#### CROSS-CURRICULAR COMPETENCIES IN PISA TOWARDS A FRAMEWORK FOR ASSESSING PROBLEM-SOLVING SKILLS

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#### Introduction

1. The OECD Programme for International Student Assessment (PISA) seeks to progressively integrate the assessment of cross-curricular competencies into the survey instrument. The assessment of problem-solving skills has been stated as a goal for the second survey cycle.

2. Problem-solving is a central educational objective within every country. Educators and policy makers are especially concerned about students' competencies for solving problems in real-life settings. These competencies involve recognising a problem, formulating the exact nature of the problem, using this knowledge to plan a strategy for solving the problem, effectively executing the strategy, reflecting on the tentative solution, making adjustments, and communicating the solution to others. The processes of problem-solving, so conceived, are found across the curriculum, in mathematics, in the sciences, in the language arts, in the social sciences as well as in many other areas of schooling. Problem-solving provides a basis for future learning, for effectively participating in society, and for conducting personal activities.

3. The INES Network A has, through an expert committee, explored the development of a framework for the assessment of problem-solving skills. This document is a result of this work and seeks to:

- provide an overview of the domain of problem-solving;
- outline what an assessment of problem-solving might contain; and,
- describe approaches PISA might take in assessing problem-solving and suggests related test specifications.

#### **Overview of the domain of problem-solving**

It seems that all cognitive activities are fundamentally problem-solving in nature. The basic argument is that human cognition is always purposeful, directed to achieving goals and to removing obstacles to those goals.

John R. Anderson (1985, p. 199)

#### Conceptions of problem-solving

4. As stated by John Anderson, problem-solving is an ever-present human activity. Central to the development of a framework for the assessment of problem-solving skills among 15-year-olds in the various countries participating in the PISA assessment is a clear vision of what constitutes problem-solving. Several writers have observed that there is no agreed-upon comprehensive definition of problem-solving (e.g., Frensch & Funke, 1995; O'Neil, 1998). Yet, there is a large body of literature on learning and work that discusses problem-solving, often without explicit definition of the term in context.

5. The nature of problem-solving is often described in the literature via the cognitive processes required. As the proposed PISA assessment of problem-solving is restricted to in real-world or discipline-based contexts, the following process components are of interest:

- Problem representation: This component includes searching for information, structuring it, and integrating it into a mental representation of the problem, taking into consideration the information present in the context of the situation;
- *Constructing a solution*: This component includes various kinds of reasoning, based on the representation, as well as the planning of actions and other solution steps; and,
- *Execution and evaluation of a solution*: Solution steps have to be executed and evaluated. The problem solver has to monitor and regulate his/her activities, constantly attending to the context of the problem. In dynamic environments, the problem solver must continuously process external information and feedback.

6. These problem-solving components make use of some very basic psychological mechanisms such as:

- retrieving, considering, and evaluating contextual information, as well as recalling and using general knowledge;
- applying mental tools (e.g., diagrams which may help in representing a problem or envisioning a solution) as well as using cultural tools (pencil and paper, calculator, computer, etc), and culturally provided systems of representations (language, symbol systems, etc);
- using various kinds of inductive and deductive reasoning;
- relating previous experiences and known strategies to new problem situations (e.g., by drawing analogies, by using metaphors); and,
- regulating emotional and motivational factors with cognitive factors.

7. Thus, problem-solving is the combination of many different cognitive and motivational processes that are orchestrated to achieve a certain goal that could not be reached by simply applying a well-known routine, or algorithm. Problem-solving competence is the capability to do this kind of orchestration within a certain range of tasks and situations. Problem-solving assessment aims at identifying the processes used and measuring the quality or the products of problem-solving activities.

8. In studying problem-solving, PISA's assessments obviously have to concentrate on the range of contexts and tasks observable via large-scale problem-solving assessments. As such assessments must necessarily depend on context- or domain-specific knowledge and strategies, every measure of problem-solving competence will, to some extent, be context- and domain-specific. Therefore, the domains, contexts, and situations in which problem-solving is assessed have to be selected very carefully.

9. However, research on problem-solving in differential psychology has also shown that on the level of (latent) abilities, problem-solving competence, defined operationally by the degree to which a person can successfully solve problems, will always be close to general reasoning.

#### Directions in defining problem-solving

10. The existing bodies of work on problem-solving do not lend themselves to a simple taxonomy of perspectives on problem-solving. Nevertheless, the literature may be organised roughly into two sets of work: "Academic" and "Applied." Each of these sets of work may be grouped into further categories, some of which may be likened to "schools" or "traditions." These categories may be identified with certain researchers and writers as shown in the following table. Given the absence of any accepted classification of work on problem-solving, the table and on the next page and the following elaboration of its contents are necessarily tentative and possibly controversial, and should be viewed in that light.

