A Family of Topology–Preserving 3D Parallel 6–Subiteration Thinning Algorithms

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Abstract. Thinning is an iterative layer-by-layer erosion until only the skeleton-like shape features of the objects are left. This paper presents a family of new 3D parallel thinning algorithms that are based on our new sufficient conditions for 3D parallel reduction operators to preserve topology. The strategy which is used is called subiteration-based: each iteration step is composed of six parallel reduction operators according to the six main directions in 3D. The major contributions of this paper are: 1) Some new sufficient conditions for topology preserving parallel reductions are introduced. 2) A new 6–subiteration thinning scheme is proposed. Its topological correctness is guaranteed, since its deletion rules are derived from our sufficient conditions for topology preservation. 3) The proposed thinning scheme with different characterizations of endpoints yields various new algorithms for extracting centerlines and medial surfaces from 3D binary pictures.

Keywords: shape representation, skeletonization, thinning, topology preservation.

1 Introduction

Skeleton-like shape features (i.e., centerline, medial surface, and topological kernel) extracted from 3D binary images play an important role in numerous applications of image processing and pattern recognition [19].

Parallel thinning algorithms [4] are capable of extracting skeleton-like shape descriptors in a topology preserving way [6]. Their iteration steps are composed of some parallel reduction operators: some object points having value of “1” in a binary image that satisfy certain topological and geometric constrains are deleted (i.e., changed to “0” ones) simultaneously, and the entire process is repeated until no points are deleted.

An object point is simple if its deletion does not alter the topology of the image [6]. In a phase of a parallel thinning algorithm, a set of simple points is deleted simultaneously that may not preserve the topology. A possible approach to overcome this problem is to use subiteration-based thinning (often referred to as directional or border sequential strategy) [4]: each iteration step is composed of k subiterations ($k \geq 2$), where only border points of certain kind are deleted.

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Since there are six major directions in 3D, most of existing parallel 3D directional thinning algorithms use six subiterations [3,14].

Object points having value of "1" in a binary image are endpoints if they provide important geometrical information relative to the shape of the objects to be represented. Surface-thinning algorithms are to extract *medial surfaces* by preserving *surface-endpoints*, curve-thinning algorithms produce *centerlines* by preserving *curve-endpoints*, and *topological kernels* (i.e., minimal structures which are topologically equivalent to the original objects) can be generated if no endpoint characterization is considered during the thinning process [2]. Medial surfaces are usually extracted from general shapes, tubular structures can be represented by their centerlines, and extracting topological kernels are useful in topological description.

The deletion rules of existing parallel thinning algorithms are generally given by matching templates with specific and "built-in" endpoint characterizations [1,3,8,9,10,11,14,15,16,20] with the exceptions of some 3D fully parallel algorithms [17] and some 3D subfield-based thinning algorithms [12,13]. In this paper, we introduce a general scheme for 6-subiteration 3D parallel thinning that is based on our new sufficient conditions for topology preservation. The proposed scheme coupled with different types of endpoints yields various topology preserving thinning algorithms.

The rest of this paper is organized as follows. Section 2 gives the basic notions of 3D digital topology. Then in Section 3 we propose our sufficient conditions for 3D parallel reduction operators to preserve topology. Section 4 presents a family of new 6-subiteration 3D parallel thinning algorithms. Finally, Section 5 gives five variations for the proposed thinning scheme by considering five different characterizations of endpoints.

2 Basic Notions and Results

Let p be a point in the 3D digital space $\mathbb{Z}^3$. Let us denote $N_j(p)$ (for $j = 6, 18, 26$) the set of points that are *j-adjacent* to point p (see Fig. 1a).

The *3D binary* $(26, 6)$ *digital picture* $\mathcal{P}$ is a quadruple $\mathcal{P} = (\mathbb{Z}^3, 26, 6, X)$ [6], where each element of $\mathbb{Z}^3$ is called a *point* of $\mathcal{P}$, each point in $X \subseteq \mathbb{Z}^3$ is called a *black point* and it has a value of "1", each point in $\mathbb{Z}^3 \backslash X$ is called a *white point* and value of "0" is assigned to it. 26-connectivity (i.e., the reflexive and transitive closure of the 26-adjacency relation) is considered for black points forming the objects, and 6-connectivity (i.e., the reflexive and transitive closure of the 6-adjacency) is considered for white points [6] (see Fig. 1a). Maximal 26-connected components of black points are called *objects*.

A black point is called a *border point* in a $(26, 6)$ picture if it is 6-adjacent to at least one white point. A border point p is called a **U**-*border point* if the point marked **U**$= u(p)$ in Fig. 1a is a white point. We can define **D**-, **N**-, **E**-, **S**-, and **W**-border points in the same way. A black point is called an *interior point* if it is not a border point. There are three *opposite pairs* **U**-**D**, **N**-**S**, and **E**-**W** in $N_6(p)\backslash\{p\}$.

