1. Fitting a demand model function to each of the $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$ historical demand time series of EOL spare parts $(i = 1, \ldots, m)$.

Step 2. Transforming the demand models to standardized demand models the domains and ranges of which is the interval $[0, 1]$.

Step 3. Clustering the standardized demand models based on their parameters by applying fuzzy c-means clustering. The clustering results in the cluster characteristic (typical) SDMs.

3.2. Knowledge application phase

Step 4. Predicting demand for active spare parts using the typical SDMs and the known demand history of active spare parts.

Output. Long-term forecast for active spare parts demand.

4. Modeling demand time series

4.1. Construction of demand model function

The parametric model function that we wish to fit to each historical demand time series $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$ of EOL spare parts ($i = 1, \ldots, m$) is based on the following $g(x)$ function.

$$g_{\mu,\omega} : [0,1] \to [0,1], x \mapsto g_{\mu,\omega}(x)$$

$$g_{\mu,\omega}(x) = \begin{cases} 0, & \text{if } (x = 0 \text{ and } \omega > 0) \\ & \text{or } (x = 1 \text{ and } \omega < 0) \\ \dfrac{1}{1 + \left(\frac{\mu}{1-\mu}\frac{1-x}{x}\right)^{\omega}}, & \text{if } 0 < x < 1, \omega \neq 0 \\ 1, & \text{if } (x = 0 \text{ and } \omega < 0) \\ & \text{or } (x = 1 \text{ and } \omega > 0) \end{cases}, \tag{1}$$

where $0 < \mu < 1$.

$g_{\mu,\omega}(x)$ is derived from Dombi's kappa function that can be used as a unary operator in fuzzy theory (Dombi, 2012a,b). It can be seen that function $g_{\mu,\omega}(x)$ is monotonously increasing from 0 to 1 if the parameter ω is positive, and it is monotonously decreasing from 1 to 0 if ω is negative. The function has the value of 0.5 in the locus μ. As

$$\left.\frac{\mathrm{d}g(x)}{\mathrm{d}x}\right|_{x=\mu} = \omega \left.\frac{g(x)(1-g(x))}{x(1-x)}\right|_{x=\mu} = \frac{\omega}{4}\frac{1}{(1-\mu)\mu}, \tag{2}$$

the slope of the function curve in the $(\mu, 0.5)$ point is proportional to parameter ω if μ is fixed.

If $|\omega| \neq 1$, then the curve has an inflection point in the interval $(0, 1)$. If $|\omega| = 1$, then $g_{\mu,\omega}(x)$ is either convex or concave, or linear in the interval $(0, 1)$, depending on the value of μ. If $\omega = 0$, then $g_{\mu,\omega}(x)$ is constant with the value of 0.5. Main properties of function $g_{\mu,\omega}(x)$ are summarized in Table 1. Figure 2 depicts different examples of curves of function $g_{\mu,\omega}(x)$.

Table 1: Main properties of function $g_{\mu,\omega}(x)$

ω	μ	monotony	shape in the interval $(0, 1)$
$0 < \omega < 1$	$0 < \mu < 1$	increasing	turns from concave to convex
$\omega = 1$	$0 < \mu < 0.5$	increasing	concave
$\omega = 1$	$\mu = 0.5$	increasing	line
$\omega = 1$	$0.5 < \mu < 1$	increasing	convex
$\omega > 1$	$0 < \mu < 1$	increasing	turns from convex to concave
$-1 < \omega < 0$	$0 < \mu < 1$	decreasing	turns from convex to concave
$\omega = -1$	$0 < \mu < 0.5$	decreasing	convex
$\omega = -1$	$\mu = 0.5$	decreasing	line
$\omega = -1$	$0.5 < \mu < 1$	decreasing	concave
$\omega < -1$	$0 < \mu < 1$	decreasing	turns from concave to convex

The following $l(t)$ and $r(t)$ functions may be derived from function $g_{\mu,\omega}(x)$ by applying linear transformations. The function $l(t)$ is given by

$$l(t) = \begin{cases} A_l, & \text{if } t = t_{s,l} \\ A_l + \dfrac{B_l - A_l}{1 + \left(\frac{t_{\mu,l} - t_{s,l}}{t_{e,l} - t_{\mu,l}} \frac{t_{e,l} - t}{t - t_{s,l}}\right)^{\omega_l}}, & \text{if } t_{s,l} < t < t_{e,l} \\ B_l, & \text{if } t = t_{e,l}, \end{cases} \tag{3}$$

where $0 < t_{s,l} < t_{\mu,l} < t_{e,l}$; $0 < A_l < B_l$; $\omega_l > 0$.

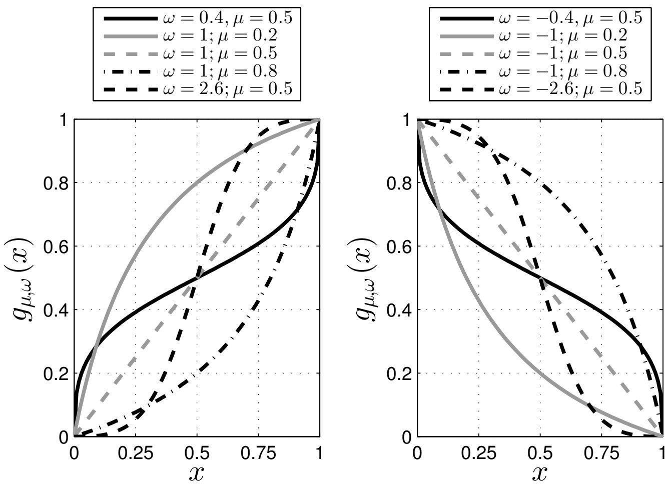

Figure 2: Examples of curves of function $g_{\mu,\omega}(x)$

The function $r(t)$ is given by

$$r(t) = \begin{cases} B_r, & \text{if } t = t_{s,r} \\ A_r + \dfrac{B_r - A_r}{1 + \left(\frac{t_{\mu,r}-t_{s,r}}{t_{e,r}-t_{\mu,r}} \frac{t_{e,r}-t}{t-t_{s,r}}\right)^{-\omega_r}}, & \text{if } t_{s,r} < t < t_{e,r} \\ A_r, & \text{if } t = t_{e,r}, \end{cases} \tag{4}$$

where $0 < t_{s,r} < t_{\mu,r} < t_{e,r}$; $0 < A_r < B_r$; $\omega_r > 0$.

We use the notation t for the independent variable to indicate that functions $l(t)$ and $r(t)$ are defined in the time domain, namely, in the intervals $[t_{s,l}, t_{e,l}]$ and $[t_{s,r}, t_{e,r}]$, respectively. Function $l(t)$ increases from A_l to B_l, while $r(t)$ decreases from B_r to A_r. The derivatives of $l(t)$ and $r(t)$ in the locus $t_{\mu,l}$ and $t_{\mu,r}$, respectively are as follows:

$$\left.\frac{\mathrm{d}l(t)}{\mathrm{d}t}\right|_{t=t_{\mu,l}} = \frac{\omega_l}{4} \frac{(B_l - A_l)(t_{e,l} - t_{s,l})}{(t_{e,l} - t_{\mu,l})(t_{\mu,l} - t_{s,l})} \tag{5}$$

$$\left.\frac{\mathrm{d}r(t)}{\mathrm{d}t}\right|_{t=t_{\mu,r}} = \frac{\omega_r}{4} \frac{(B_r - A_r)(t_{e,r} - t_{s,r})}{(t_{e,r} - t_{\mu,r})(t_{\mu,r} - t_{s,r})}. \tag{6}$$

These mean that the slope of $l(t)$ at $t_{\mu,l}$ is proportional to ω_l and the slope of $r(t)$ is proportional to ω_r if $t_{\mu,l}$ and $t_{\mu,r}$ are fixed. Figure 3 shows an example of each of the functions $l(t)$ and $r(t)$.

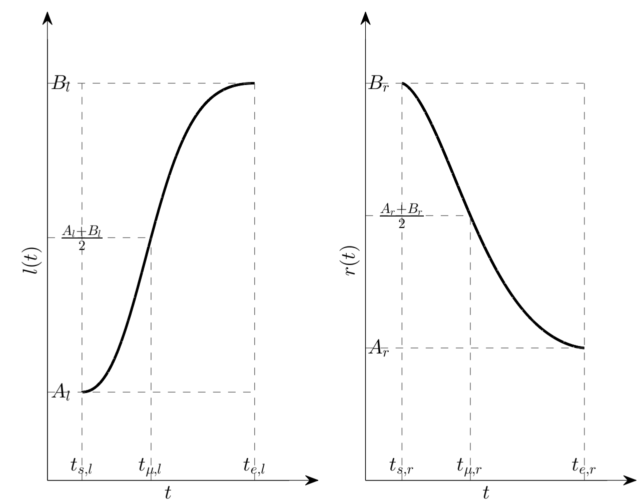

Figure 3: Examples of curves of functions $l(t)$ and $r(t)$

It is worth mentioning that there is a property of function $l(t)$ that is related to the semantics of its parameters. For the sake of easier readability, we will use the following simplified notation to introduce this property: $a = t_{s,l}$, $b = t_{e,l}$, $A = A_l$, $B = B_l$, $\mu = t_{\mu,l}$, $\omega = \omega_l$. Using these notations, $l(t)$ may be written as

$$l(t) = A + \frac{B - A}{1 + \left(\frac{\mu-a}{b-\mu}\frac{b-t}{t-a}\right)^{\omega}}. \tag{7}$$

It can be proven that if $a < t < b$, then

$$\frac{B-M}{M-A}\frac{l(t)-A}{B-l(t)} = \left(\frac{b-\mu}{\mu-a}\frac{t-a}{b-t}\right)^{\omega}, \tag{8}$$

where

$$M = \frac{A+B}{2}, \tag{9}$$

$a < \mu < b; A < B; \omega > 0$. This property of $l(t)$ may be interpreted as follows. Let a and b be the start and end times of a demand growth, respectively, and A and B the demands at a and b, respectively. Furthermore, let $D(t)$ denote the demand at time t. If $D(t) = l(t)$, that is, if the demand growth is given by function $l(t)$, then for any time t between a and b, the demand difference $l(t) - A$ divided by the demand difference $B - l(t)$ is proportional to the power of fraction of the corresponding time differences $t - a$ and $b - t$. In our interpretation, the exponent of the power is ω, while the proportion factor is

$$\frac{1}{\frac{B-M}{M-A}} \left(\frac{b-\mu}{\mu - a} \right)^{\omega}. \tag{10}$$

Note that function $r(t)$ has a similar property; that is, it can be proven that there is an equation which has the form of (8) with a negative ω.

Using functions $l(t)$ and $r(t)$ with the original parameter notations and with the $B = B_l = B_r > 0$ settings, we define the Demand Model Function (DMF) $f(t)$ as follows. The Demand Model Function $f(t)$ is given by

$$f(t) = \begin{cases} A_l, & \text{if } t = t_{s,l} \\ A_l + \dfrac{B - A_l}{1 + \left(\frac{t_{\mu,l} - t_{s,l}}{t_{e,l} - t_{\mu,l}} \frac{t_{e,l} - t}{t - t_{s,l}} \right)^{\omega_l}}, & \text{if } t_{s,l} < t < t_{e,l} \\ B, & \text{if } t_{e,l} \le t \le t_{s_r} \\ A_r + \dfrac{B - A_r}{1 + \left(\frac{t_{\mu,r} - t_{s,r}}{t_{e,r} - t_{\mu,r}} \frac{t_{e,r} - t}{t - t_{s,r}} \right)^{-\omega_r}}, & \text{if } t_{s,r} < t < t_{e,r} \\ A_r, & \text{if } t = t_{e,r} \end{cases} \tag{11}$$

where $0 < t_{\mu,l} < t_{e,l} < t_{s,r} < t_{\mu,r} < t_{e,r}$; $0 < A_l, A_r < B$; $\omega_l, \omega_r > 0$.

The demand model function $f(t)$ describes all the three characteristic parts of a typical purchase life-cycle curve of an electronic spare component.

The first two cases in the definition of $f(t)$ correspond to the first increasing phase of the purchase life-cycle curve, B represents the constant second phase of it, while the last two cases in definition of $f(t)$ describe the third declining phase of the purchase life-cycle curve. It is important to emphasize that each

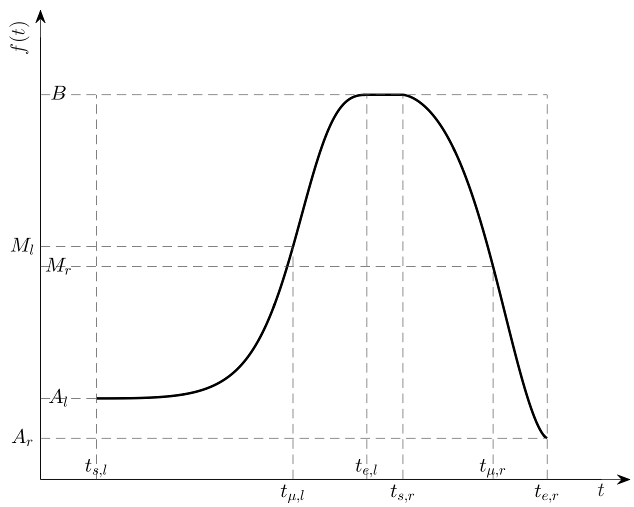

Figure 4: Curve of a demand model function

parameter of $f(t)$ has a geometric interpretation, that is, parameters of $f(t)$ determine its shape and so they may be viewed as geometric properties of the purchase life-cycle curve modeled by $f(t)$. Figure 4 shows an example of the curve of function $f(t)$. Semantics of the parameters of model function $f(t)$ are as follows.

$t_{s,l}$: left end of domain of $f(t)$ (start time of the life-cycle curve)

A_l: value of $f(t)$ in the locus $t_{s,l}$ (left end value of $f(t)$)

$t_{e,r}$: right end of domain of $f(t)$ (end time of the life-cycle curve)

A_r: value of $f(t)$ in the locus $t_{e,r}$ (right end value of $f(t)$)

B: maximum of function $f(t)$ (constant value of $f(t)$ in the second phase of the life-cycle curve)

$t_{\mu,l}$: the locus in which $f(t) = M_l = (A_l + B)/2$

$t_{e,l}$: locus of the end of the left-hand side curve (end of the first phase of the life-cycle curve)

ω_l: slope of the left-hand side curve of $f(t)$ in locus $t_{\mu,l}$ is proportional to ω_l (determines the growth speed of the left-hand side of the life-cycle curve)

$t_{s,r}$: locus of the start of the right-hand side curve (start of the third phase of the life-cycle curve)

$t_{\mu,r}$: the locus in which $f(t) = M_r = (A_r + B)/2$

ω_r: slope of the right-hand side curve of $f(t)$ in locus $t_{\mu,r}$ is proportional to ω_r (determines the declining speed of the right-hand side of the life-cycle curve)

4.2. Fitting demand model functions to historical demand time series

Let

$$A_{l,i}, B_i, t_{s,l,i}, t_{\mu,l,i}, t_{e,l,i}, \omega_{l,i}, A_{r,i}, t_{s,r,i}, t_{\mu,r,i}, t_{e,r,i}, \omega_{r,i} \tag{12}$$

denote the parameters of the demand model function $f_i(t)$ that we wish to fit to the demand time series $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$ of the ith end-of-life spare part, where $0 < t_{\mu,l,i} < t_{e,l,i} < t_{s,r,i} < t_{\mu,r,i} < t_{e,r,i}$; $0 < A_{l,i}, A_{r,i} < B_i$; $\omega_{l,i}, \omega_{r,i} > 0$; $i = 1, 2, \ldots, m$. We determine the unknown model parameters of $f_i(t)$ by minimizing the

$$\sum_{j=1}^{n_i} \left(f_i(j) - D_{i,j}\right)^2 \tag{13}$$

quantity using the so-called GLOBAL method which is a stochastic global optimization procedure introduced by Csendes (see Csendes (1988);

Csendes et al. (2008)). The GLOBAL method was implemented in the MATLAB 2017b numerical computing environment. The following boundaries are set for the unknown parameters of model function $f_i(t)$ to minimize the objective function in (13):

$$\begin{gathered} 0 < A_{l,i} < \infty;\ 0 < B_i < \infty;\ t_{s,l,i}^{(0)} \leq t_{s,l,i} \leq t_{s,l,i}^{(0)};\ t_{s,l,i}^{(0)} < t_{\mu,l,i} < t_{e,r,i}^{(0)}; \\ t_{s,l,i}^{(0)} < t_{e,l,i} < t_{e,r,i}^{(0)};\ 0 < \omega_{l,i} < \infty;\ 0 < A_{r,i} < \infty;\ t_{s,l,i}^{(0)} < t_{s,r,i} < t_{e,r,i}^{(0)}; \\ t_{s,l,i}^{(0)} < t_{\mu,r,i} < t_{e,r,i}^{(0)};\ t_{e,r,i}^{(0)} \leq t_{e,r,i} \leq t_{e,r,i}^{(0)};\ 0 < \omega_{r,i} < \infty. \end{gathered} \tag{14}$$

Figure 5 shows how the model function $f(t)$ can be fitted to variously shaped demand time series by applying the GLOBAL method. Each model parameter is also shown in Figure 5.