	Fields of work*	Associated researchers/writers/ organisations*#
	Artificial intelligence	Newell & Simon; Hinrichs
ACADEMIC	Cognitive psychology (including Complex Problem-solving in disciplines)	Gestalt School; Glaser; Chi; Mayer; Dörner; Funke; Frensch; Sternberg
	Developmental Psychology/Epistemology	Piagetian; Vygotsky
	Situated cognition	Brown, Collins and Duiguid; Scribner; Lave; Rogoff; Greeno
	Learning in subject disciplines, e.g., math, physics, reading, writing, history	Shoenfeld; Chi; Stanovich, Flower; Weinberg; Klieme, Voss
Applied	Vocational education, e.g., medicine, electronics, mechanics	Barrows; Lesgold; Gitomer; Lurch
	Generic work skills	Carnevale; SCANS (US); Key Qualifications (Germany); Key Competencies (Australia)
	Business	Arlen; Brightman
	Life skills	Hyman

\* These lists offer examples only. No claim is made for their completeness.

# The named researchers would not necessarily choose to be labelled as they are here. For example, Jean Lave describes herself as an anthropologist although her research forms a crucial part of the foundations of situated cognition.

#### Academic and applied studies of problem-solving

11. "Academic" work here refers to research designed to further understanding of problem-solving. This work includes German Gestalt tradition, the Piagetian (Swiss-French) and Vygotskian (Russian) developmental perspectives to problem-solving, the German project tradition in problem-solving, and finally work in North America originally conceived via the information processing paradigm and more recently under the paradigm of situated cognition.

12. "Applied" work includes the research on problem-solving and the use of concepts related to problem-solving in a variety of fields ranging from school mathematics to vocational education to business more generally. This work bridges the academic and the applied categories by pursuing an understanding of problem-solving within the context of improving learning in specific disciplines. In doing so, it treats the process of problem-solving itself as relatively unproblematic, proposes standard sequences of problem-solving steps, and explores the conditions for effective use of problem-solving to get work done.

13. This categorisation of problem-solving studies offers a way of viewing the extant work on problem-solving. However, all such categorisations are problematic. Obvious examples of this are the distinction between "cognitive psychology" and "learning in the subject disciplines," since the key researchers identified with these groupings are widely recognised as leaders in advancing knowledge about problem-solving in ways that apply beyond their specific disciplinary interests. This is also true, in some extent, in making a distinction between those working in artificial intelligence and those working in cognitive psychology.

# Academic Approaches

14. While there are a distinct number of approaches to the study of problem-solving, few differences exist about the nature of problem-solving itself. While the language used to describe these elements and the connections among them differs, most individuals involved in the study of problem-solving are in general agreement with the following definition of the essence of problem-solving, derived from Mayer and Wittrock, (1996):

Problem-solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver.

15. However, there are differences among the schools of work on the various components of problem-solving.

The Problem Situation (the recognition of an situation needing attention)		
The Context (the setting in which the problem-solving takes place)		
The Nature of the Task (the tools which can be used, the cultural boundaries, time constraints,)		
The Problem Solver		
(the content and procedural knowledge of the person undertaking the problem-solving, his/her ability to monitor progress towards achieving the goal, and his/her familiarity with the problem or similar problems)		
The Problem-solving Process		
(the interaction between problem solver, problem, and context)		

16. The components of problem-solving might be described as shown in the following table:

17. The work in each category acknowledges the existence, more or less explicitly, of all of these components of problem-solving. But each set of work pays particular attention to one or two of these

elements. The following table identifies these differing emphases. Once again, this formulation is tentative and possibly controversial, and should be viewed in that light.

18. The academic fields of work on problem-solving reveal marked distinctions in the emphases placed on the elements of problem-solving. Their starting point is essentially the same, however. Each field is founded upon efforts to understand the nature of human problem-solving.

	Fields of work	Components of problem- solving emphasised
Academic	Artificial intelligence	process; prior knowledge (Schank); situated perspectives ( Winograd)
	Cognitive psychology (including Complex Problem-solving)	process; problem solver's knowledge (expert vs. novice)
	Situated cognition	context; problem solver's knowledge and the tools provided by the culture
Applied	Learning in subject disciplines, e.g., math, physics, reading, writing, history	process, problem solver (expert vs. novice)
	Vocational education, e.g., medicine, electronics, mechanics,	process, problem solver (expert vs. novice)
	Generic work skills	problem solver, process
	Business	problem, process
	Life skills	? (maybe context)*

\* The question mark here indicates lack of sufficient information so far to warrant an entry.