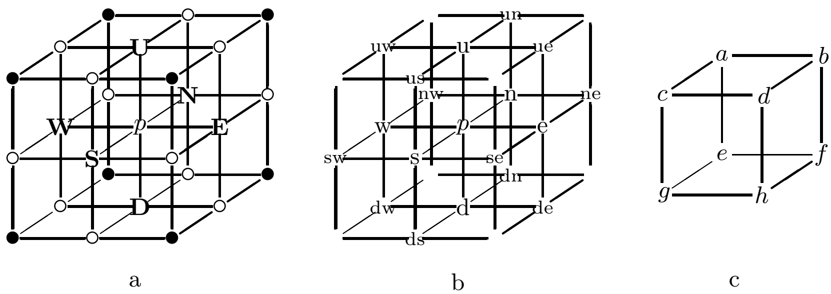


Fig. 1. Frequently used adjacencies in $\mathbb{Z}^3$ (a). The set $N_6(p)$ contains point p and the six points marked **U**, **D**, **N**, **E**, **S**, and **W**. The set $N_{18}(p)$ contains $N_6(p)$ and the twelve points marked "○". The set $N_{26}(p)$ contains $N_{18}(p)$ and the eight points marked "●". Notation for the points in $N_{18}(p)$ (b). The $2 \times 2 \times 2$ cube that contains an object (c)

A *parallel reduction operator* changes a set of black points to white ones (which is referred to as deletion). A 3D parallel reduction operator does *not* preserve topology if any object is split or is completely deleted, any cavity (i.e., maximal 6-connected component of white points) is merged with another cavity, a new cavity is created, or a hole (that donuts have) is eliminated or created.

A black point is called a *simple* point if its deletion does not alter the topology of the image [6]. Note that simplicity of point p in $(26, 6)$ pictures is a local property that can be decided by investigating the set $N_{26}(p)$ [6].

Parallel reduction operators delete a set of black points and not only a single simple point. Ma gave some *sufficient conditions* for 3D parallel reduction operators to preserve topology [7]. Those conditions require some additional concepts to be defined. Let $\mathcal{P}$ be a $(26, 6)$ picture. The set $D = \{d_1, \ldots, d_k\}$ of black points is called a *simple set* of $\mathcal{P}$ if D can be arranged in a sequence $\langle d_{i_1}, \ldots, d_{i_k} \rangle$ in which d_{i_1} is simple and each d_{i_j} is simple after $\{d_{i_1}, \ldots, d_{i_{j-1}}\}$ is deleted from $\mathcal{P}$, for $j = 2, \ldots, k$. (By definition, let the empty set be simple.) A *unit lattice square* is a set of four mutually 18-adjacent points in $\mathbb{Z}^3$; a *unit lattice cube* is set of eight mutually 26-adjacent points in $\mathbb{Z}^3$.

Theorem 1. [7] *A 3D parallel reduction operator is topology preserving for (26,6) pictures if all of the following conditions hold:*

1. *Only simple points are deleted.*
2. *If two, three, or four black corners of a unit lattice square are deleted, then these corners form a simple set.*
3. *No object contained in a unit lattice cube is deleted completely.*

3 New Sufficient Conditions for Topology Preserving Parallel Reductions

Theorem 1 provides a general method of verifying that a parallel thinning algorithm preserves topology [5]. In this section, we present some new sufficient conditions for topology preservation as a basis for designing 3D 6-subiteration parallel thinning algorithms. In order to introduce our new sufficient conditions for topology preserving parallel reductions that delete **U**-border points, we define two special kinds of point sets.

Definition 1. *Let* $p \in X$ *be a black point in picture* $(\mathbb{Z}^3, 26, 6, X)$ *and let* $S(p) \subseteq X \setminus \{p\}$ *be a set of black points such that* $S(p) \cup \{p\}$ *is contained in a unit lattice square. The set* $S(p)$ *is called a* **U**-square-considerable set *if for any point* $s \in S(p) \cup \{p\}$, $u(s) \notin S(p) \cup \{p\}$.

We can define **D**-, **N**-, **E**-, **S**-, and **W**-square-considerable sets in the same way. Let us state some properties of **U**-square-considerable sets.