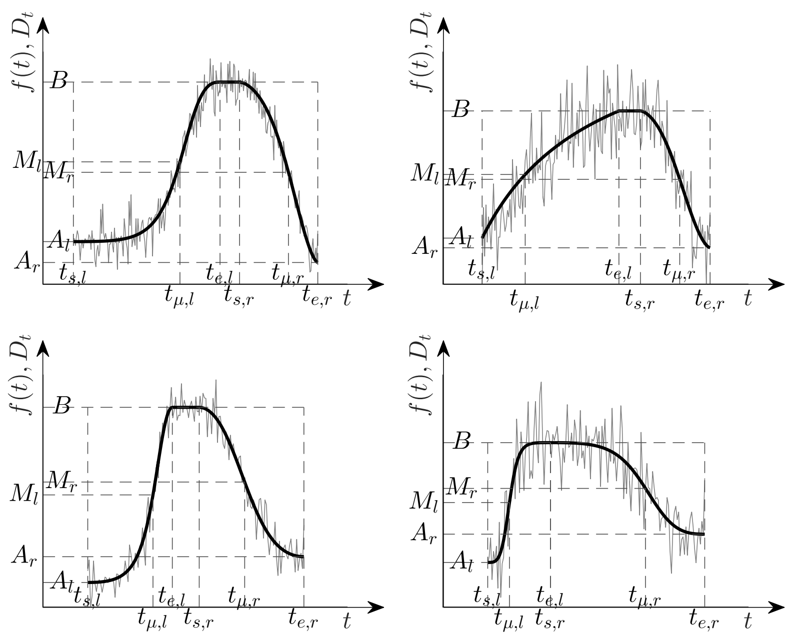

Figure 5: Examples of demand model curves fitted to various demand time series

Note that the objective function in (13) can also be minimized by using an interior point algorithm (see e.g. Waltz et al. (2006)), however, it may result in just a local minimum. In order to find the global minima by this method, certain heuristics would be required to determine the appropriate initial values of the model parameters.

4.3. Standardizing the fitted demand models

Once the parameters of $f_i(t)$ for the demand time series $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$ have been identified, the model $f_i(t)$ can be standardized to the $s_i : [0,1] \to [0,1]$, $x \mapsto s_i(x)$ function by applying the following transformation:

$$x = \frac{t-1}{n_i - 1} \tag{15}$$

$$s_i(x) = \frac{f\left((n_i - 1)\,x + 1\right) - \min\{A_{l,i}, A_{r,i}\}}{B_i - \min\{A_{l,i}, A_{r,i}\}}. \tag{16}$$

Applying the transformation given by (15) and (16) to the model $f_i(t)$ results in the following standardized parameters:

$$y_{l,i} = \frac{A_{l,i} - \min\{A_{l,i}, A_{r,i}\}}{B_i - \min\{A_{l,i}, A_{r,i}\}} \tag{17}$$

$$y_{B_i} = \frac{B_i - \min\{A_{l,i}, A_{r,i}\}}{B_i - \min\{A_{l,i}, A_{r,i}\}} = 1 \tag{18}$$

$$x_{s,l,i} = \frac{t_{s,l,i} - 1}{n_i - 1} = \frac{1-1}{n_i - 1} = 0 \tag{19}$$

$$x_{\mu,l,i} = \frac{t_{\mu,l,i} - 1}{n_i - 1} \tag{20}$$

$$x_{e,l,i} = \frac{t_{e,l,i} - 1}{n_i - 1} \tag{21}$$

$$y_{r,i} = \frac{A_{r,i} - \min\{A_{l,i}, A_{r,i}\}}{B_i - \min\{A_{l,i}, A_{r,i}\}} \tag{22}$$

$$x_{s,r,i} = \frac{t_{s,r,i} - 1}{n_i - 1} \tag{23}$$

$$x_{\mu,r,i} = \frac{t_{\mu,r,i} - 1}{n_i - 1} \tag{24}$$

$$x_{e,r,i} = \frac{t_{e,r,i} - 1}{n_i - 1} = \frac{n_i - 1}{n_i - 1} = 1. \tag{25}$$

It can be seen that the transformation given by (15) and (16) does not modify ω_l and ω_r. Taking into account that $y_{B_i} = 1$, $x_{s,l,i} = 0$ and $x_{e,r,i} = 1$, function $s_i(x)$ may be given by the parameters $y_{l,i}, x_{\mu,l,i}, x_{e,l,i}, \omega_{l,i}, y_{r,i}, x_{s,r,i}, x_{\mu,r,i}, \omega_{r,i}$:

$$s_i(x) = \begin{cases} y_{l,i}, & \text{if } x = x_{s,l,i} \\ y_{l,i} + \dfrac{1 - y_{l,i}}{1 + \left(\frac{x_{\mu,l,i}}{x_{e,l,i} - x_{\mu,l,i}} \frac{x_{e,l,i} - x}{x}\right)^{\omega_{l,i}}}, & \text{if } 0 < x < x_{e,l,i} \\ 1, & \text{if } x_{e,l,i} \le x \le x_{s_r,i} \\ y_{r,i} + \dfrac{1 - y_{r,i}}{1 + \left(\frac{x_{\mu,r,i} - x_{s,r,i}}{1 - x_{\mu,r,i}} \frac{1 - x}{x - x_{s,r,i}}\right)^{-\omega_{r,i}}}, & \text{if } x_{s,r,i} < x < 1 \\ y_{r,i}, & \text{if } x = 1. \end{cases} \tag{26}$$

$s_i(x)$ is the standardized demand model (SDM) of the historical demand time series $D_{i,1}, D_{i,2}, \dots, D_{i,n_i}$. Note that each parameter of function $s_i(x)$ has the same geometric interpretation as the corresponding parameter of function $f_i(t)$. Henceforward, we will use the $s_{\mathbf{p}_i}(x)$ notation for the SDM of the historical demand time series $D_{i,1}, D_{i,2}, \dots, D_{i,n_i}$, where the parameter vector $\mathbf{p}_i$ is

$$\mathbf{p}_i = (p_{i,1}, p_{i,2}, \dots, p_{i,8}) = (y_{l,i}, x_{\mu,l,i}, x_{e,l,i}, \omega_{l,i}, y_{r,i}, x_{s,r,i}, x_{\mu,r,i}, \omega_{r,i}). \tag{27}$$

Owing to the standardization, one of the parameters $y_{l,i}$, $y_{r,i}$ is zero and the other one is positive. Function $s_{\mathbf{p}_i}(x)$ may be viewed as a primitive that represents the life-cycle curve of the demand time series $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$. Due to the construction of the standardized demand model functions, each parameter of a primitive has a geometric interpretation, that is, parameters of a primitive determine the shape of its curve. This property of the standardized demand model functions allows us to cluster them based on their parameters so that the clustering results in typical standardized demand models (SDMs). Figure 6 shows the curve of a standardized demand model function. y_{M_l} and y_{M_r} are the standardized values of M_l and M_r, respectively, that is $y_{M_l} = (y_l + 1)/2$, $y_{M_r} = (y_r + 1)/2$.

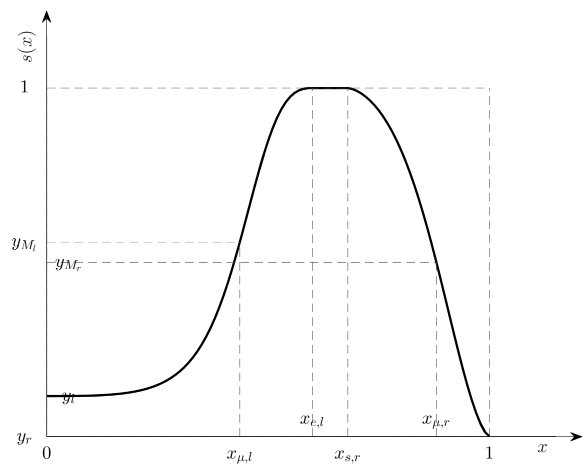

Figure 6: An example of the curve of a standardized demand model function

4.4. Clustering the standardized demand models

In order to identify typical standardized demand models, we cluster the $s_{\mathbf{p}_i}(x)$ models based on their parameter vectors $\mathbf{p}_i$ by applying the fuzzy c-means clustering algorithm (Bezdek, 1981). As we employ fuzzy c-means clustering, cluster $\mathbf{C}_q$ is defined as a set of those $\mathbf{p}_i$ vectors of which membership values in $\mathbf{C}_q$ are the highest among all the clusters. That is,

$$\mathbf{C}_q = \left\{ \mathbf{p}_i : \mu_q(\mathbf{p}_i) = \max_{t=1,\ldots,m} \mu_t(\mathbf{p}_i), i \in \{1, 2, \ldots, m\} \right\}, \tag{28}$$

where $\mu_j(\mathbf{p}_i)$ is the membership value of vector $\mathbf{p}_i$ in fuzzy cluster $\mathbf{C}_j$, and if a vector $\mathbf{p}_i$ has a 0.5 membership in two different clusters, then it is in the cluster with lower index $(j, q \in 1, 2, \ldots, N)$. Let us assume that the clusters $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_N$ $(N \leq m)$ of standardized demand models are formed, and let $\mathcal{I}_q$ be the index set of standardized demand models $s_{\mathbf{p}_i}(x)$ that belong to cluster $\mathbf{C}_q$ $(q \in 1, 2, \ldots, N)$, that is,

$$\mathcal{I}_q = \{ i : \mathbf{p}_i \in \mathbf{C}_q, i \in \{1, 2, \ldots, m\} \} \tag{29}$$

and furthermore let $\mathbf{c}_q$ be the parameter vector of the *cluster characteristic standardized demand model* $s_{\mathbf{c}_q}(x)$, that is, $\mathbf{c}_q$ is the centroid of vectors $\mathbf{p}_i$ for which $i \in \mathcal{I}_q$. The function $s_{\mathbf{c}_1}(x), s_{\mathbf{c}_2}(x), \ldots, s_{\mathbf{c}_N}(x)$ represent the *typical standardized demand models*, and as such may be viewed as representative models for the purchase life-cycles of the historical demand time series $D_{i,1}, D_{i,2}, \ldots, D_{i,n_i}$ of end-of-life spare parts $(i = 1, 2, \ldots m)$.

The typical SDMs are generated from historical time series of end-of-life spare parts, that is, they represent historical knowledge on past demands. In case of consumer electronic goods, the full purchase life-cycles of spare parts of the same component commodity (such as motherboards, power-supply units, etc.) show certain similarities (Stipan et al., 2000; Groen et al., 2004). These similarities, on the one hand, lay the foundation of the clustering method we discussed so far. On the other hand, they allow us to assume that the unknown future demand for active spare parts of a component commodity may follow similar life-cycles to some of the typical historical purchase life-cycles of EOL spare parts of the same component commodity. Based on it, a potential application of the typical standardized demand models is the prediction of future demands for active spare parts.

4.5. Using the typical standardized demand models for demand prediction

The demand time series of active spare parts, for which orders are given, are fractional ones, that is, they will be continued in the future. Let $d_{F,1}, \ldots, d_{F,M}$ denote the fractional demand time series of an active spare part. For each typical SDM $s_{\mathbf{c}_q}(x)$, we wish to identify the parameters $\alpha_q \geq M$, $\beta_q \geq 0$ and $\gamma_q > 0$ of function $g_q : [1, \alpha_q] \rightarrow \mathbb{R}^+ \cup 0$, $t \mapsto g_q(t)$

$$g_q(t) = \gamma_q s_{\mathbf{c}_q}\left(\frac{t-1}{\alpha_q}\right) + \beta_q \tag{30}$$

for which

$$\varepsilon_q = \sum_{k=1}^{M} \left(g_q(k) - d_{F,k}\right)^2 \rightarrow \min \tag{31}$$

$(q = 1, 2, \ldots, N)$. Solution for each fitting problem described in (30) and (31) can be found by applying the same GLOBAL method that was referenced in section 4.2. The initial values $\alpha_q^{(0)}$, $\beta_q^{(0)}$ and $\gamma_q^{(0)}$ of α_q, β_q and γ_q, respectively, are set as

$$\alpha_q^{(0)} = M; \beta_q^{(0)} = 0; \gamma_q^{(0)} = 1. \tag{32}$$

The boundaries for α_q, β_q and γ_q are set as

$$M \leq \alpha_q < \infty; \quad 0 \leq \beta_q < \infty; \quad 0 < \gamma_q < \infty. \tag{33}$$

Let δ_q be defined as

$$\delta_q = \frac{\varepsilon_q}{\sum_{i=1}^{N} \varepsilon_i}. \tag{34}$$

Each δ_q is in the $[0, 1]$ interval, and expresses the distance between function $g_q(t)$ and the fractional demand time series $d_{F,1}, \ldots, d_{F,M}$ $(q = 1, 2, \ldots, N)$. For each q, assuming that $\delta_q > 0$, let the weight w_q be defined as

$$w_q = \frac{\frac{1}{\delta_q}}{\sum_{i=1}^{N} \frac{1}{\delta_i}}. \tag{35}$$

w_q expresses the similarity between function $g_q(t)$ and the fractional demand time series $d_{F,1}, \ldots, d_{F,M}$, and let α_{max} be given by

$$\alpha_{max} = \max_{q=1,\ldots,N} (\alpha_q). \tag{36}$$

Then we compute function $\mathcal{F}(t)$ as

$$\mathcal{F}(t) = \sum_{q=1}^{N} w_q g_q^*(t), \tag{37}$$

where

$$g_q^*(t) = \begin{cases} g_q(t), & \text{if } 0 \leq t \leq \alpha_q \\ 0, & \text{if } \alpha_q < t \leq \alpha_{max} \end{cases} \tag{38}$$

$(q = 1, 2, \ldots, N)$. The $\mathcal{F}(M+1), \ldots, \mathcal{F}(\lfloor \alpha_{max} \rfloor)$ values may be viewed as the forecasts of the unknown $d_{F,M+1}, \ldots, d_{F,\lfloor \alpha_{max} \rfloor}$ values, respectively. The w_q weight determines how much the typical standardized demand model $s_{\mathbf{c}_q}(x)$ is considered in the forecast through the function $g_q(t)$, while α_{max} determines the length of time frame in which $\mathcal{F}(t)$ is not identically equal to zero.

5. A demonstrative example

Based on the introduced methodology, a software application was developed for modeling and forecasting purchase life-cycles of electronic spare parts. In this example it is to be presented how our method was applied to real-life demand time series.