#### Artificial intelligence

19. Work on artificial intelligence focuses on the analysis of the processes people use to solve problems and the development of intelligent programmes that can emulate these processes, even to the extent of using their programmed heuristics to solve problems that were not part of their initial programming. While some of this work focuses on the characteristics of different kinds of problems and use is made of studies of expert vs. novice problem solvers to identify the problem-solving strategies that separate experts from novices, the main focus is on the problem-solving process. That is, the interaction between the problem solver and the problem.

20. This research suggests that problem-solving is a search through a problem in an attempt to close the gap between an existing state and a desired goal state, where the gap constitutes the problem. The problem solver develops a mental representation of the problem and adopts strategies to move from the existing to the goal state. Such strategies may be strong or weak and more or less strategic, depending on the knowledge demands of the problem, the knowledge of the problem solver and the problem solver's experience with solving similar problems. Search strategies (or heuristics) identified through this research include:

#### - "trial-and-error";

- adopting a backwards-and-forwards strategy of moving forward until a barrier is encountered, then backtracking to a point at which forward movement is again possible, adopting a modified strategy, and so on; and
- means-ends analysis, leading to a strategy such as adoption of a goal recursion strategy in which the problem solver develops a representation of the problem and the goal state that will allow him/her to establish and achieve intermediate goals while keeping the ultimate goal in mind.

#### Cognitive psychological research

21. The major interest of mainstream North American cognitive psychological research has been to uncover the processes by which people think and learn (Bransford, Brown, & Cocking, 1999). The role problem-solving has played in this research is that it has provided a vehicle for examining cognition. As such, the problems selected for use in cognitive psychological research studies have been selected more for their capacity to help reveal thinking processes than for their capacity to illustrate the kinds of problems people solve on an everyday basis. A classic example of such a problem is the "Tower of Hanoi" problem used by Newell and Simon. Such studies have helped to identify heuristics (strategies, "rules of thumb") that problem solvers use to construct mental models of a problem situation and the goal to be achieved. They also deal with methods to identify how solvers find their way through the related problem space (the thinking space encompassing a problem, the goal state, and the pathway(s) between the problem and the goals(s)), in order to arrive at a solution. A further common feature of this research has been the study of different kinds of problem solvers as they tackle certain problems.

22. Other research in cognitive psychology has focused on the differences between experts and novices in specific content areas. The purpose of this research has been to uncover differences in their approaches with a view to establishing the implications for learning; that is, what novices in a given field need to learn in order to respond to problems in ways that are similar to how experts respond. Such research overlaps with the "applied" research on learning in the subject disciplines (e.g., mathematics, physics) and in vocational education (e.g., electronics, medicine) in comparing the problem-solving of experts and novices in these knowledge-rich domains. The results have been used to inform research on learning in these disciplines and in vocational occupations. The move towards concentrating problem-solving research in specific subject domains resulted from the lack of findings in early studies identifying approaches to problem-solving that would generalise to wide realms of application. As with the work on artificial intelligence, this research suggested that problem-solving heuristics that generalise across domains are usually weak and do not provide much assistance in the solution of real-world problems that demand strong, domain-specific heuristics.

23. Expert problem solvers, because of their superior knowledge and experience, demonstrate greater capacity than novices in a number of areas related to problem-solving. They are better at constructing a mental representation of the problem space, organising knowledge and relevant information into structures that will facilitate solving the problem; and in selecting and adapting more strategic and efficient heuristics for arriving at the goal state (Chi, 1989; Siegler, 1989).

24. Included in this work are the findings of those researchers associated with the school of Complex Problem-solving. Their orientation arose from different motivations than the North American focus on learning. Like the North Americans, the Europeans realised that the results from research on solving simple problems did not generalise to more complex and life-like problems. But rather than move, as the North American research did, towards domain-specific problem-solving, the European research focused on exploring the nature of complex problems.

25. From this point, two main approaches developed in Europe — the work initiated by Broadbent in Great Britain and the work initiated by Dörner in Germany. The two approaches share an emphasis on relatively complex, semantically rich tasks that are similar to real-life problems. Both approaches make extensive use of computerised tasks. In fact, the kinds of tasks used in this research would be almost impossible to construct without the technical capacity provided by computer programmes. The tradition

initiated by Broadbent, however, emphasises the distinction between the cognitive problem-solving processes that operate under awareness compared with those that operate outside of awareness. The tradition initiated by Dörner, on the other hand, focuses on the interplay among the cognitive, motivational, and social components of problem-solving. The problems designed to support this research utilise very complex scenarios with as many as 2000 interconnected variables. The most widely known example of such a problem is the "LOHNHAUSEN" problem designed by Dörner. This problem involves a scenario in which the assessment subjects take on the role of being "a good mayor" of a simulated town.