Proposition 1. *The following 33 sets may be* **U***-square-considerable ones (see Fig.* 1b*):*
$\emptyset$, $\{un\}$, $\{ue\}$, $\{us\}$, $\{uw\}$, $\{nw\}$, $\{n\}$, $\{ne\}$, $\{w\}$, $\{e\}$, $\{sw\}$, $\{s\}$,
$\{se\}$, $\{dn\}$, $\{de\}$, $\{ds\}$, $\{dw\}$, $\{nw,n\}$, $\{nw,w\}$, $\{n,w\}$, $\{ne,n\}$,
$\{ne,e\}$, $\{n,e\}$, $\{sw,s\}$, $\{sw,w\}$, $\{s,w\}$, $\{se,s\}$, $\{se,e\}$, $\{s,e\}$,
$\{nw,n,w\}$, $\{ne,n,e\}$, $\{sw,s,w\}$, $\{se,s,e\}$.

Proposition 2. *Any subset of a* **U***-square-considerable set is a* **U***-square-considerable set as well.*

These properties are obvious by careful examination of the points in $N_{18}(p)$ (see Fig. 1b).

Definition 2. *Let* $C \subseteq X$ *be an object of picture* $(\mathbb{Z}^3, 26, 6, X)$ *that is contained in a unit lattice cube.* C *is called a* **U**-cube-considerable object *if all of the following conditions hold:*

1. $\#(C) \geq 2$ *(where* $\#(C)$ *denotes the number of elements in* C*).*
2. *For any point* $c \in C$, $u(c) \notin C$ *(i.e.,* C *must contain* **U***-border points).*
3. C *is not contained in a unit lattice square.*

We can define **D**-, **N**-, **E**-, **S**-, and **W**-cube-considerable objects in the same way. Let us state the two most important properties of **U**-cube-considerable objects.

Proposition 3. *For any* **U***-cube-considerable object* C, $\#(C) \leq 4$.

It is easy to see that any object contained in a unit lattice cube that contains 5, 6, 7, or 8 points, must contain at least one element that is not a **U**-border point (i.e., it must contain a pair of points p and $u(p)$).

Proposition 4. *There are 32 possible* **U***-cube-considerable objects.*

The possible **U**-cube-considerable object are listed as follows (see Fig. 1c):
$\{a,h\}$, $\{a,h,b\}$, $\{a,h,b,c\}$, $\{a,h,b,g\}$, $\{a,h,c\}$, $\{a,h,c,f\}$, $\{a,h,f\}$, $\{a,h,f,g\}$, $\{a,h,g\}$, $\{b,g\}$, $\{b,g,a\}$, $\{b,g,a,d\}$, $\{b,g,d\}$, $\{b,g,d,e\}$, $\{b,g,e\}$, $\{b,g,e,h\}$, $\{b,g,h\}$, $\{c,f\}$, $\{c,f,a\}$, $\{c,f,a,d\}$, $\{c,f,d\}$, $\{c,f,d,e\}$, $\{c,f,e\}$, $\{c,f,e,h\}$, $\{c,f,h\}$, $\{d,e\}$, $\{d,e,b\}$, $\{d,e,b,c\}$, $\{d,e,c\}$, $\{d,e,f\}$, $\{d,e,f,g\}$, $\{d,e,g\}$.

The lexicographical order relation "$\prec$" between two distinct points $p = (p_x, p_y, p_z)$ and $q = (q_x, q_y, q_z)$ is defined as follows:

$$p \prec q \quad \Leftrightarrow \quad (p_z < q_z) \vee (p_z = q_z \wedge p_y < q_y) \vee (p_z = q_z \wedge p_y = q_y \wedge p_x < q_x).$$

Definition 3. *Let $C \subseteq \mathbb{Z}^3$ be a set of points. Point $p \in C$ is the* smallest element *of C if for any $q \in C \backslash \{p\}$, $p \prec q$.*

We are now ready to state our new sufficient conditions for topology preserving parallel reductions that delete **U**-border points. Note that sufficient conditions for simultaneous deletion of **D**-, **N**-, **E**-, **S**-, and **W**-border points can be given in the same way.

Theorem 2. *Let T be a parallel reduction operator. Let p be any black point in any picture $(\mathbb{Z}^3, 26, 6, X)$ such that point p is deleted by T. Operator T is topology preserving for $(26, 6)$ pictures if all of the following conditions hold:*

1. *Point p is a simple and* **U***-border point in picture $(\mathbb{Z}^3, 26, 6, X)$.*
2. *For any* **U***-square-considerable set $S(p)$ that contains simple and* **U***-border points in $(\mathbb{Z}^3, 26, 6, X)$, p is a simple point in picture $(\mathbb{Z}^3, 26, 6, X \backslash S(p))$.*
3. *Point p is not the smallest element of any* **U***-cube-considerable object.*

Proof. To prove it, we show that the parallel reduction operator T satisfies all conditions of Theorem 1.