Empirical demand time series of 120 end-of-life laptop motherboard types were used to generate the standardized demand models. Each of the empirical demand time series used as an input to our method represents the weekly demands for a laptop motherboard type that is utilized as a spare part in electronic repair services. It should be emphasized that in our example each motherboard type is specific to a laptop type or to a set of laptop types being released to the market together. This specificity of the motherboards makes us assuming that their purchase life-cycles exhibit unimodal curves and so our modeling method is applicable to this case. Notice that based on the properties of our demand model function, it is not suitable to model multimodal purchase life-cycle curves. 110 time series out of the 120 were used as training samples, while 10 time series were used for testing the method. The standardized demand models generated from the historical demand time series were clustered into 7 clusters by applying the fuzzy c-means algorithm. Here, the Davies-Bouldin criterion values (Davies and Bouldin, 1979) were used to determine the optimal number of clusters. The parameters of the typical standardized demand models are shown in Table 2. Figure 7 shows the curves of the clustered standardized demand models (gray lines) and the curves of the cluster centroids that are the typical standardized demand models (black lines). It can be seen from this figure that the cluster centroids

Table 2: Parameters of cluster centroids

q	y_l	$x_{\mu,l}$	$x_{e,l}$	ω_l	y_r	$x_{s,r}$	$x_{\mu,r}$	ω_r
1	0.0000	0.1474	0.3533	1.5075	0.0000	0.5627	0.8498	1.5022
2	0.1887	0.1434	0.5843	1.4091	0.0000	0.7151	0.8418	0.9196
3	0.0000	0.4083	0.6047	1.5008	0.0000	0.8037	0.9488	0.3055
4	0.2307	0.3414	0.6896	3.4135	0.0000	0.7517	0.9045	0.6289
5	0.0000	0.2529	0.4492	2.6908	0.0000	0.6431	0.8476	1.1070
6	0.0000	0.2341	0.4985	0.8191	0.0000	0.7481	0.8397	2.1251
7	0.1120	0.1841	0.2982	2.4222	0.0000	0.6491	0.8578	2.8680

represent well the corresponding standardized demand models.

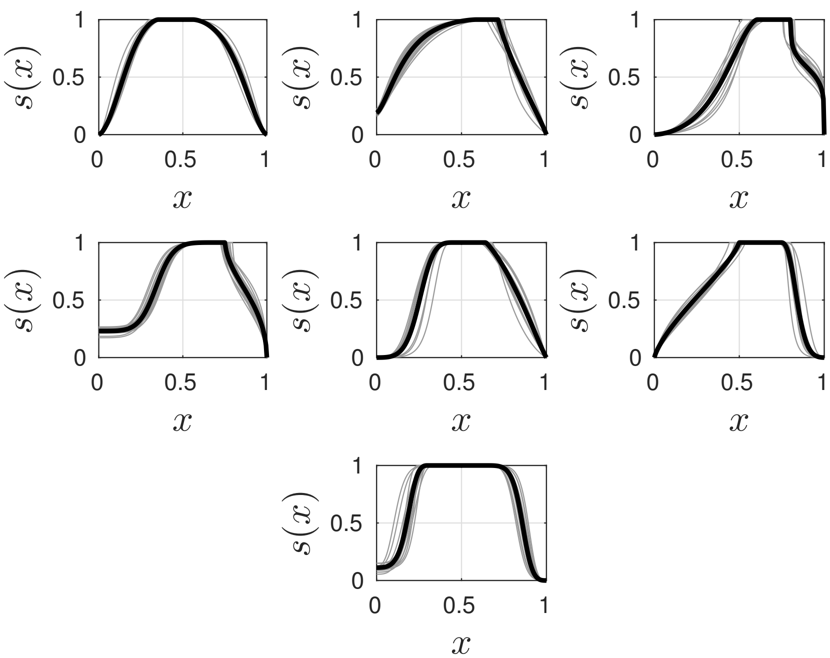

Figure 7: Clusters of the standardized demand models

The typical standardized demand models were used to predict future demands for spare parts based on known fractions of their empirical demand time series. Note that the historical demand time series of the studied active spare parts are from the test sample; that is, these were not used for establishing the standardized demand models. Figure 8 shows the modeled and predicted $\mathcal{F}(t)$ values (thick black line), the known fraction (thin black line) and the unknown future values (grey line) for one of the test time series. The unit of the time scale in Figure 8 is one week. The dashed vertical lines in Figure 8 indicate the border between the known and unknown parts of the demand time series. Similar plots generated for the other 9 test times series are available in the Appendix.

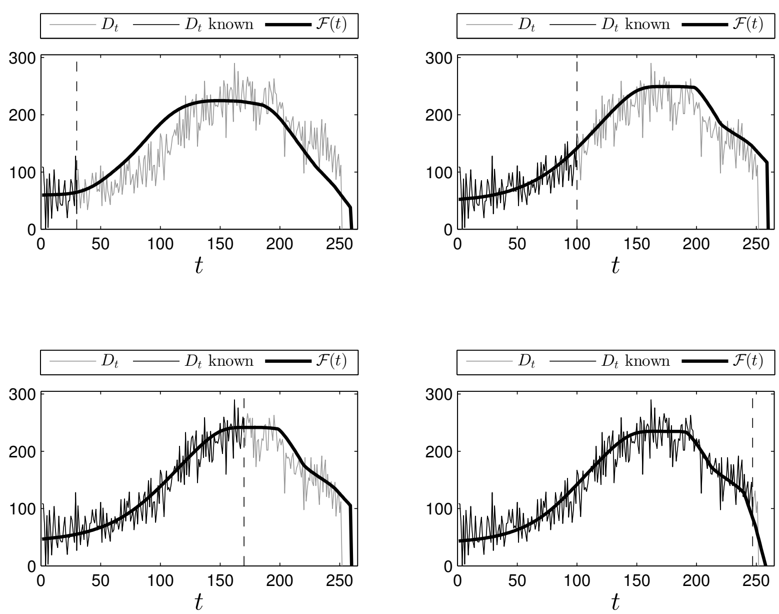

Figure 8: An example of modeling and predicting

There are a couple of properties of our modeling and predicting method that are substantial from perspective of companies that provide spare parts logistics as a service. Here, these properties are to be demonstrated by using the example in Figure 8, however, the models and predictions generated for the other 9 test time series have very similar properties (see the Figures A.1–A.9 in the Appendix). That is, the properties discussed here may be viewed as the generic characteristics of our modeling and forecasting method. The first three subplots (2 in the first row and the left hand side one in the second row) in Figure 8 (and in each of the Figures A.1–A.9 in the Appendix) show the cases when the demand values are known for the first 30, 100, 170 weeks, respectively. The right bottom subplot in Figure 8 (and in each of the Figures A.1–A.9 in the Appendix) is for the case, when the demand values are unknown only for the last 5 weeks.

Figure 8 shows how the model $\mathcal{F}(t)$ is developing as more and more information on the historical demand is getting available. The left upper graph in Figure 8 shows the case when the demand values of the studied active spare part are known for the first 30 weeks. When the demand is in the increasing phase of the purchase life-cycle, like in this case, the most exciting practical question is when the life-cycle curve will reach the end of this phase. (Note, that in this particular case, the life-cycle is at the beginning of its increasing phase.) The curve of demand time series (gray line) is turning from its increasing phase to its quasi constant phase at around week 145. The curve of model function $\mathcal{F}(t)$ is turning from increasing to quasi constant approximately 10 weeks earlier, that is, the model function $\mathcal{F}(t)$ can indicate the turning point between the first two phases of the life-cycle curve. This graph also demonstrates that the demand time series is turning from its quasi constant phase to its decreasing phase at around week 190, while the model function $\mathcal{F}(t)$ is doing so at around week 185. It means that our method is able to indicate the turning points of the life-cycle curve relatively well at early stages of the life-cycle. We also need to add that the model function $\mathcal{F}(t)$ slightly overestimates the real demand in the flat phase, and it underestimates the real demand in the decreasing phase.

The right upper graph in Figure 8 shows the result of our life-cycle curve modeling and predicting method in the case when the demand values of the studied active spare part are known for the first 100 weeks. In this case, the life-cycle curve is still in the increasing phase, but here, compared to the previous case, we have more historical information about the demand. On the one hand, in this case, the model predicts the turning point between the increasing and flat phases of the life-cycle curve more accurately than in the case when the demands only for the first 30 weeks were known. On the other hand, our model slightly overestimates the turning point between the flat and decreasing phases of the life-cycle curve. It can be seen from this graph that the model function $\mathcal{F}(t)$ slightly overestimates the real demands, but at the same time, it is following the demand trend well. It should also be added that the predicted turning point between the flat and decreasing phases did not change much compared to that in the left upper plot.

In the left bottom graph, the demand values are known for the first 170 weeks, and the demand life-cycle curve is in its quasi constant phase. Here, the predicted turning point, in which the demand time series starts to decrease, does not change compared to its previous prediction that was based on the first 100 known demand values. It can be seen from this graph that the model function $\mathcal{F}(t)$ follows the demand trend well, but slightly overestimates that.

The right bottom graph shows the case when almost the complete demand life-cycle is known. Here, the model function $\mathcal{F}(t)$ matches the time series quite well.

By comparing the four model functions, it can be seen that the total length of the predicted life-cycle does not change significantly, it is around 260. This observation is in line with the expectation that the stability of the predicted life-cycle should be increasing as more information about the historical demand is getting available. Although our method seems to perform well in estimating the final demand date, we should also mention that there is a structural resemblance among the examined time series that contributes to the goodness of estimation. Namely, in our example, the time series which fall into the same cluster have similar length and typically there is only one or just a few clusters represented with significant weights in the aggregate model function $\mathcal{F}(t)$.

5.1. Results and discussion

The $\mathcal{F}(t)$ function-based forecast results for the active spare parts in the 10 test samples were compared to the results of ARIMA and exponential smoothing-based forecasts as well as to prediction results of two soft computational methods that we developed in MATLAB software and applied for long-term demand forecasting. These latter two methods utilize Adaptive Neuro Fuzzy Inference Systems (ANFIS) and Feedforward Neural Networks (FNN).

The default weight for the exponential smoothing applied here was computed by fitting an ARIMA (0,1,1) model to the data, and back-casting was used to calculate the initial smoothed value. The Hyndman and Khandakar (2008) algorithm was used to identify the best fitting ARIMA model.

The training data set for the ANFIS and FNN methods was the same one that we used to generate the typical standardized demand models. Both for the ANFIS and FNN methods, each demand time series of the 110 end-of-life electronic spare parts were split into five period long segments, and a linear regression was applied to each of these segments. Note that setting the segment size to five periods was based on the experience that this selection yielded the best modeling results. Owing to this approach, after data normalization each demand time series was described by a series of slope and intersection pairs which were used as inputs and outputs for training the ANFIS and FNN systems. Based on this, an ANFIS and an FNN model was generated for each demand time series. The ANFIS model-based forecast for each fractional demand time series of the test samples was generated as the weighted average of the predictions given by the 110 individual ANFIS models. The weights, with which the models were considered, had been computed based on the similarities between the normalized fractional demand time series of the active spare part and the model inputs used for training. A similar approach was used for generating the FNN model-based forecasts.

For each fractional demand time series of the test samples, 30-week-ahead forecasts were generated by each of the above mentioned methods ($\mathcal{F}(t)$ function-based, ARIMA, exponential smoothing, ANFIS, FNN), and the mean squared error (MSE) values of the forecasts were used to compare the methods. These results for the first test time series (the one in Figure 8) are summarized in Table 3. The results for the other 9 test samples are in the Tables A.1–A.9 in the Appendix.

Table 3: MSE values of forecasts

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...50	51...80	inc.	inc.	1013.2	919.6	2636.9	923.2	1027.4
2	1...100	101...130	inc.	inc.	1142.8	976.1	8239.5	1150.3	1197.3
3	1...130	131...160	inc.	inc./flat	903.4	2241.5	3356.9	942.1	963.2
4	1...150	150...180	flat	flat	652.1	703.4	692.1	640.3	670.2
5	1...180	181...210	flat	flat/dec.	1013.5	2341.6	4491.3	1251.8	1233.6
6	1...210	211...240	dec.	dec.	1124.6	1098.8	8408.4	1109.8	1132.8

The PLC Phase columns in Table 3 indicate in which phase of the purchase life-cycle the last known demand and the predicted demands are (inc., flat and dec. stand for the increasing, flat and decreasing phases, respectively).

Table 3 demonstrates that our method is able to indicate relatively well the turning points of the life-cycle curve at early stages of the purchase life-cycle. It is worth mentioning that the traditional statistical forecasting techniques, such as the ARIMA or the exponential smoothing do not have such a capability. These methods are able to give reliable predictions within certain phases of the purchase life-cycle curve, however, they cannot predict the turning points of the curve and so they are not suitable for long-term forecasting of demand for spare parts in spare part logistics businesses. We should also add that this may be due to the fact that our method exploits the similarities among the parts of a family, while the ARIMA and exponential smoothing methods do not assume any family. These properties of the forecasts generated by the ARIMA and exponential smoothing methods are also visible in Table 3.

When the current demand for the studied active spare part is in its first, increasing phase and the time period of prediction also belongs to this phase (row 1 and row 2 in Table 3), the ARIMA method gives the most accurate prediction. (Note that the current demand is interpreted as the latest known demand.) Similarly, when the current demand and the time period of forecast are both in the decreasing phase of the life-cycle, the ARIMA method performs well (row 6 in Table 3). At the same time, when the current demand is in the first increasing phase of the purchase life-cycle and the time period of forecast includes the first turning point of the life-cycle curve (row 3 in Table 3), the ARIMA method does not perform well as it cannot predict the turning point. A similar conclusion on the behavior of the ARIMA-based forecast in the third phase of the purchase life-cycle may be drawn from row 5 in Table 3 when the time period of forecast includes the second turning point of purchase life-cycle.

The exponential smoothing method gives a constant forecast, and so this is suitable to model the second, quasi constant phase of the purchase life-cycle. When the current demand is in the second phase of the purchase life-cycle and the time period of forecast is also in this phase (row 4 in Table 3), the exponential smoothing gives accurate predictions compared to the other studied methods. In other cases, the exponential smoothing method gives weaker forecast results than the other studied methods.

The methods founded on machine learning techniques, fuzzy inference systems, artificial neural networks, or their combinations are more appropriate for long-term predictions. On the one hand, these methods have the ability to indicate the turning points of the purchase life-cycle curve relatively well (row 2 and row 5 in Table 3). On the other hand, these techniques give similarly good forecast results as the ARIMA does when the current demand and the demand in the time period of forecast are both in the same phase of the purchase life-cycle (row 1, row 2, row 4 and row 6 in Table 3).

Our method may be viewed as a hybrid one that utilizes analytic curve fitting to identify the trends of historical demand time series and fuzzy clustering to recognize typical model functions. Similar to the ANFIS and FNN methods, the $\mathcal{F}(t)$ function-based forecasts also have the capability to indicate the turning points of the life-cycle curve relatively well. It can be seen from Table 3 that our method has similar forecasting performance to those of the ANFIS and FFN methods. It is worth mentioning that besides these properties of the $\mathcal{F}(t)$ function-based forecasts, the typical standardized demand models, which our forecast method is founded on, embody historical knowledge on the past purchase life-cycles, and this knowledge is linked to the semantics of parameters of the typical standardized demand models. The forecast results for the other 9 test time series (in the Tables A.1–A.9 in the Appendix) also support the above-discussed properties of the examined methods.