26. According to Frensch and Funke (1995), Complex Problem-solving occurs in individuals to overcome barriers between a given state and a desired goal state by means of behavioural and/or cognitive, multi-step activities. The given state, goal state, and barriers between given state and goal state are complex, change dynamically during problem-solving, and are intratransparent. The exact properties of the given state, goal state, and barriers are unknown to the solver at the outset. Complex Problem-solving implies the efficient interaction between a solver and the situational requirements of the task, and involves a solver's cognitive, emotional, and social abilities and knowledge.

27. Complex Problem-solving focuses on how people deal with novel and complex tasks. This emphasis on task novelty further distinguishes Complex Problem-solving from the North American mainstream tradition. Considerable work has been devoted to identifying factors that affect the complexity of a problem. These include:

- the number of variables in the problem and their interconnectivity;
- the dynamic aspects of problem situations; that is, the extent to which the conditions affecting the situation are subject to change over time or as a consequence of changes in related variables;
- the intra-transparency or opaqueness of the situation; that is the extent to which the problem and its characteristics can be clearly discerned;
- the number of goals and the need to select priorities and balance possibly competing or contradictory goals. (Funke, 1998)

28. Over time, two strategies have been adopted for studying how people interact with complex, novel problems. One has been to use naturalistic scenarios that embody everyday problems and to try to identify individual differences in the ways subjects respond to the problems. The other approach has been to create well-defined tasks with known characteristics (e.g., in terms of variables and their interconnectivity) and to systematically manipulate features of the task environment to test how people acquire and use knowledge in interacting with these tasks (Buchner, 1995).

#### Situated cognition

29. Situated cognition offers an alternative orientation towards problem-solving. It emphasises the relationship between the problem solver and the setting in which the problem-solving takes place. Drawing on studies of cognition by ordinary people in everyday situations, this work argues that knowledge is situated, being in part a product of the activity, context, and culture in which it is developed and used. Such a perspective has significant implications for learning. One aspect of this relates to the kinds of activities selected to support students' learning. Authentic tasks are advocated over "school" activities. Another is the concept of cognitive apprenticeship in which students are enculturated into the ways of thinking in a given domain through a process of situated modelling and coaching. A third is the importance of collaborative approaches to learning. It is argued that learning in a group setting is important, not just because groups offer a convenient way to accumulate the individual knowledge of their members, but because social interaction is an essential part of the development of knowledge. (Brown, Collins, & Duguid, 1989).

# Applied approaches

Learning in subject disciplines and vocational education

30. The emphasis in the applied fields of research on learning in subject disciplines and vocational education mirrors closely the emphasis on the problem-solving process described above with reference to the North American tradition in cognitive psychology. In fact, in large part, this work owes its existence to the failure of earlier work to identify general problem-solving strategies that would apply effectively across subject domains.

# Business and work skills

31. Work on problem-solving with business applications includes both empirical research and training programmes and other personnel management advice. These reflect an emphasis on the nature of the problem and the problem-solving process. Other dimensions of these problems include factors such as the operating level (on a continuum from day-to-day problems that must be solved but do not usually have long-term effects to strategic problems that are critical issues with long-term effects) and problem awareness (a continuum from crisis problems requiring reactive problem-solving to opportunity problems that provide opportunities for proactive problem-solving) (Brightman, 1980; Barton, 1999).

32. Work in the business field often identifies standard sequences of problem-solving steps, such as: identify the need; plan the project; collect facts; analyse data; develop alternatives; present recommendation; and implement the decision (King, 1981). There are also findings in this field of study that would support an emphasis on the situational aspects of the problem. Meacham and Emont (1989) argue that problem-solving is essentially a social activity, suggesting some connections to situated cognition.

# Identifying components of problem-solving

33. The literature identifies and describes a range of characteristics of each of the components of problem-solving. Some of these characteristics overlap (e.g., problems may be described as being knowledge-rich or knowledge-lean and may be described as being domain-specific or non-domain specific). Some of these characteristics may produce interactions among those components (e.g., the knowledge demands of a problem may interact with the knowledge possessed by the problem solver).