1. Operator T may delete simple points by Condition 1 of Theorem 2. Hence Condition 1 of Theorem 1 is satisfied.
2. Since operator T may delete **U**-border points (by Condition 1 of Theorem 2), it is sufficient to deal with the 33 possible **U**-square-considerable sets (see Definition 1, Proposition 1, and Proposition 2). The following points have to be checked:
   (a) Suppose that $S(p) = \emptyset$ $(\#(S(p)) = 0)$. Since Condition 1 of Theorem 2 holds, point p is simple in $(\mathbb{Z}^3, 26, 6, X) = (\mathbb{Z}^3, 26, 6, X \backslash S(p))$. Therefore, Condition 2 of Theorem 1 is satisfied.
   (b) Let a and b be two corners of a unit lattice square that are deleted by T. If $p = b$ and $S(p) = \emptyset$, then b is a simple point in $(\mathbb{Z}^3, 26, 6, X)$ by case (a). Suppose that $p = a$ and $S(p) = \{b\}$. Since Condition 2 of Theorem 2 holds, point a is simple in $(\mathbb{Z}^3, 26, 6, X \backslash S(p))$. Consequently, $\{a, b\}$ is a simple set. Therefore, Condition 2 of Theorem 1 is satisfied.
   (c) Let a, b, and c be three corners of a unit lattice square that are deleted by T. In this case b and c are two corners of a unit lattice square and $\{b, c\}$ is a simple set by case (b). Suppose that $p = a$ and $S(p) = \{b, c\}$. Since Condition 2 of Theorem 2 holds, point a is simple in $(\mathbb{Z}^3, 26, 6, X \backslash S(p))$. Consequently, the set $\{a, b, c\}$ is simple. Therefore, Condition 2 of Theorem 1 is satisfied.
   (d) Let a, b, c, and d be four corners of a unit lattice square that are deleted by T. In this case b, c and d are three corners of a unit lattice square

and $\{b, c, d\}$ is a simple set by case (c). Suppose that $p = a$ and $S(p) = \{b, c, d\}$. Since Condition 2 of Theorem 2 holds, point a is simple in $(\mathbb{Z}^3, 26, 6, X \setminus S(p))$. Consequently, the set $\{a, b, c, d\}$ is simple. Therefore, Condition 2 of Theorem 1 is satisfied.

3. Let us consider object C that is contained in a unit lattice cube. The following points have to be checked:
   (a) Suppose that $\#(C) = 1$, $C = \{a\}$. In this case, a is an isolated point that is not simple. Since Condition 1 of Theorem 2 holds, point a cannot be deleted by T. Therefore, Condition 3 of Theorem 1 is satisfied.
   (b) Suppose that $\#(C) = 2$, $C = \{a, b\}$. If a and b are two corners of a unit lattice square, then C cannot be deleted completely by Condition 2 of Theorem 2. If C contains a point that is not a **U**-border point, then C cannot be deleted completely by Condition 1 of Theorem 2. Otherwise C is a **U**-cube-considerable object and its smallest element cannot be deleted by Condition 3 of Theorem 2. Therefore, Condition 3 of Theorem 1 is satisfied.
   (c) Suppose that $\#(C) = 3$, $C = \{a, b, c\}$. If a, b, and c are three corners of a unit lattice square, then C cannot be deleted completely by Condition 2 of Theorem 2. If C contains a point that is not a **U**-border point, then C cannot be deleted completely by Condition 1 of Theorem 2. Otherwise C is a **U**-cube-considerable object and its smallest element cannot be deleted by Condition 3 of Theorem 2. Therefore, Condition 3 of Theorem 1 is satisfied.
   (d) Suppose that $\#(C) = 4$, $C = \{a, b, c, d\}$. If a, b, c, and d are four corners of a unit lattice square, then C cannot be deleted completely by Condition 2 of Theorem 2. If C contains a point that is not a **U**-border point, then C cannot be deleted completely by Condition 1 of Theorem 2. Otherwise C is a **U**-cube-considerable object and its smallest element cannot be deleted by Condition 3 of Theorem 2. Therefore, Condition 3 of Theorem 1 is satisfied.
   (e) Suppose that $\#(C) > 4$. In this case, C must contain at least one point that is not a **U**-border point by Proposition 3. That point cannot be deleted by Condition 1 of Theorem 2. Therefore, Condition 3 of Theorem 1 is satisfied. □

4 The New 6-Subiteration Thinning Algorithms

Now we propose a set of new 6-subiteration 3D parallel thinning algorithms. Their deletable points are derived directly from Theorem 2.

Let us consider an arbitrary characterization of endpoints that is called as type $\mathcal{E}$. The algorithm denoted by **6SI-**$\mathcal{E}$ is our 6-subiteration 3D parallel thinning algorithm that preserves endpoints of type $\mathcal{E}$ (see Algorithm 1).