Table 4 contains summary statistics of the modeling results based on the MSE values of all the examined forecasts which are detailed in Table 3 and in the Tables A.1–A.9 in the Appendix. Table 4 shows for each method, in how many cases out of the total 50 cases it performed the best, second best, third

Table 4: Summary statistics of the modeling results

	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
# of times the best	22	21	0	6	1
# of times the 2nd best	14	3	1	25	7
# of times the 3rd best	11	5	0	17	16
# of times the 4th best	3	21	0	2	25
# of times the 5th best	0	0	49	0	1
Sum of ranks	95	126	247	115	168

best, etc. The sums of ranks in this table are computed based on the ranks from 1 through 5 representing an increasing order of the MSE values of the forecasts. For example, the $\mathcal{F}(t)$ function-based method was ranked as first, second, third, fourth and fifth in 22, 14, 11, 3 and 0 cases, respectively, and so its sum of ranks is 22*1+14*2*11*3+3*4+0*5 = 95. Based on the sum of ranks metric, we can conclude that the $\mathcal{F}(t)$ function-based method had the best overall forecasting performance, while the ANFIS and ARIMA methods proved to have the second and third best performances, respectively. We should also note that if we take into account solely the number of times a method had the best forecasting performance, then the $\mathcal{F}(t)$ function-based method is the best (22 times), the ARIMA method is close to that (21 times) and the ANFIS method is the third one (6 times). Notice that the $\mathcal{F}(t)$ function-based, the ANFIS and the ARIMA methods were among the three best methods in 47, 49 and 29 cases, respectively. This result tells us that in the case of our example, the $\mathcal{F}(t)$ function-based and ANFIS methods had similar overall forecasting performance.

6. Conclusions

In this paper, a hybrid technique for long-term forecasting of electronic spare parts for aftermarket repair services is presented. The proposed modeling methodology is founded on the assumption that the purchase life-cycles of spare parts can be described by a curve with short-term fluctuations around it, and this model curve has the following characteristics:

(i) the curve is unimodal; that is, first increasing, then a plateau and finally decreasing

(ii) the curve can have maximum two inflexion points, one in the increasing and one in the decreasing phase

(iii) the curve can be zero or positive, both at the start and at the end

(iv) transformations allow this curve to fit required heights and locations.

Long-term planning of spare part purchasing is essential for spare part logistics providers in order to manage their inventory levels. Moreover, long-term outlook of spare part demand should be incorporated into stocking decisions as well. Based on the empirical results from our demonstrative example, we can conclude that our method is mainly advantageous in long-term forecasting, it can be especially useful in supporting purchase planning decisions in the ramp-up and declining phases of purchase life-cycles. Similar to fuzzy and neuro-fuzzy systems based methods the presented forecasting methodology is able to indicate the turning points of the life-cycle curve well. In addition to that, for the motherboard data examined in our example, the forecasting performance of the introduce method was as good as the performance of the studied ANFIS-based method.

The real novelty of the introduced approach is the semantics of model parameters of typical standardized demand models which the proposed forecasting methodology is built upon. Although the results obtained from our modeling are encouraging and it has the potential to be a suitable alternative forecasting technique, we should mention that our method is mainly applicable to model purchase life-cycles of product specific spare parts. This conclusion follows from the fact that our model function is unimodal and from the phenomenon that for product specific spare parts the demand function is likely to be unimodal and for general spare parts it may be multi-modal. We should also add that our conclusions are founded on the empirical results from a demonstrative example in which we examined one spare part family. Namely, 120 motherboard types as laptop specific spare parts were used to test our method. As part of a future research, we plan to test our method on other electronic spare part families. These additional investigations will allow us to obtain more information about the characteristics of our method and draw more generic conclusions.

The introduced model function, which is used to grasp the purchase life-cycles of spare parts, has certain mathematical and computational properties that lay the foundation for future researches in its applications to spare parts demand planning. Function $f(t)$, which is used as a demand model, can also be considered as a filter for the studied demand time series. An advantage of using this function as a time series filter is that function $f(t)$ is multiple differentiable, and so results of the filtering can be used for further mathematical analyses. $f(t)$ with its parameters is a function family that we use for curve fitting. Function parameters have certain semantics that support describing and interpreting the demand trend as function of time. The convexity or concavity of the increasing and decreasing phases of the fitted model function $f(t)$ carry information that can be useful in practical demand planning.

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7. Appendix

Modeling and predicting results for the test samples

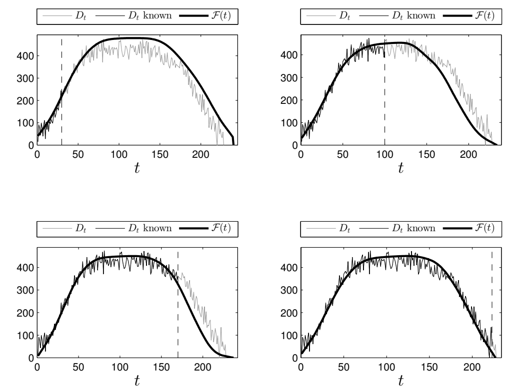

Figure A.1: Test sample #2

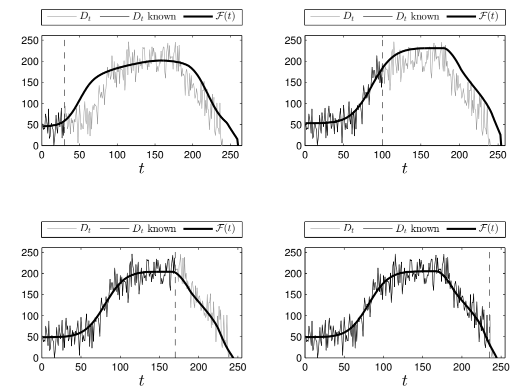

Figure A.2: Test sample #3

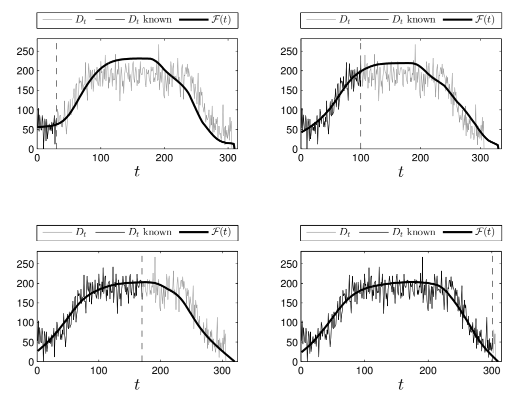

Figure A.3: Test sample #4

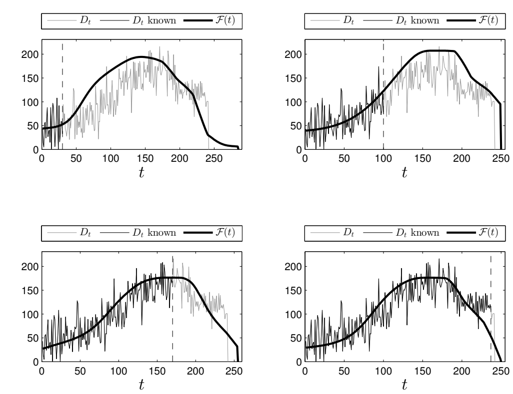

Figure A.4: Test sample #5

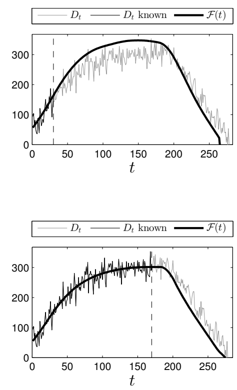

Figure A.5: Test sample #6

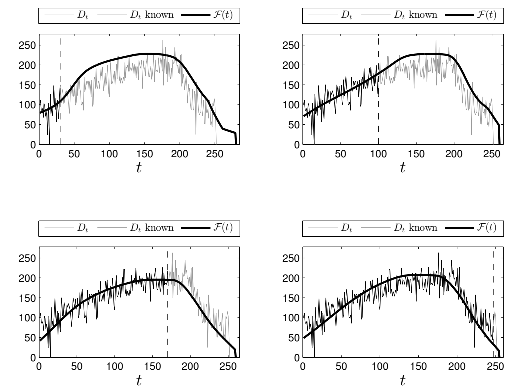

Figure A.6: Test sample #7

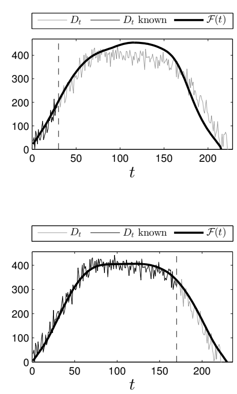

Figure A.7: Test sample #8

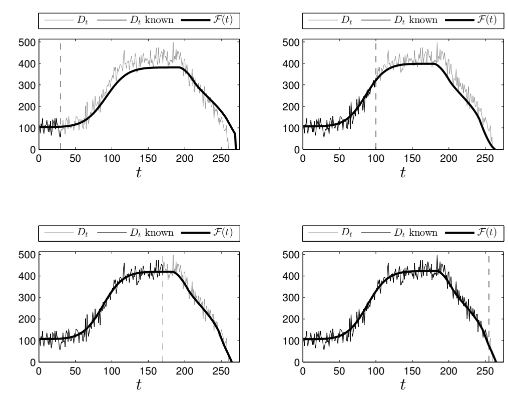

Figure A.8: Test sample #9

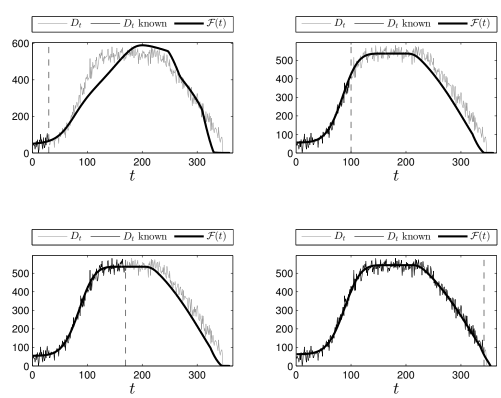

Figure A.9: Test sample #10

Table A.1: MSE values of forecasts for test sample #2

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc./flat	1244.5	2572.4	9764.9	1322.2	1389.4
2	1...90	91...120	flat	flat	903.8	876.1	915.2	887.3	912.6
3	1...130	131...160	flat	dec.	1127.4	2473.7	7281.2	1052.5	1170.3
4	1...160	161...190	dec.	dec.	564.5	602.1	8931.1	567.1	605.2
5	1...190	191...220	dec.	dec.	703.2	698.2	6931.1	712.1	735.2

Table A.2: MSE values of forecasts for test sample #3

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc.	976.5	992.5	8852.4	1003.1	1032.4
2	1...90	91...120	inc.	inc./flat	1245.2	3008.1	9011.3	1212.4	1297.6
3	1...130	131...160	flat	flat	842.4	866.1	902.8	871.5	880.2
4	1...160	161...190	flat	flat/dec.	712.2	3556.1	7993.2	772.4	794.2
5	1...190	191...220	dec.	dec.	603.2	598.2	7955.2	608.4	638.3
6	1...220	221...240	dec.	dec.	563.1	602.2	7331.7	582.8	611.4

Table A.3: MSE values of forecasts for test sample #4

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...50	51...80	inc.	inc.	852.2	812.5	9417.1	881.8	897.4
2	1...80	81...110	inc.	inc./flat	1133.6	3321.8	9262.6	1098.4	1184.2
3	1...110	111...140	flat	flat	816.4	831.8	887.2	851.5	867.1
4	1...210	211...240	flat	flat/dec.	703.2	3608.2	7541.4	721.5	734.1
5	1...250	251...280	dec.	dec.	631.4	601.2	7023.1	615.4	652.1

Table A.4: MSE values of forecasts for test sample #5

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc.	1021.2	1144.3	5324.3	1089.2	1123.2
2	1...100	101...130	inc.	inc.	1057.6	1034.8	6818.3	1042.1	1083.5
3	1...130	131...160	inc.	inc./flat	1112.4	2218.8	5841.8	1098.3	1122.8
4	1...160	161...190	flat	flat/dec.	1204.2	3818.1	4227.8	1252.1	1281.1
5	1...200	201...230	dec.	dec.	1289.4	1202.2	1371.1	1259.2	1297.8

Table A.5: MSE values of forecasts for test sample #6

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc.	1021.2	987.2	8961.4	1089.2	1123.2
2	1...100	101...130	inc.	inc.	963.2	991.7	6321.8	981.2	1027.6
3	1...130	131...160	inc.	inc./flat	1021.2	2482.5	2519.8	1055.4	1048.8
4	1...160	161...190	flat	flat/dec.	992.5	1542.8	2976.8	1059.1	1103.2
5	1...200	201...230	dec.	dec.	876.4	801.3	7344.2	856.2	891.7

Table A.6: MSE values of forecasts for test sample #7

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc.	923.4	908.5	3918.2	955.2	987.6
2	1...100	101...130	inc.	inc.	1067.2	1031.7	4045.8	1044.7	1103.8
3	1...130	131...160	inc.	inc./flat	986.3	1014.2	1347.8	1005.2	1038.6
4	1...160	161...190	flat	flat/dec.	1129.5	1323.1	1976.8	1187.1	1163.5
5	1...200	201...230	dec.	dec.	1048.4	973.3	7718.1	1003.2	1067.3

Table A.7: MSE values of forecasts for test sample #8

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc./flat	872.6	1368.2	9241.7	908.3	937.2
2	1...90	91...120	flat	flat	963.2	925.4	1089.1	947.2	982.6
3	1...150	151...180	flat	flat/dec.	934.1	1292.7	14227.2	963.5	958.3
4	1...180	181...210	dec.	dec.	492.5	403.6	8431.1	448.2	486.3

Table A.8: MSE values of forecasts for test sample #9

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...80	81...110	inc.	inc.	961.2	927.2	12013.6	991.6	956.2
2	1...110	111...140	inc.	flat	863.4	1251.1	8809.8	885.2	907.3
3	1...140	141...170	flat	flat	811.3	796.4	1127.2	802.4	843.2
4	1...170	171...200	flat	dec.	956.2	1478.2	8695.2	998.4	978.3
5	1...200	201...230	dec.	dec.	461.2	421.8	7188.1	455.2	483.4

Table A.9: MSE values of forecasts for test sample #10

	Period		PLC Phase		MSE				
	Known	Forecast	Known	Forecast	$\mathcal{F}(t)$	ARIMA	Exp. S.	ANFIS	FNN
1	1...60	61...90	inc.	inc.	441.5	502.4	17042.9	481.2	438.4
2	1...110	111...140	inc.	flat	911.8	1876.1	1115.2	897.3	952.1
3	1...170	171...200	flat	flat	744.4	721.7	738.2	742.3	778.6
4	1...200	201...230	flat	dec.	663.5	989.4	8990.1	682.1	679.1
5	1...230	231...260	dec.	dec.	400.2	388.1	8371.4	406.1	421.9