34. Research findings from academic cognitive psychology and Complex Problem-solving studies, and to some extent from applied business problem-solving research, yield information about the characteristics of each of the elements of problem-solving. These results may be an artefact of the generally fine-grained nature of these studies. However, the search for information at this level of detail is not complete, and it is possible that other fields may yet yield relevant material.

- 35. The purpose of unpacking these components and characteristics is to:
  - highlight the complex nature of each component; and,
  - illustrate the variety of dimensions of problem-solving that need to be considered in the design of an assessment instrument.

36. The listing of problem characteristics here is not comprehensive, nor is the discussion of each characteristic complete. A study of problem-solving needs to be sensitive to the wide variety of characteristics that can be associated with problems. Problems can, by themselves, have several dimensions, such as the degree to which they are open- or close-ended. The actual conditions under which the students are asked to solve the problems and the resources available to them during that time constitute

another set of considerations. One also has to consider the previous experiences of the problem solver-to see to what degree have they experienced similar problems or questions. A fourth area of considerations is the degree to which specific problems may require unique strategies or not. Additional characteristics may be added or removed as the work on this framework progresses.

#### Problem dimensions

#### Degree of problem definition

37. Problems can be viewed as "well-defined" or "ill-defined." Problems vary both in terms of the completeness of problem specificity (thus, how well the problem is understood) and the certainty with which the correct or optimal solution can be recognised (Arlin, 1989). Problems also vary according to the extent to which a problem provides values for its "parts." By considering the known, partially known, and unknown information it is possible to differentiate among well-defined and ill- defined problems, as well as to develop solution paths. Well- defined problems have relatively specific values while ill-defined problems have one or more unknown values.

38. Well-defined problems relate to repetitive and routine tasks which are well defined and can be solved by standardised/automated procedures (Brightman, 1986). These problems are more like "puzzles" (Luszcz, 1989) that have one specific goal or answer and are solved using explicit rules and algorithms. Because of their explicitness, the problem space for well-structured problems is a closed system.

39. Ill-defined problems, on the other hand, are less specific, harder to grasp, may have multiple goals and solutions. Their solutions are more difficult to attain because the problem space for ill-defined problems is an open system (Luszcz, 1989). Ill-defined problems are novel, elusive, often ambiguous "out-of-focus messes" (Brightman, 1986). These types of problems are solved using judgement, creativity, problem-solving processes, and heuristics.

#### Degree of domain-specificity

40. The degree of domain specificity refers to the extent to which a problem might be inserted into the specific practices of a certain area of expertise. Domain specificity deals with the degree to which the problem, because of the knowledge required, must be categorised in one or a small number of academic domains. For example, a problem dealing with the solution of a Diophantine equation would fall with in the domain of elementary number theory in mathematics. In the literature, this characteristic is also referenced by terms such as knowledge-rich vs. knowledge-lean; high vs. low knowledge of relevant subject matter.

Nature of and relationship among variables

41. The nature of and relationship among variables deals with the extent to which the values of the variables involved in a problem are fixed, static, both over time and in relation to the other variables. That is, does the relationship between variables and the actual values of the variables themselves remain constant, static, or shift, dynamically, as problem-solving is occurs.

42. Another major characteristic involving variables is whether the problem is univariate or requires multiple variables. Research has shown that the complexity of problem-solving is positively correlated with the number of variables involved. Multivariate problems can be further categorised by whether the values of one variable are dependent on those of another variable, or by whether the values of the variables, or specified subsets of the variables, interact with one another in a variety of ways.

# Problem intensity

43. Arlin (1989) defines problem intensity as the motivational attraction of the problem where the problem solver is sufficiently engaged that it is worthwhile to attempt a solution. Brightman points out that problem intensity can effect the problem-solving processes in business situations. High intensity, or "crisis," problems are forest fires that cannot be avoided and require immediate action, while opportunity problems exploit possibilities for action and are more long-term endeavours. Both crisis and opportunity problems are important, motivating, and must be solved but the immediate nature of crisis problems leads to reactive types of actions and problem-solving while opportunity problems are solved in a proactive manner.

# Degree of realism

44. This problem characteristic applies to presented (assessment, instruction, research) problems. Problem temporality, as this characteristic is sometimes called, is the perception of whether or not the problem is one that may be encountered by the problem solver (Arlin, 1989). If the problem presented is one that is unlikely to be experienced, what is the motivation for expending effort toward a solution? If the problem is perceived as one that may be encountered in the present or near future then there might be a higher motivational level associated with an attempt to reach a solution.

# Nature of solution

45. Problems vary in their expected, or actual, solutions. Some problems, especially those that can be modelled by an equation, have a unique, or deterministic, solutions. Other problems, for example problems involving design and construction issues, have a multitude of possible solutions. The degree to which a problem has a very narrow solution space or a very expansive solution space affects the manner in which students approach and handle problems.