The usual ordered list of the deletion directions $\langle\ \mathbf{U}, \mathbf{D}, \mathbf{N}, \mathbf{E}, \mathbf{S}, \mathbf{W}\ \rangle$ [3,14] is considered in Algorithm **6SI-**$\mathcal{E}$. Note that subiteration-based thinning algorithms are not invariant under the order of deletion directions (i.e., choosing different orders may yield various results).

Algorithm 1

```
Input: picture (ℤ³, 26, 6, X)
Output: picture (ℤ³, 26, 6, Y)
Y = X
repeat
  // one iteration step
  for each i ∈ {U, D, N, E, S, W} do
    // subiteration for deleting some i-border points
    D(i) = { p | p is an i-ℰ-deletable point in Y }
    Y = Y \ D(i)
until D(U) ∪ D(D) ∪ D(N) ∪ D(E) ∪ D(S) ∪ D(W) = ∅
```

In the first subiteration of our 6-subiteration thinning algorithms, the set of **U**-$\mathcal{E}$-deletable points are deleted simultaneously, and the set of **W**-$\mathcal{E}$-deletable points are deleted in the last (i.e., the 6th) subiteration. Now we lay down **U**-$\mathcal{E}$-*deletable* points. We can define **D**-, **N**-, **E**-, **S**-, and **W**-$\mathcal{E}$-*deletable* points in the same way.

Definition 4. *A black point p in picture* $(\mathbb{Z}^3, 26, 6, X)$ *is* **U**-$\mathcal{E}$-deletable *if all of the following conditions hold:*

1. *Point p is a simple and* **U***-border point, but it is not an endpoint of type $\mathcal{E}$ in picture* $(\mathbb{Z}^3, 26, 6, X)$.
2. *For any* **U***-square-considerable set $S(p)$ composed of simple points and* **U***-border points, but not endpoints of type $\mathcal{E}$ in picture* $(\mathbb{Z}^3, 26, 6, X)$*, point p remains simple in picture* $(\mathbb{Z}^3, 26, 6, X \setminus S(p))$.
3. *Point p is not the smallest element of any* **U***-cube-considerable object.*

We can state our main theorem.

Theorem 3. *Algorithm* **6SI-**$\mathcal{E}$ *is topology preserving for* $(26, 6)$ *pictures for arbitrary characterization of endpoints.*

Proof. It can readily be seen that Condition i of Definition 4 satisfies Condition i of Theorem 2 ($i = 1, 2, 3$). Consequently, the first subiteration of Algorithm **6SI-**$\mathcal{E}$ is a topology preserving parallel reduction for $(26, 6)$ pictures for arbitrary characterization of endpoints.

Similarly, it can be seen that the five parallel reductions assigned to the remaining five subiterations of Algorithm **6SI-**$\mathcal{E}$ are topology preserving as well. Hence, the entire algorithm composed of topology preserving reductions is topology preserving too.

Note that the proof of Theorem 2 does not consider the applied type of endpoints $\mathcal{E}$. Hence arbitrary characterizations of endpoints yield topologically correct 6-subiteration thinning algorithms. □

5 Examples of the New 6-Subiteration Thinning Algorithms

In Section 3, we defined the deletable points of the proposed 6-subiteration thinning algorithm **6SI-$\mathcal{E}$** that preserves endpoints of type $\mathcal{E}$. We stated that various characterizations of endpoints yield different algorithms. Here, we define four types of endpoints (**C1**, **C2**, **S1**, and **S2**) that determine four new thinning algorithms (**6SI-C1**, **6SI-C2**, **6SI-S1**, and **6SI-S2**). Furthermore, if no endpoints are preserved, then we get topological kernels. Therefore, no restriction is applied to an "endpoint" of type **TK**, which leads to the algorithm called **6SI-TK**.

Definition 5. *A "1" point p in picture* $(\mathbb{Z}^3, 26, 6, X)$ *is a curve-endpoint of type* **C1** *if* $(N_{26}(p)\setminus\{p\}) \cap X = \{q\}$ *(i.e., p is 26-adjacent to exactly one "1" point).*

Definition 6. *A "1" point p in picture* $(\mathbb{Z}^3, 26, 6, X)$ *is a curve-endpoint of type* **C2** *if* $(N_{26}(p)\setminus\{p\}) \cap X = \{q\}$ *and the number of elements in* $(N_{26}(q)\setminus\{q\}) \cap X$ *is less than or equal to 2.*

Definition 7. *A "1" point p in picture* $(\mathbb{Z}^3, 26, 6, X)$ *is a surface-endpoint of type* **S1** *if there is no interior point in the set* $N_6(p) \cap X$.