Figure_1

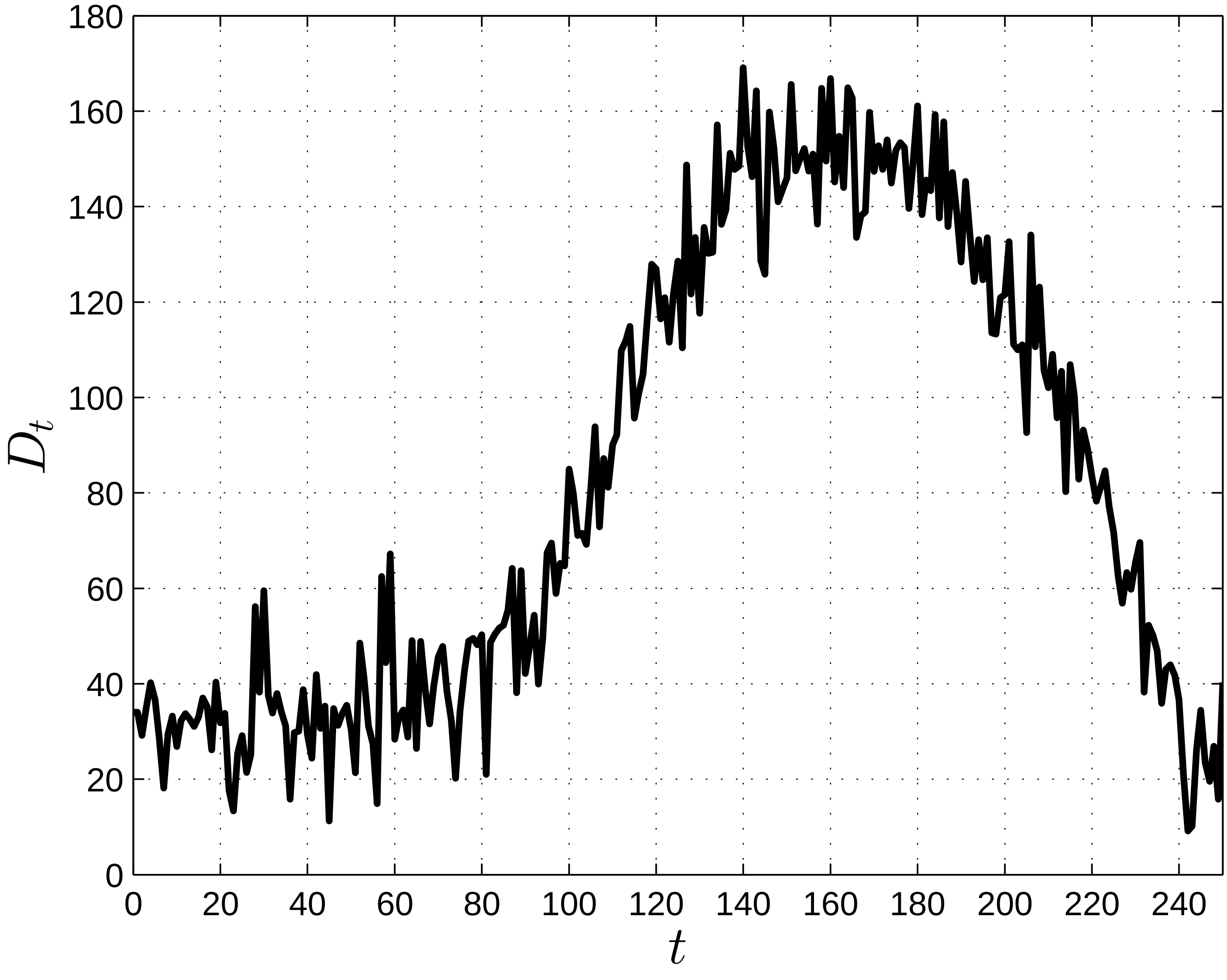

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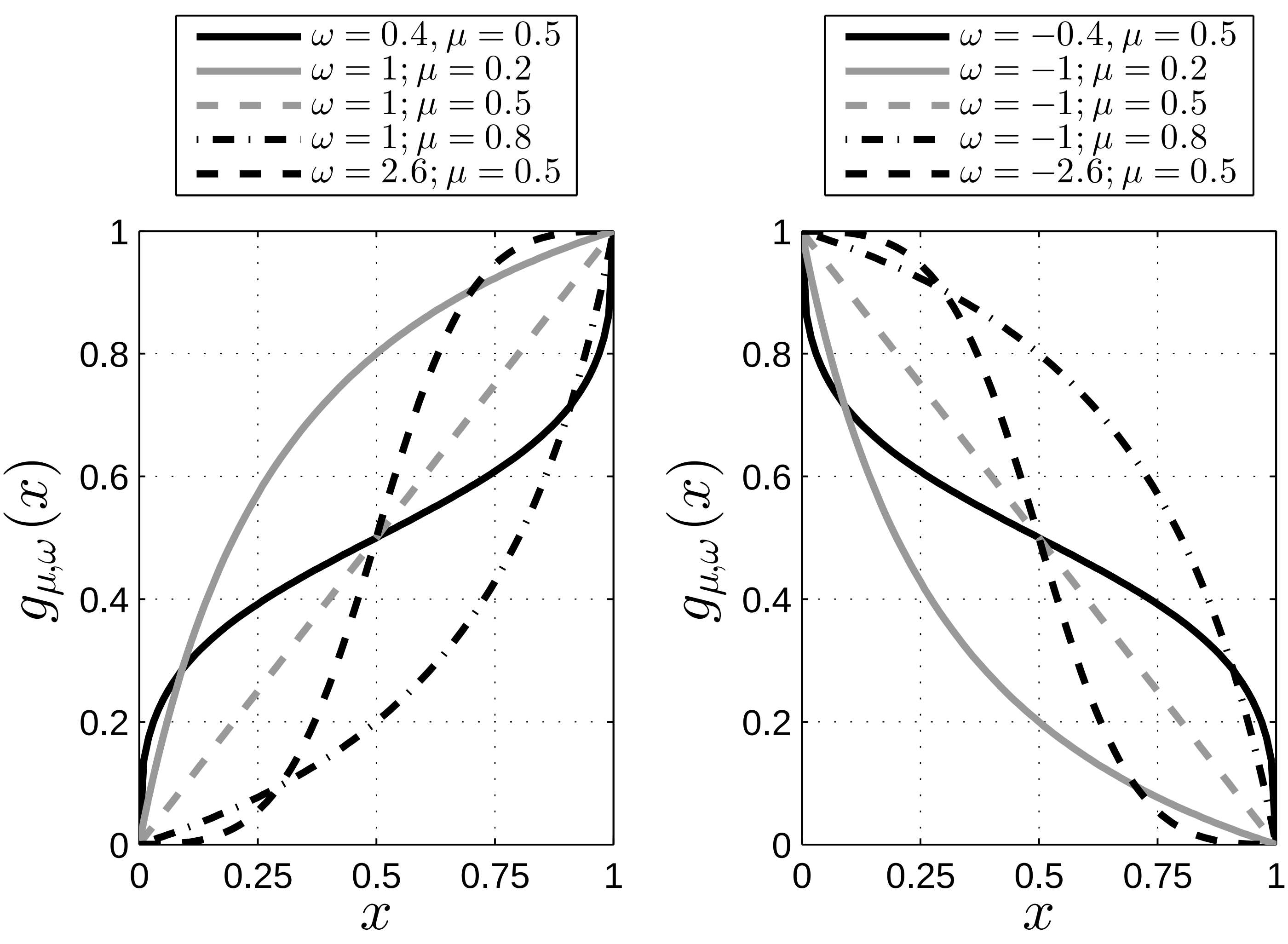
Figure_2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$\omega = 1; \mu = 0.2$
$\omega = 1; \mu = 0.5$
$\omega = 1; \mu = 0.8$
$\omega = 2.6; \mu = 0.5$
$\omega = -0.4, \mu = 0.5$
$\omega = -1; \mu = 0.2$
$\omega = -1; \mu = 0.5$
$\omega = -1; \mu = 0.8$
$\omega = -2.6; \mu = 0.5$
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x
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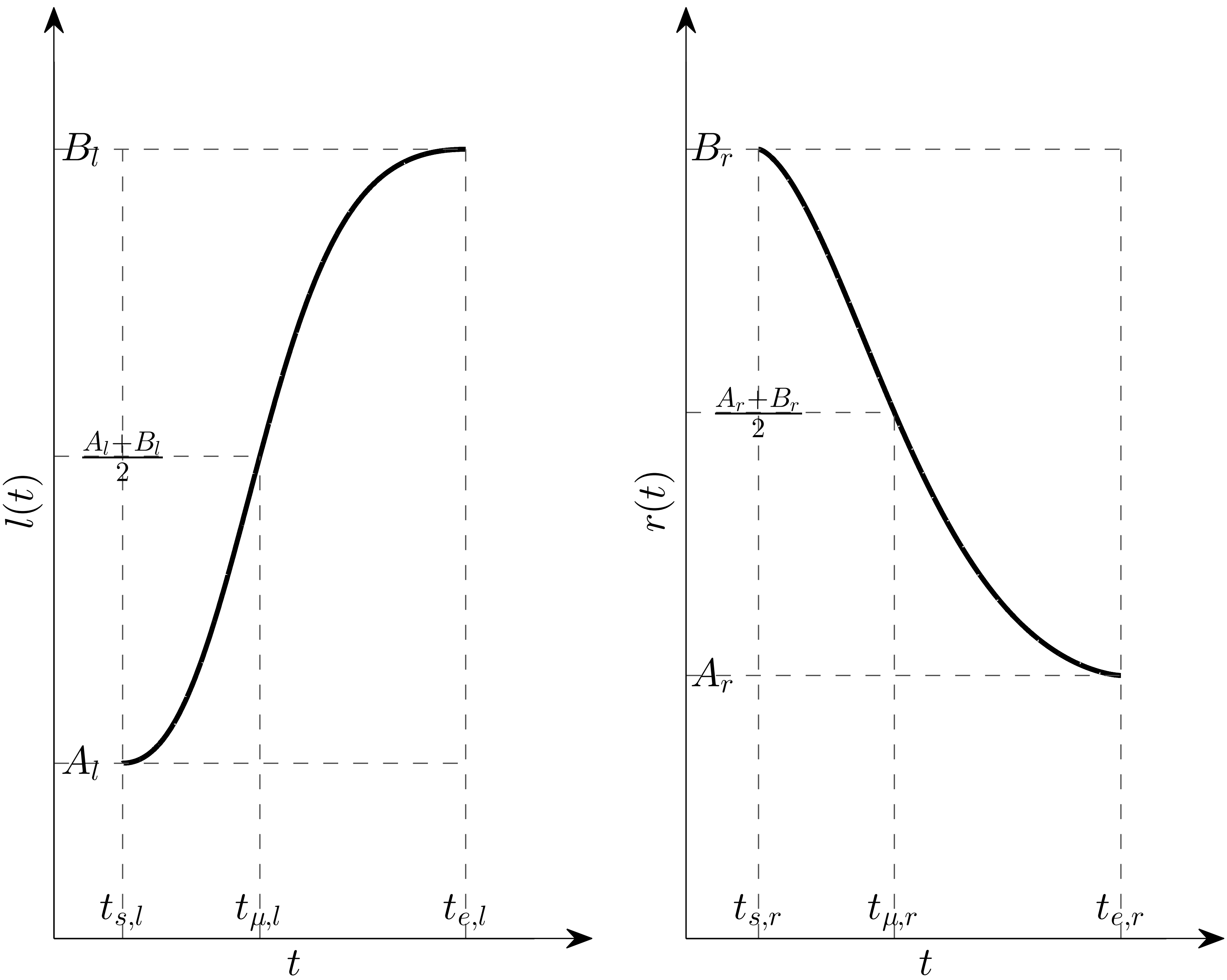
0.75

Figure_3



$l(t)$
B_l
$\frac{A_l+B_l}{2}$
A_l
$t_{s,l}$
$t_{\mu,l}$
$t_{e,l}$
t
$r(t)$
B_r
$\frac{A_r+B_r}{2}$
A_r
$t_{s,r}$
$t_{\mu,r}$
$t_{e,r}$
t

Figure_4

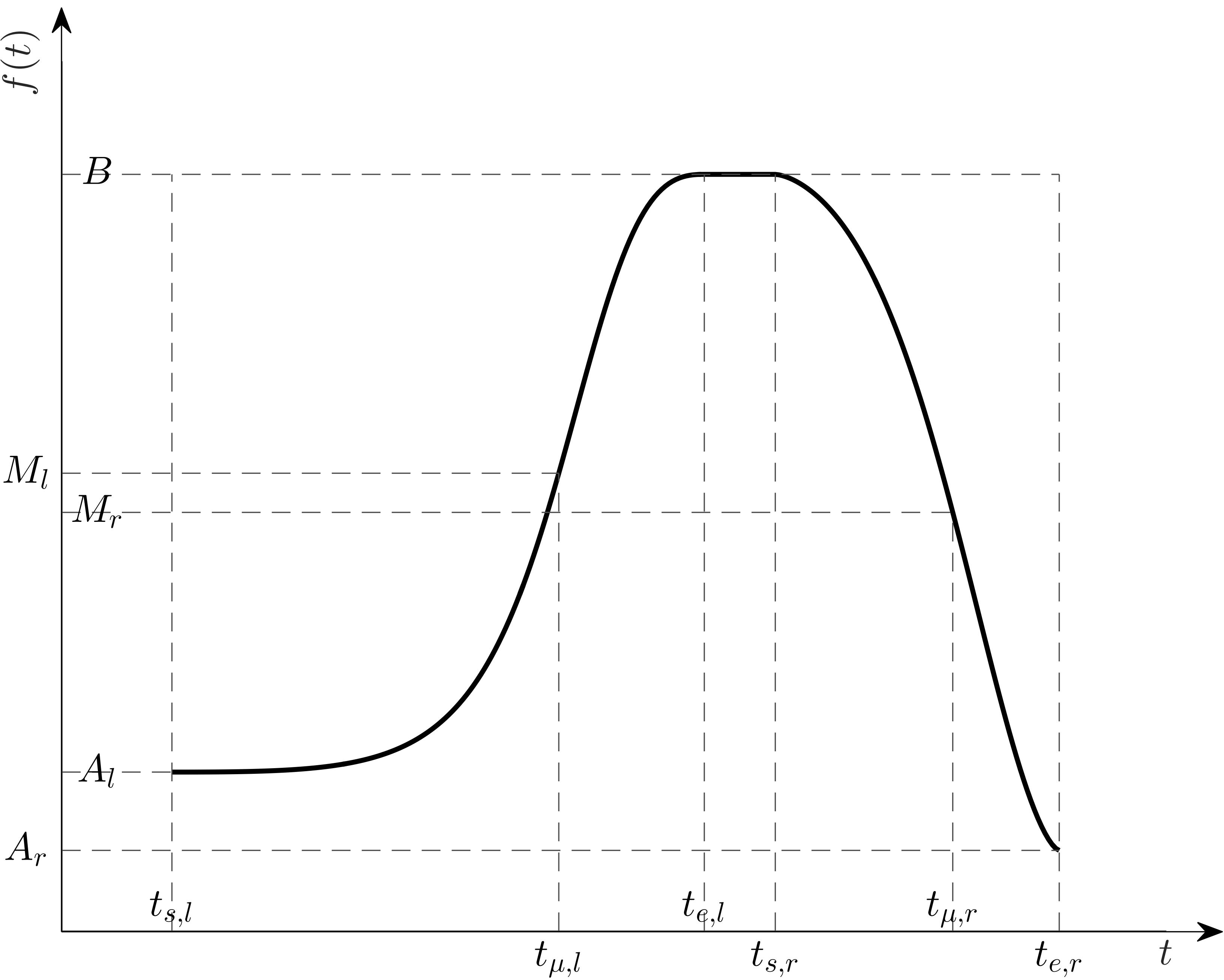
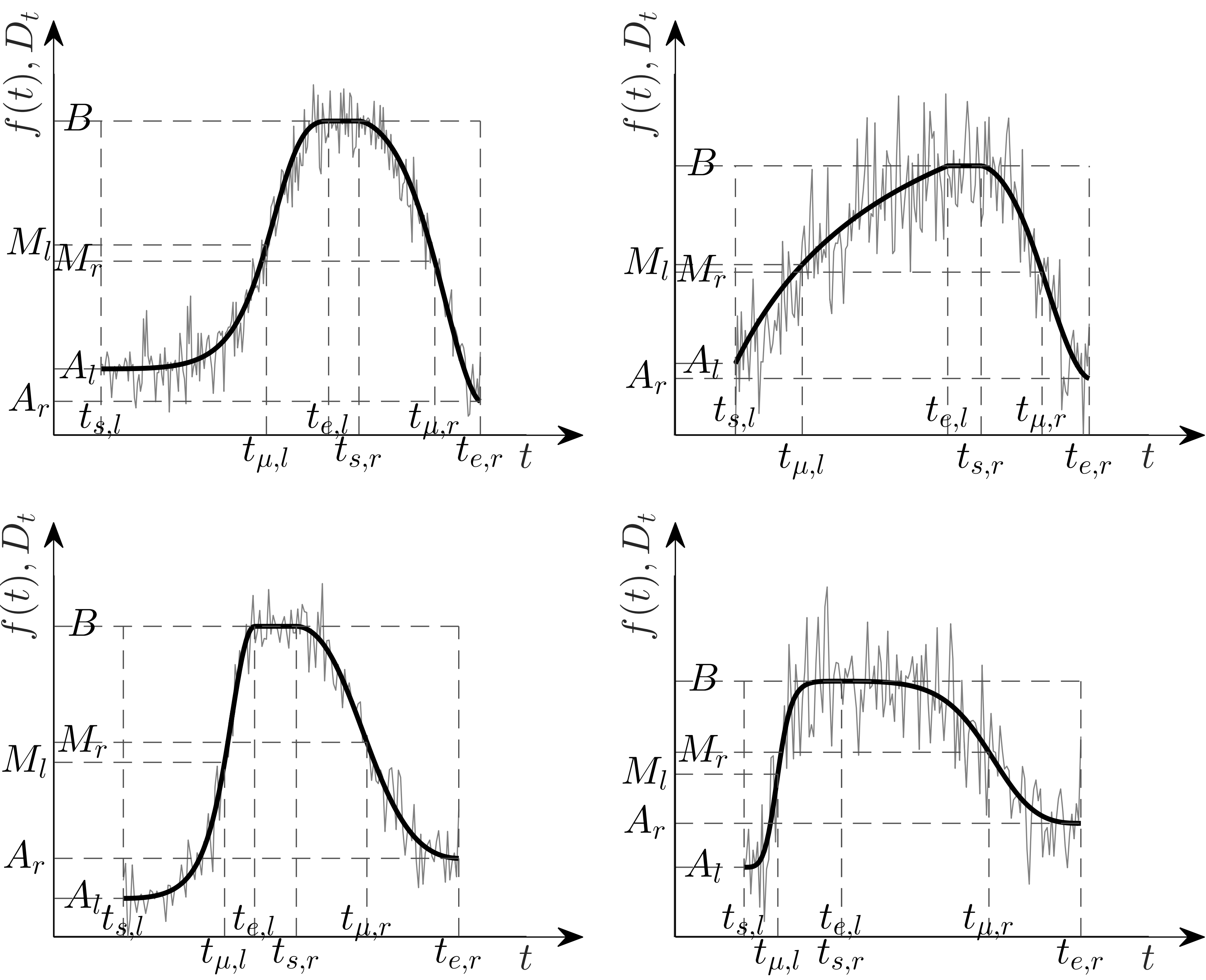