#### Required response mode of problem

46. Another characteristic of problems is whether they require the problem solver to select a solution or to create or fabricate a solution. In many cases these are now referred to as multiple choice, matching, or true false for the selection options and regular student constructed or extended student constructed problems for those requiring student work.

#### The context or setting in which the problem is to be solved

47. The administration of a problem-solving assessment brings with it additional considerations which must be factored into the design of the assessment and into the interpretation of the resulting data. Several of these issues are discussed in the following.

#### Context

48. Central to problem-solving tasks are the situations themselves and the conditions they bring to the problem solver. To what degree does the problem involve context with specific situational requirements? How many different disciplinary domains are called on for a solution to be developed? What requirements are made in terms of context-specific processing requirements?

# Task requirements

49. What logical demands does the task place on the problem solver? What requirements are there for communicating the results? Is the process required single or multi-faceted? This particularly relates to the degree of information processing required by the problem solver. How complex is the issue at hand?

#### Time constraints

50. This consideration deals with the degree to which the problem-solving assessment a timed assessment. Do students have to work under a time constraint? Will their problem-solving processes curtailed prior to showing all of the information they know and being able to construct all of the answers they could have produced? This is akin to the question of speediness in a classical assessment design.

# Access to additional resources

51. In contemporary assessments, students often have access to the technology they commonly use in regular learning settings. This may involve hand calculators, computers, or other forms of electronic information retrieval tools. It may involve access to non-human resources such as the library in a school, current newspapers, or community resources. This is contrasted with assessment administrations where students work at a desk with pencil-and-paper in the absence of any supporting resources. This also raises the issue of cultural tools and cross-cultural differences in the use of different cultural tools. There are different forms of calculation instruments and different cultural reactions to different test response formats.

# Social interaction

52. Some assessments provide for a measure of the affect of working alone, as compared to working as part of a group of problem-solvers. The *Pacesetter Mathematics* assessment of the College Board provides both individual and group assessments as part of its culminating assessment process. Vygotsky has paid a lot of attention to differences in problem-solving ability when subjects work alone versus when they work with others or with a teacher. People differ in their ability to profit from instruction. Some researchers have tried to make this issue relevant to current methods of assessment (Brown, ).

# The problem solver

53. Two of the key characteristics that must be considered in any assessment are the background and knowledge of the problem solver. These and other dimensions related to the problem solver are detailed below.

#### Knowledge of relevant subject matter

54. This dimension deals with the degree to which the problem solver might be considered an expert or novice in the field of the problem posed. What knowledge does the individual have in the specific domain if the problem is domain specific? To what degree is the problem solver's knowledge connected across disciplines and to what degree can the problem solver shift among multiple representations?

# High strategic/procedural knowledge

55. This second dimension deals with the toolkit of heuristics and algorithms the problem solver has relative to the problem posed. A problem solver with knowledge of explicit strategies and algorithms

related to the problem posed is much better off than a problem solver armed only with general strategies and disconnected knowledge of related procedures.

Familiarity with the problem

56. The degree that a problem solver is familiar with the problem is a third dimension. However, it is not only the familiarity with the problem, but also knowledge of the subject domain to which the problem belongs. What appears to one person as a novel and seemingly impenetrable problem may be little but a routine task to someone else. These differences may relate to individual differences in knowledge of the subject matter or to problem familiarity developed through numerous encounters with a variety of problems, or both. One of the characteristics that separates experts from novices is the ability of the former to recognise patterns of information relevant to a problem and quickly identify those patterns likely to lead to a satisfactory solution. For example, see the studies of chess experts and novices. These abilities derive not only from knowledge of the relevant subject matter but also from extensive experience with similar kinds of problems.

57. How well do the contents of the problem match with the "stuff" of the problem solver's everyday life (Arlin, 1989)? This is an important point when discussing cross-cultural issues and when attempting to transform contrived, knowledge restricted, or research-based problems into real-life situations.

58. A couple of concerns are immediately apparent. The first is that if a problem becomes too familiar, the problem solver may view it as a well-known, often encountered problem and therefore employ a "rule-of-thumb" solution. Thus, the problem becomes a non-problem since the solution is obvious or trivial and there is no opportunity to assess the problem solver's analytical reasoning abilities or problem-solving.

59. A second concern is that what may be familiar to one person is not necessarily familiar to another. In particular, what the problem designer might believe to be familiar content may be unfamiliar to the solver. This is an obvious concern in cross-cultural settings but it is also a critical element when dealing with age and social differences.