Note the characterization of surface-endpoints **S1** are applied in some existing thinning algorithms [1,11,16].

Definition 8. *A "1" point p in picture* $(\mathbb{Z}^3, 26, 6, X)$ *is a surface-endpoint of type* **S2** *if the set* $N_6(p)\setminus\{p\}$ *contains at least one opposite pair of "0" points.*

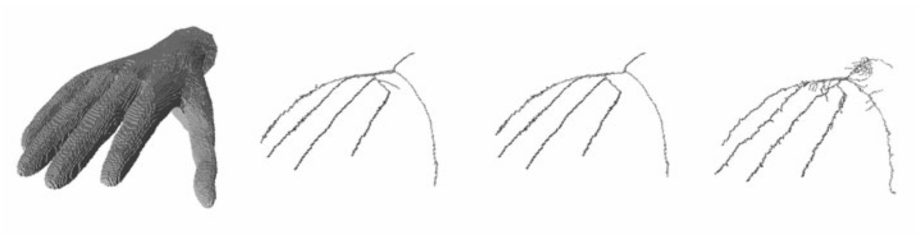
Note that the characterization of surface-endpoints **S2** is introduced in [15].

In experiments algorithm **6SI-TK** and the further algorithms based on the four types of endpoints according to Definitions 5-8 were tested on objects of different shapes. Here we present some illustrative examples below (Figs. 2-8). Our new algorithms are compared with the existing 6-subiteration curve-thinning algorithm **PK-C** [14] and surface thinning algorithm **GB-S** [3]. Numbers in parentheses mean the count of "1" points.

The tubular test objects in Figs. 2-4 are represented by their centerlines extracted by the three curve-thinning algorithms **6SI-C1**, **6SI-C2**, and **PK-C**.

We can state that algorithm **6SI-C2** produces less skeletal points than algorithm **6SI-C1** does. However, it may produces overshrunk centerlines (see the sixth short "finger" in Fig. 2) compared to algorithm **6SI-C1** which, on the other hand, extracts skeletons containing more unwanted line segments (see the earless horse in Fig. 4). It is not surprising since endpoint characterization **C2** is more restrictive than **C1**. It can be seen that the existing algorithm **PK-C** produces several unwanted side branches that are not present in the centerlines of the new algorithms **6SI-C1** and **6SI-C2**.

Note that skeletonization is rather sensitive to coarse object boundaries. The false segments included by the produced skeletons must be removed by a pruning step [18].



original (865 941) **6SI-C1** (1 022) **6SI-C2** (864) **PK-C** (1573)

Fig. 2. A 174 × 103 × 300 image of a hand and its centerlines produced by the three curve-thinning algorithms under comparison

original (378 043) **6SI-C1** (891) **6SI-C2** (888) **PK-C** (1431)

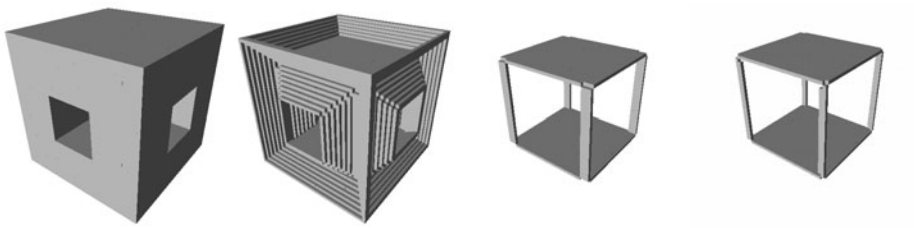
Fig. 3. A 304 × 96 × 261 image of a helicopter and its centerlines produced by the three curve-thinning algorithms under comparison

The medial surfaces of the non-tubular test objects in Figs. 5-7 were extracted by the three surface-thinning algorithms **6SI-S1**, **6SI-S2**, and **GB-S**. Note that algorithm **6SI-S2** produces much less skeletal points than algorithm **6SI-S1** does: outer "corners" and "edges", which remain connected with the inner skeletal parts, are not deleted by algorithm **6SI-S1**. It can be seen that the existing algorithm **GB-S** produces overshrunk seams between sheets.