Figure_5



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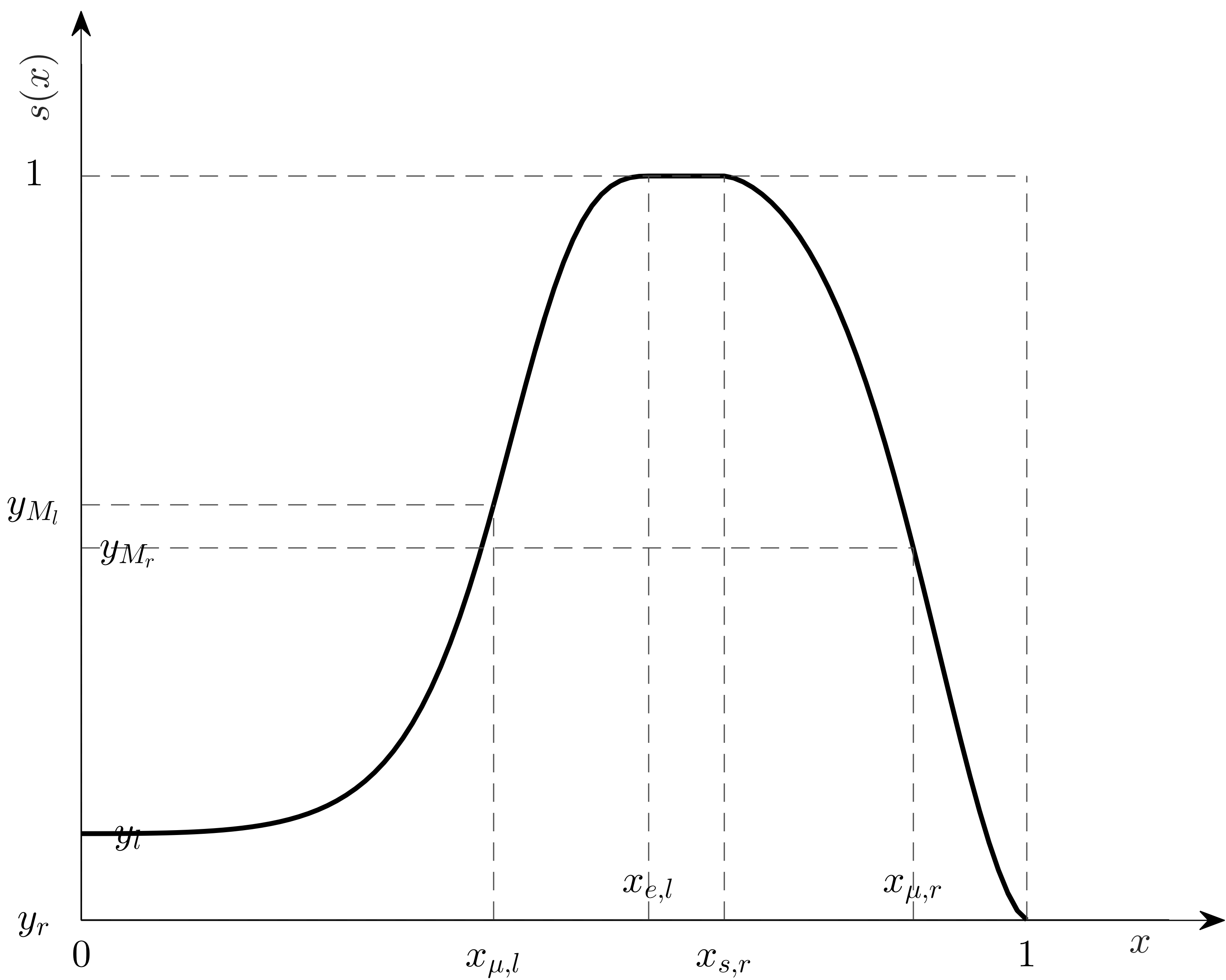
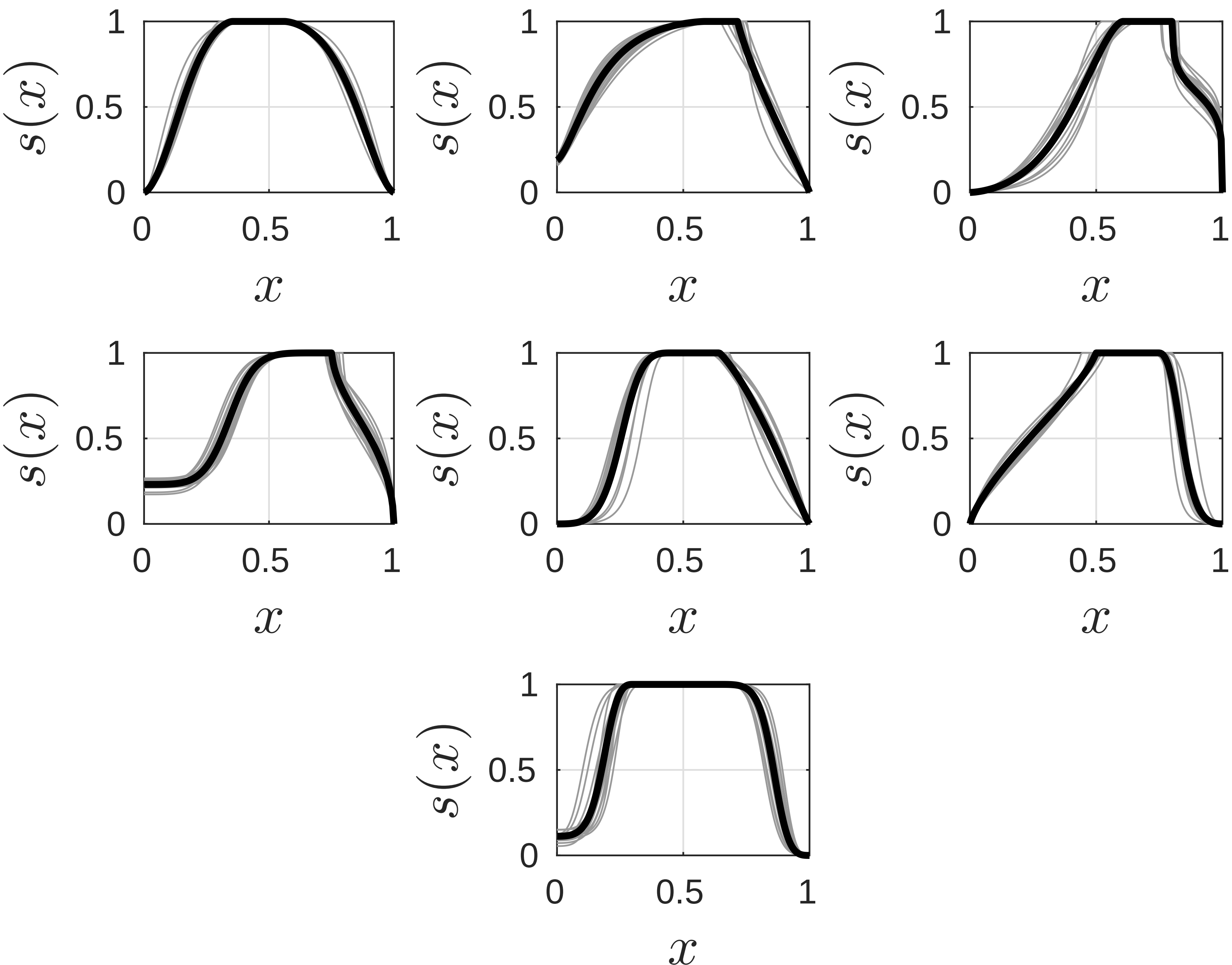


Figure_7



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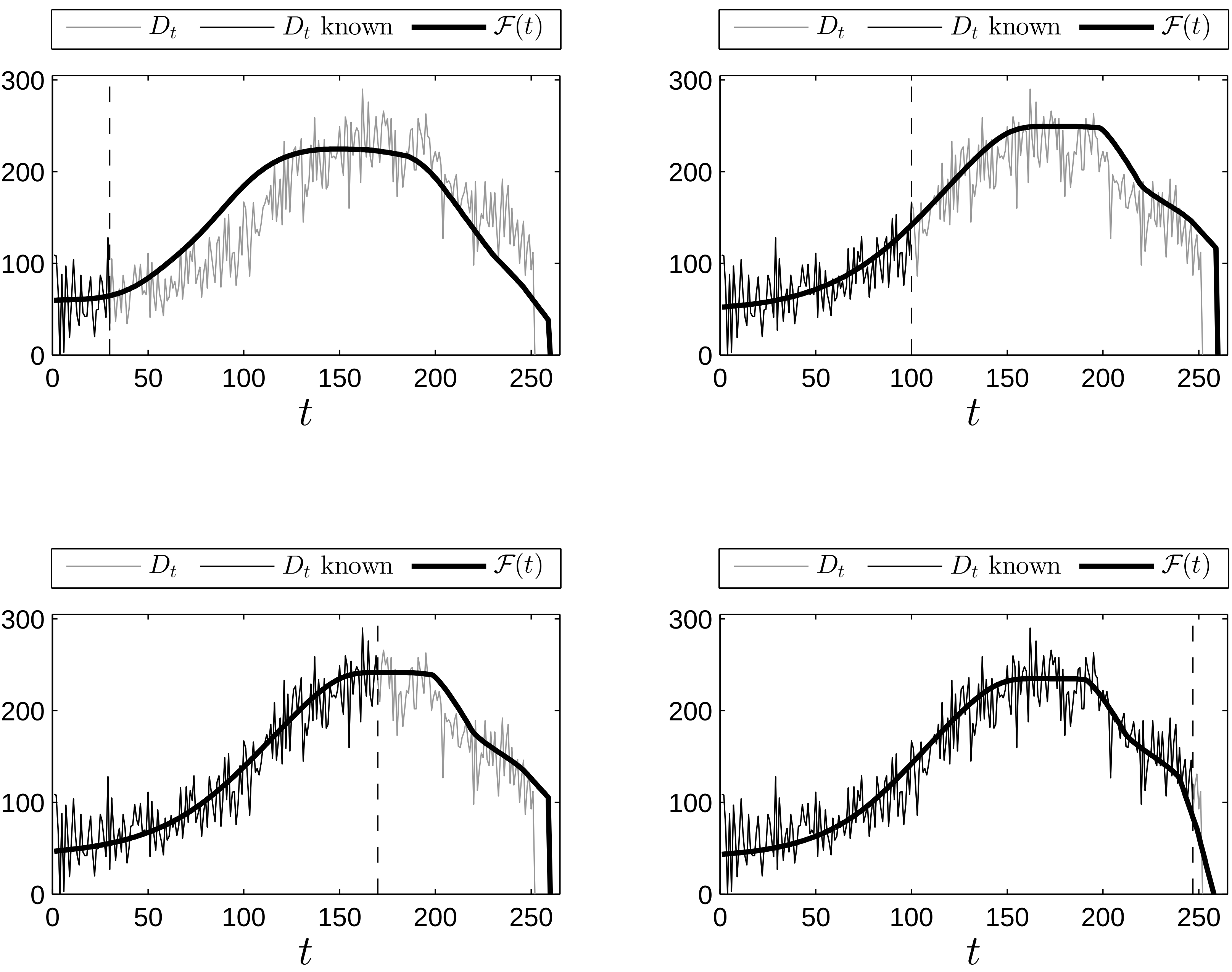