#### Motivation

60. A fourth consideration is the degree of motivation shown by the problem solver in confronting the problem. Students facing assessments that drop into their classrooms from the sky and have little consequences for them may be less motivated to show their full problem-solving powers than a student who knows that the results of the assessment have meaningful consequences. Motivation may rise and fall due to the interaction of a number of variables in a student's life. Disregarding the degree of potential motivation and effort on the part of the student can lead to drawing wrong conclusions.

61. Also tied to this area are differences in students' achievement motivation. Some are driven by intrinsic factors to succeed on assessments. Others need extrinsic factors to motivate them to achieve. When assessments appear, as the PISA assessments, for which there are no external rewards or individual scores to be reported for either the student or back to the school, personal motivations, or motivations of students from an entire ethnic group or culture, could be affected.

#### Assessing problem-solving in PISA

62. The foregoing material has provided a review of the extant knowledge concerning problemsolving from both an academic and applied viewpoint, it has reviewed some of the issues central to considering factors in problem-solving, and it has delved into aspects of what an assessment of problemsolving would have to consider in starting to measure students' work in a cross-curricular setting. Central to all of these issues is the topic of analytical reasoning—the ability of students' to reason, and reason well, in inductive, deductive, and critical/complex settings. It also deals with students' abilities to reason in settings where goals have been established for them and situations in which they must set their own goals and targets as they consider the problem at hand. It is these issues that should define the core of a problem-solving assessment in PISA.

63. Most researchers involved in the study of problem-solving, via one conception or another, generally agree that the general essence of problem-solving is as follows:

Problem-solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver (Mayer and Wittrock, 1996).

64. Problem-solving in the school curriculum is differentiated from such general problem-solving in that goals are often set for students, whereas in the real world, the goal may or may not be evident at the outset. In the PISA assessment of problem-solving both forms of problem-solving will be studied, those with externally established goals and those with goals, or sub-goals, set by individual students as their study of a problem unfolds. The paper refers to the former class of problems as goal-directed problems and the latter class as open-ended problems.

65. PISA assessments also call on students to display their problem-solving skills in a variety of settings. Each of the assessments, reading, mathematical literacy, and scientific literacy has problem-solving components listed in its framework. In reading, problem-solving is assessed through students' attempts to bring meaning to passages, to develop interpretations, and to determine voice. In mathematics, problem-solving is assessed through students' posing and solving of problems. Particular emphasis is given to students' choice of solution strategy, correctness of work and purported solution, and reflection on their work. In scientific literacy, problem-solving is assessed through students' identification of relevant knowledge and related questions and through their drawing and evaluating of conclusions (PISA Assessment, 2000).

66. The PISA assessment of problem-solving moves beyond these frameworks in establishing an additional set of measures, measures of the students' performance in problem situations where the:

- content moves from familiar material to unfamiliar settings,
- context moves from school-based topics to real-world applications, and
- complexity moves from simple and complex translation activities to situations calling for the multiple applications of processes and heuristics.

67. Such problems call on students to make executive decisions about the paths of work to follow and to determine the relative efficacy of different courses of action as they work to arrive at a solution. In particular, they involve the students' capabilities to engage in analytic and analogical reasoning settings. These allow measures to be developed to describe the efficiency of students' work, the productivity or idea generation aspects of their problem-solving approaches, and their command of the central processes of problem representation and the use of problem-solving heuristics.

68. The question of what problem-solving competencies an individual needs for life is more closely related to the solving of problems in a cross-curricular format than it is to solving problems in the single domain of a specific school subject. Everyday real-life problems call on individuals to merge knowledge and strategies from a variety of fields to reach some resolution. These problems call for individuals to move among different, but sometimes related, representations and to exhibit some degree of flexibility in the way in which they retrieve and apply their knowledge. The problems in the PISA assessment of problem solving engages students in contexts involving problems they have not seen in their school work or other examinations and, thus, provides a measure of their transfer of problem-solving and reasoning competencies to new settings.

69. This paper examines the essence of such problem-solving through both the frame of a crosscurricular assessment of problem-solving competencies and as seen through assessing problem-solving within the extant portions of the PISA assessment using released items from the first cycle. These examples are provided to draw the distinction between the nature of items used and the information gained from the cross-cultural assessment of problem-solving and the current PISA assessments of problemsolving in the areas of reading, mathematical literacy, and scientific literacy.