For the test objects without any holes or cavities in Figs. 2, 4, and 6, our algorithm **6SI-TK** produces only one isolated point as their topological kernel (which is not depicted in Fig. 8). The topological kernels of the remaining test

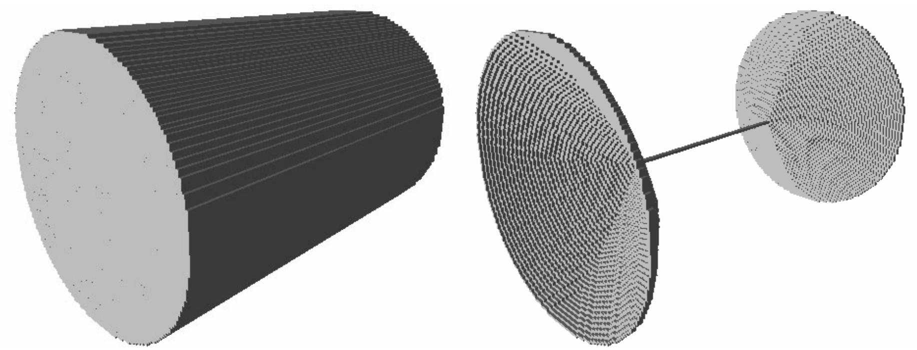
original (1 099 920) **6SI-C1** (878) **6SI-C2** (833) **PK-C** (1869)

Fig. 4. A 300 × 239 × 83 image of a horse and its centerlines produced by the three curve-thinning algorithms under comparison

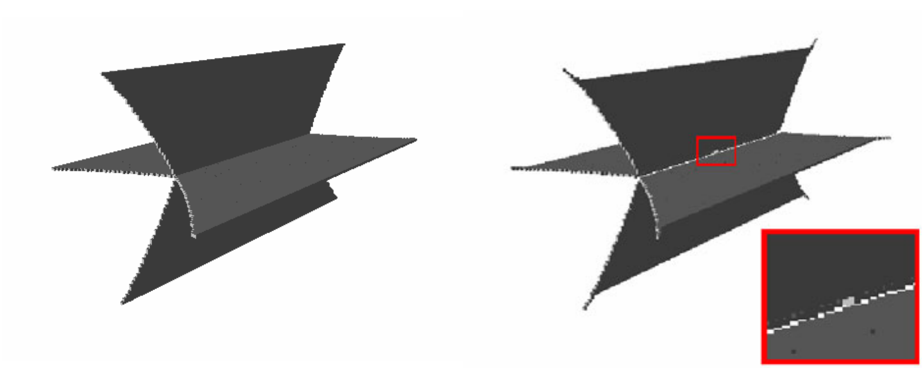


original (74 250) **6SI-S1** (15 864) **6SI-S2** (2 370) **GB-S** (2 370)

Fig. 5. A 45 × 45 × 45 cube with two holes and its medial surfaces produced by the three surface-thinning algorithms under comparison



original (1 173 750) **6SI-S1** (25 886)



6SI-S2 (16 857) **GB-S** (14 293)

Fig. 6. A 104 × 104 × 152 image of a cylinder and its medial surfaces produced by the three surface-thinning algorithms under comparison. Note that algorithm **GB-S** produced an overshrunk seam between sheets.