Figure_9

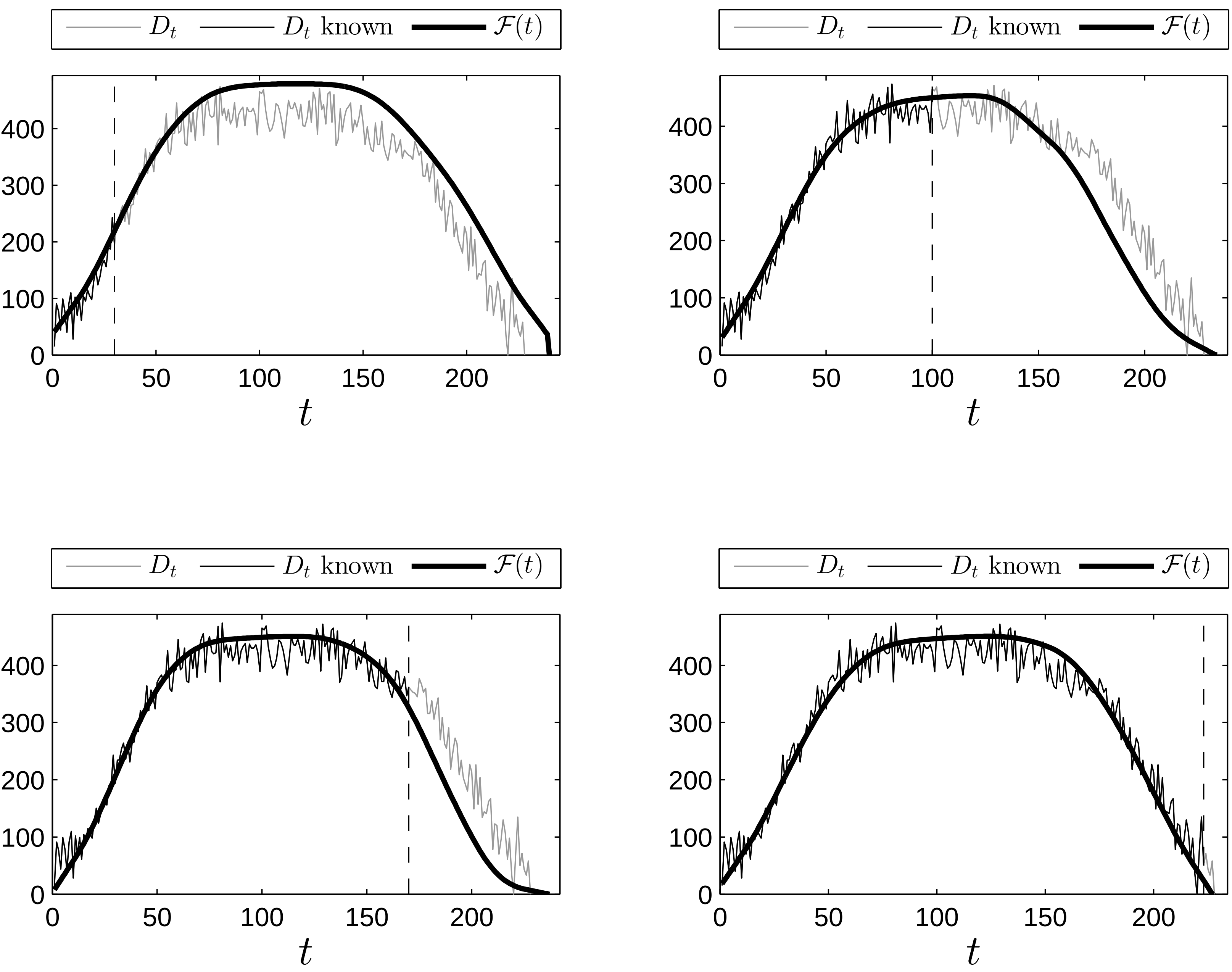


Figure_10

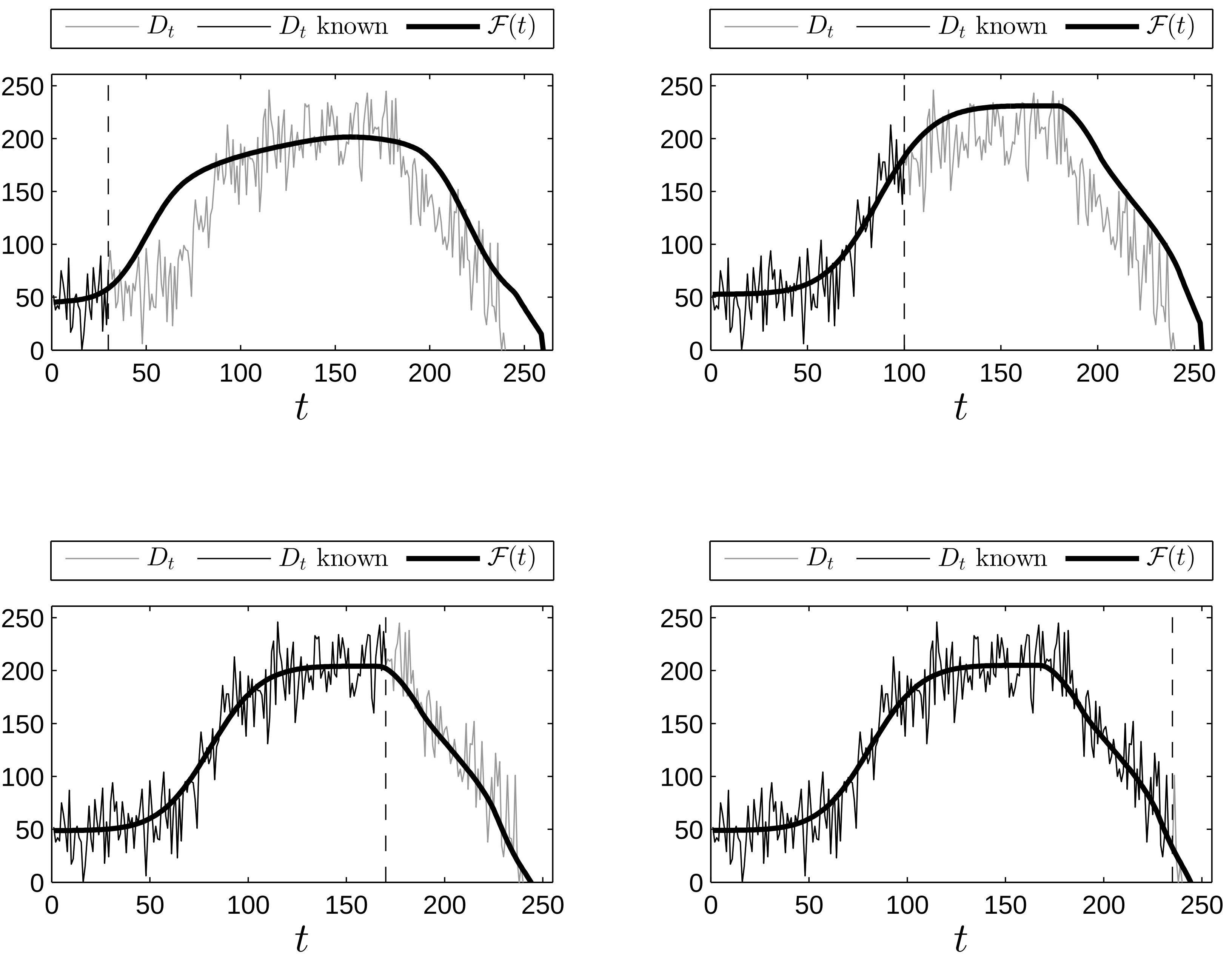


Figure_11

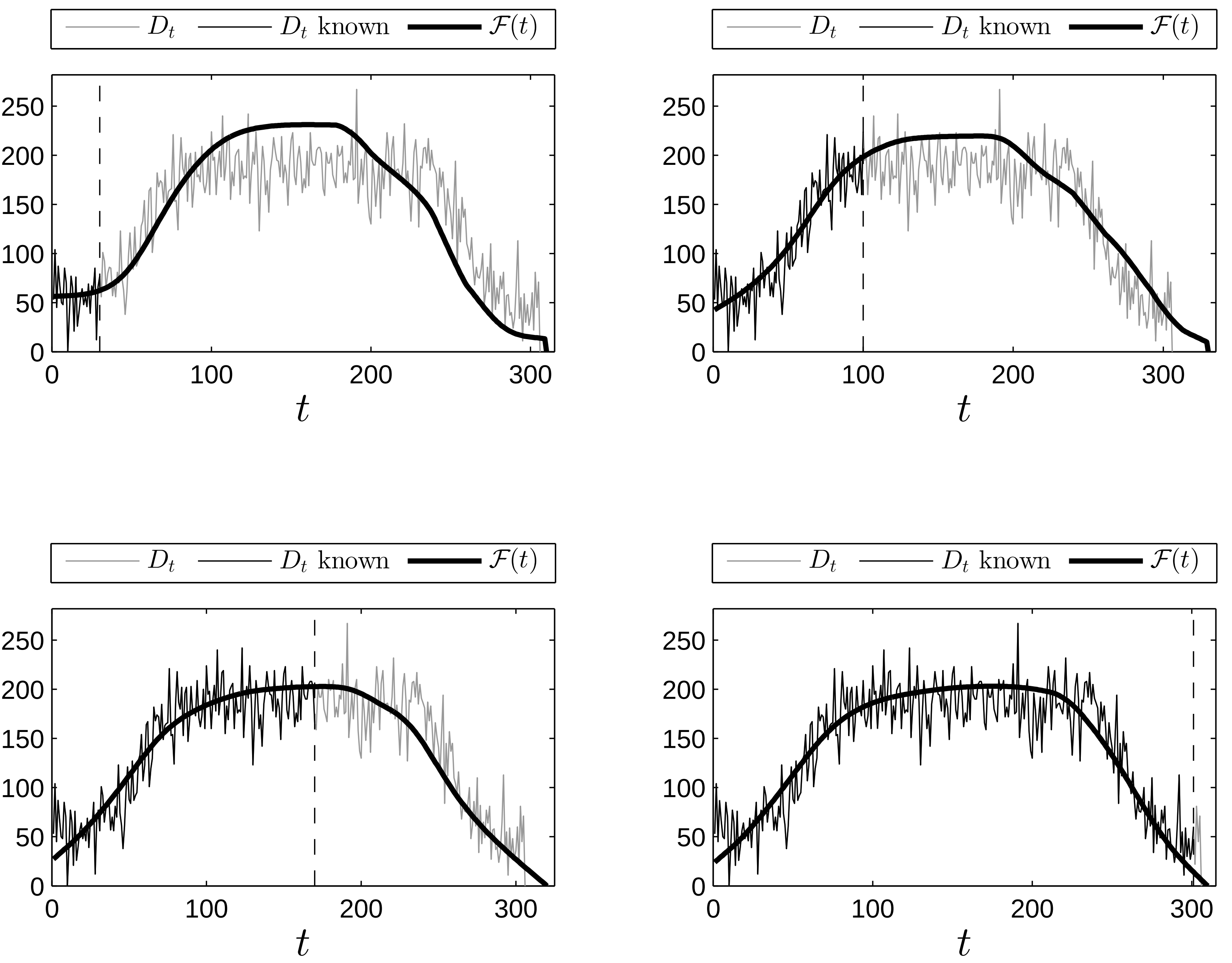


Figure_12

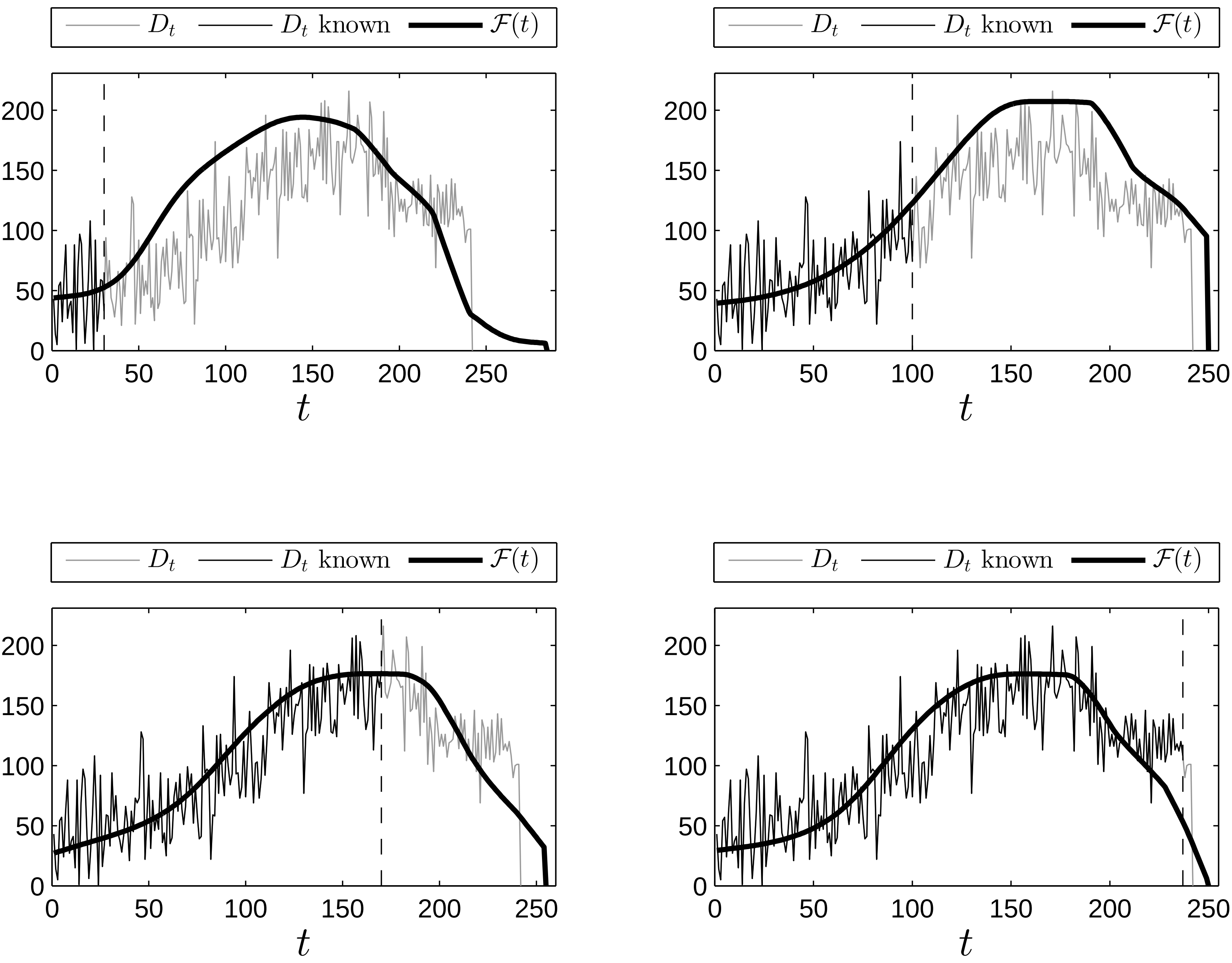


Figure_13

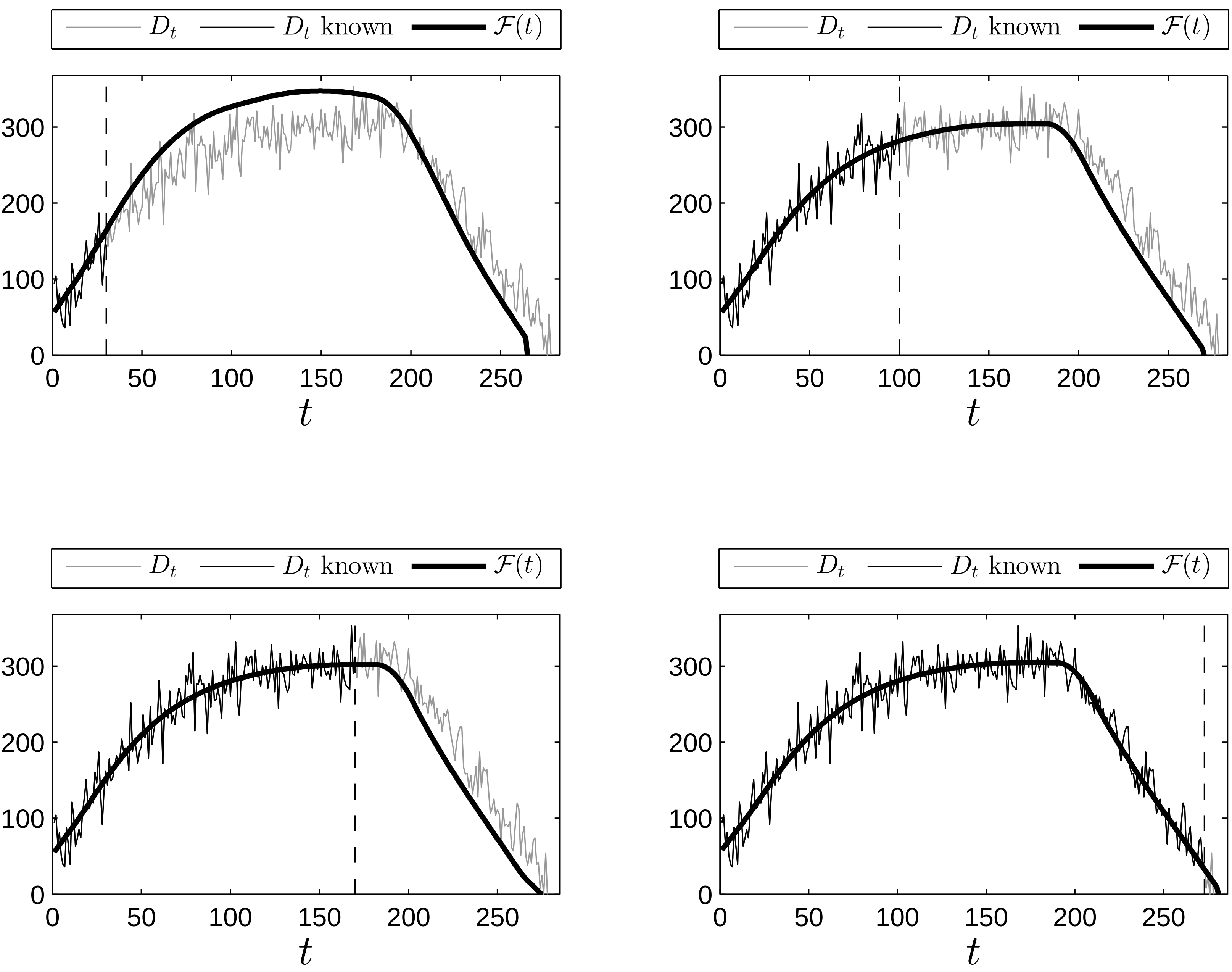
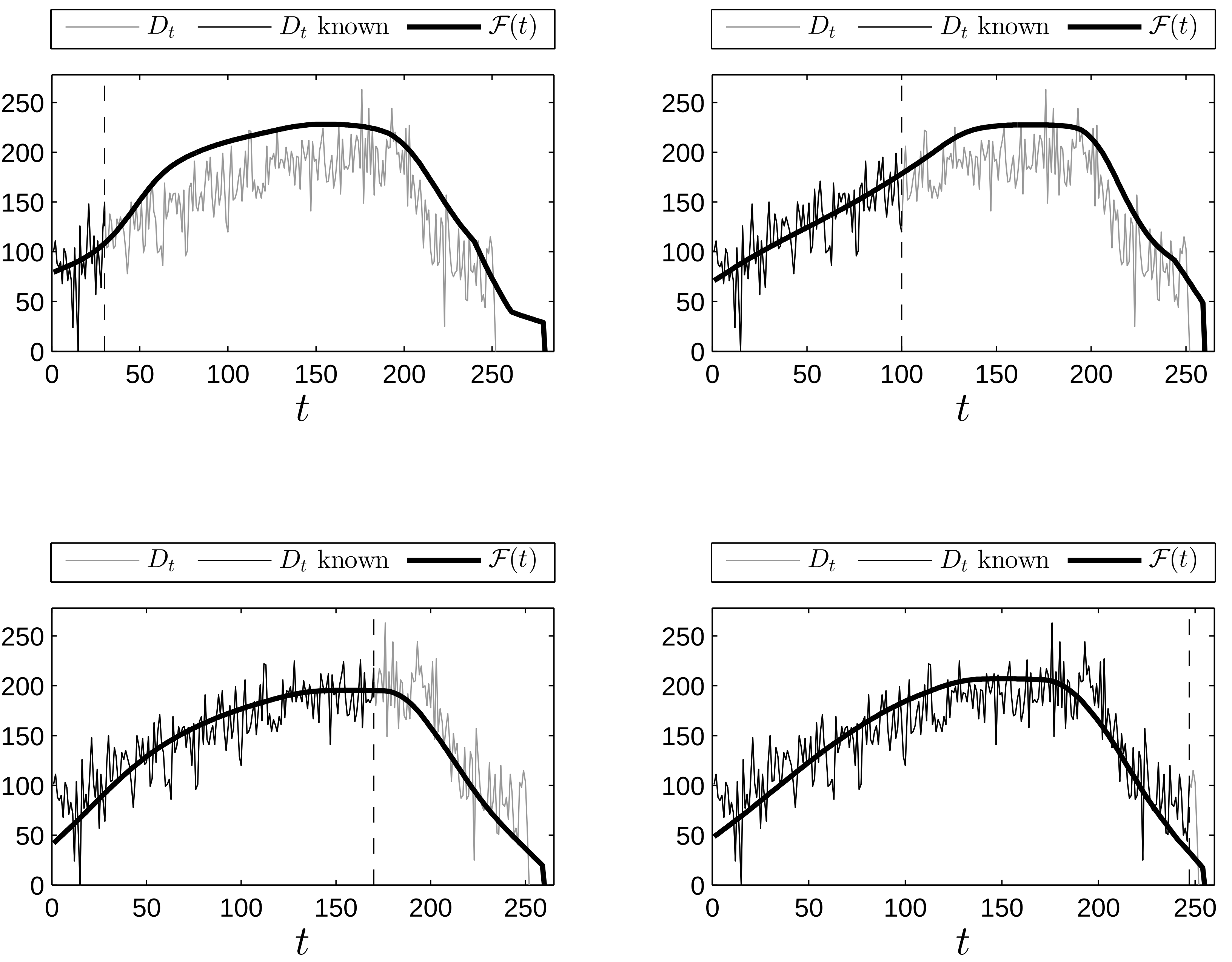


Figure_14



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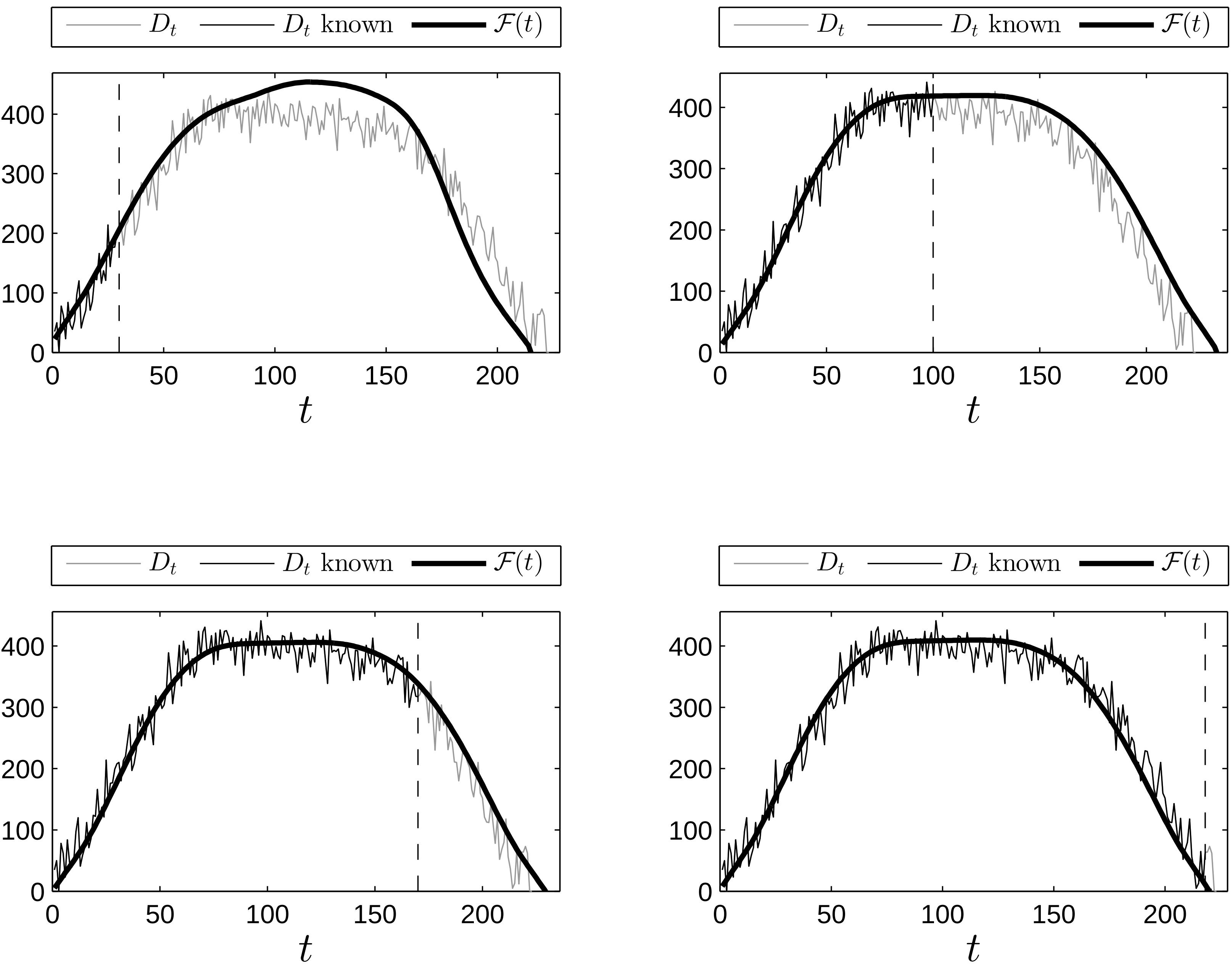


Figure_16

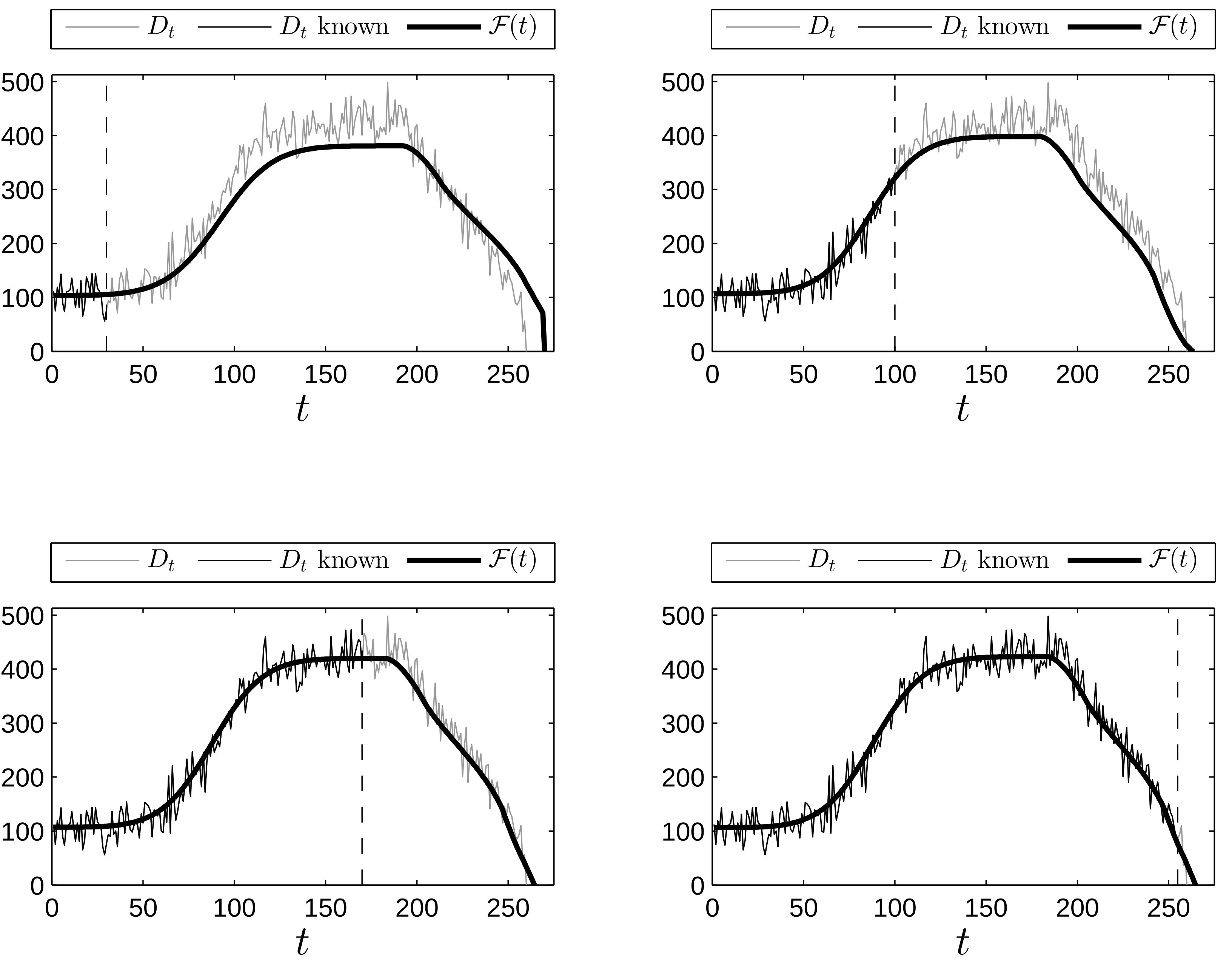
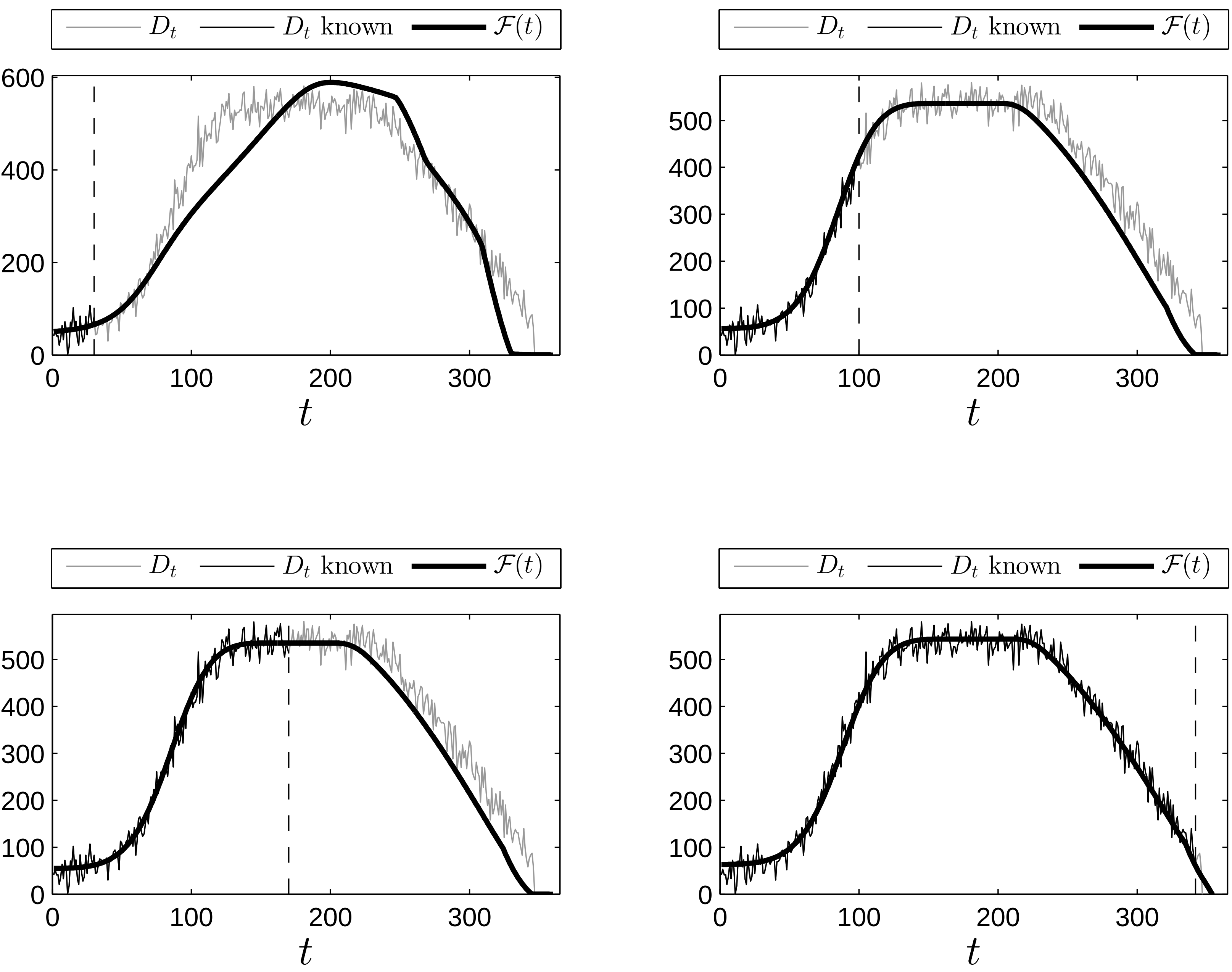

D_t
D_t known
$\mathcal{F}(t)$
0
100
200
300
400
500
0
50
100
150
200
250
t

Figure_17



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