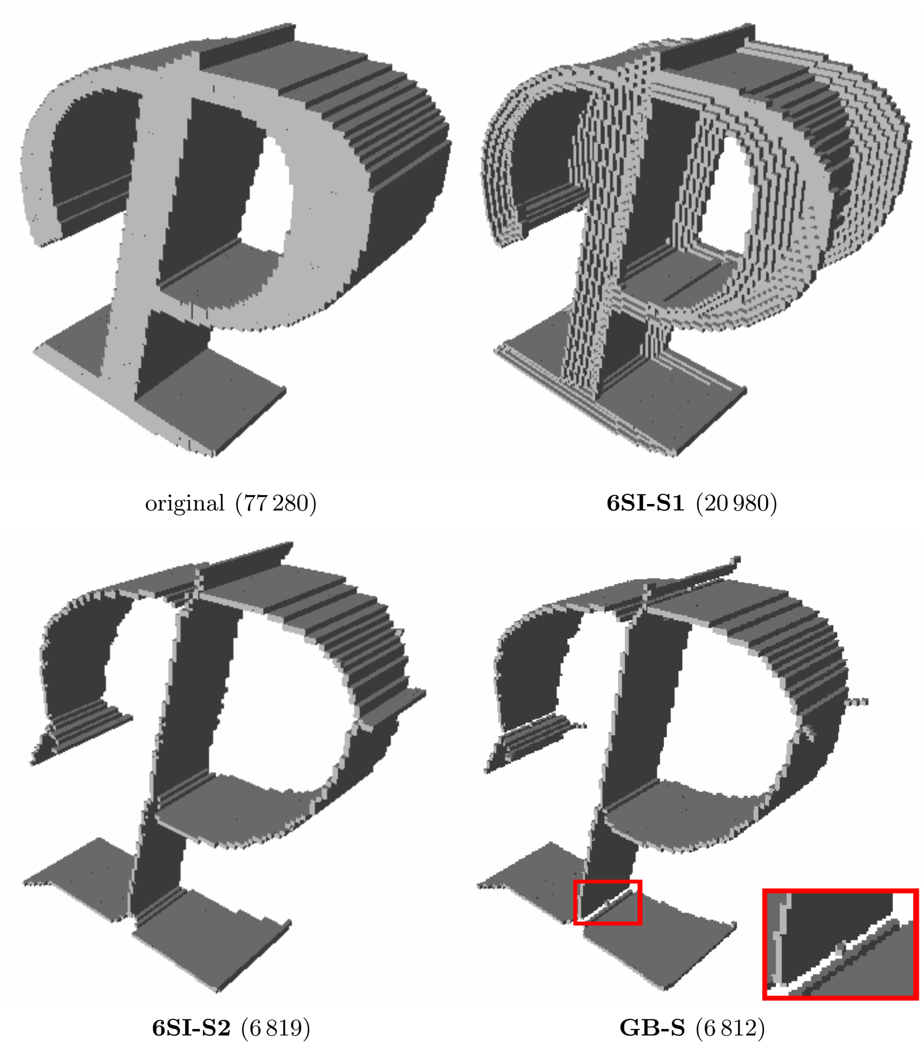


Fig. 7. A $100 \times 100 \times 30$ image of an object with a hole and its medial surfaces produced by the three surface-thinning algorithms under comparison. Note that algorithm **GB-S** produced an overshrunk seam between sheets.

objects containing some holes in Figs. 3, 5, and 7 are formed by 1-point wide closed curves (see Fig. 8).

By adapting the efficient implementation method presented in [16] our algorithms can be well applied in practice: they are capable of extracting skeleton-like features from large 3D shapes within one second on a usual PC.

The proposed implementation uses a pre-calculated look-up-table to encode the simple points. Since the simplicity of a point p can be decided by examining

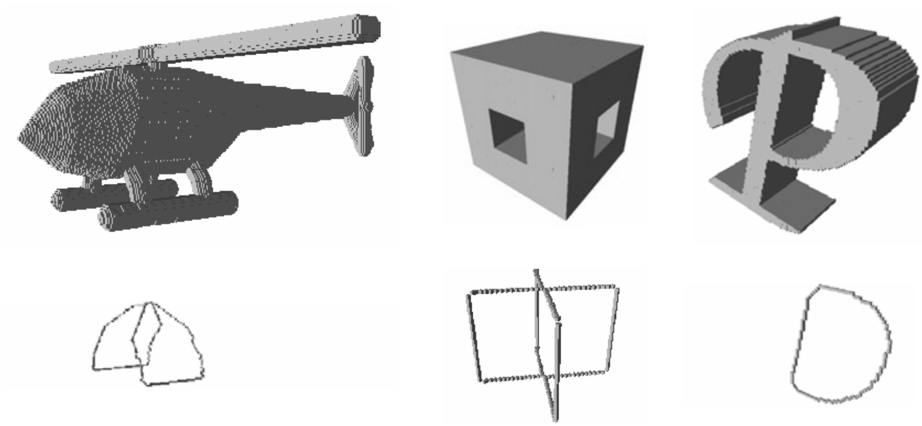


Fig. 8. Three objects with holes (upper row) and their topological kernels produced by algorithm **6SI-TK** (lower row). The extracted structures do not contain any simple points and they are topologically equivalent to the original objects.

the set $N_{26}(p)$, that look-up-table has 2^{26} entries of 1 bit in size, hence it requires just 8 MB of storage space in memory.

In addition, two lists/sets are used to speed up the process: the first one for storing the border points in the current picture. It is easy to see that thinning algorithms can only delete border-points, thus the repeated scans of the entire array storing the actual picture are not needed. The second list/set is to store all points that are "potentially deletable" in the current subiteration. At each phase of the thinning process, the deletable points are deleted, and the list of border points is updated accordingly.

The array storing the actual picture may contain five kinds of values: the value of "0" corresponds to "0" points, the value of "1" corresponds to interior points, the value of "2" is assigned to border points (that are stored in the first list/set), the value of "3" is assigned to all points that satisfy Condition 1 of Definition 4, and the value of "4" corresponds to all points that satisfy Conditions 1 and 2 of Definition 4.

6 Conclusions

Fast and reliable extraction of skeleton-like shape features (i.e., medial surface, centerline, and topological kernel) is extremely important in numerous applications for large 3D shapes. In this paper, we presented a new scheme for 6-subiteration parallel 3D thinning algorithms that is based on our new sufficient conditions for topology preservation. Hence the topological correctness of our algorithms is guaranteed. Five variations for the proposed thinning scheme were presented by considering five different characterizations of endpoints. Additional types of endpoints coupled with our general thinning scheme yield newer thinning algorithms.

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