70. The assessment of problem-solving as a cross-curricular competency calls for the development of ways to assess a student's quality of knowledge and the processes through which that knowledge is applied in non-routine problem-solving settings. The goals of this assessment are to find how the student:

- uses conceptual understanding and procedural knowledge in non-routine settings;
- applies reasoning to understand a given problem context and related information;
- employs strategies and representations to link assumptions to desired goals;
- formulates questions to understand the task at hand;
- uses a variety of forms of reasoning in new settings;
- uses knowledge from one situation to reason in another; and
- evaluates his/her current work and makes adjustments prior to citing he/she has a solution.

71. Such foci allow for the analysis of students' ability to generate possibilities, to define and search problem spaces, to generate possibilities, to validate courses of inquiry, and to both communicate and reflect on their problem-solving activities. Such opportunities are rarely followed in subject-based inquiries of student problem-solving, as they focus on students' abilities to acquire the correct answer or to apply the appropriate procedures.

72. In the PISA assessment of problem-solving, the major feature of interest is the analytical reasoning that a student applies in addressing new and non-routine problems. Content specific problem strategies are developed within the school subject fields and are assessed there. What is specific to cross-curricular problem-solving is the analytical reasoning competencies that students develop throughout their schooling and which they are able to apply in new and non-routine situations which ask them to integrate their knowledge from individual fields of study. Problems measuring such competence should involve both a new context and levels of expectation of student performance than what would be found in a content-based area. Specific examples will be provided later. With analytical reasoning at the core of the cross-curricular competencies in problem-solving, it will be important to separate these reasoning behaviours from those normally associated with problem-solving in content areas that are more discipline-based.

73. The assessment of students' cross-curricular competencies in problem-solving call for students to reason in situations where the problems:

- are novel, but presented in meaningful situations;
- are based around real-life contexts, distinguished from those seen in instructional settings;
- involve the application of integrated curricular content from mathematics, science, civics, history, economics, sociology, or other area; and,
- require students to apply analytical, including analogical, reasoning skills.

74. The important feature is that the situation posed must call on the student to integrate their knowledge and understanding in new and unique ways.

75. In doing so, the assessment must provide opportunities to see students' reflective, self-regulating, problem-solving behaviour as much as possible, to monitor and evaluate the strategic approaches they employ, and to note the formal models and representations they employ in their work.

76. On the other hand, an assessment students' disciplinary problem-solving competencies in the mathematics framework would attempt to examine many of the same features. However, the focus would be more on specific strategies developed for use in situations involving quantities, spatial knowledge, chance, or data. The contexts involved would:

- be more familiar from a curricular standpoint;
- employ more familiar problem-solving approaches;
- be placed in novel, but related to familiar, settings; and,
- be restricted to mathematical situations.

77. The focus would be on connecting and integrating knowledge within mathematics as a discipline and in developing new mathematical knowledge through thinking, generalisation, and insight as defined in the PISA framework for mathematical literacy (OECD, 1999).

78. Each discipline has specific problem-solving strategies that students are expected to master and apply to a wide variety of exercises within the discipline. It is analytical reasoning that ties these cognitive approaches together with their new representations and interpretations. The student who is successful in employing inductive, deductive, and critical reasoning in cross-curricular settings is the one who is able to note the given information, abstract the structure out from the context, reformulate it, and act on it to produce new knowledge or structures.

79. Analytical reasoning strategies go beyond students' opportunity to learn logic and reasoning in the classroom. They are the ultimate problem-solving tools that transcend disciplinary boundaries. Items selected for the cross-curricular assessment must allow for students to show their reasoning. While problems are classified with respect to being goal-directed or open-ended, it is the way students approach these problems that is of interest. While problems are also classified with respect to whether they require inductive approaches, deductive approaches, or more mixed critical thinking strategies, it is the nature and quality of student thinking that is of primary interest.

80. Analytical reasoning was selected as the core of the PISA problem-solving assessment efforts because such reasoning is the heart of the problem-solving process. The nature by which one represents problems, selects strategies for attacking them, transforms their contents to equivalent formats, and solves them involves a carefully welding together of strategies and knowledge developed with individual disciplines. The levels of use of inductive, deductive, and critical reasoning skills that separate the expert problem solver from a novice are well known, but these have rarely been examined in school settings in such a large-scale assessment. The inductive and deductive strategies are used to manipulate and investigate certain concepts. The critical thinking strategies operate at a higher level in examining concepts. However, they are also used in testing statements purporting to explain various issues or to provide persuasive explanations.

81. This reasoning is often reserved for study by advanced or 'gifted" students. Most students actually "catch" it from their teachers, rather than have it formally taught to them. However, when students must reason in non-routine real-world contexts involving social, political, ethical, or personal issues and, at the same time, apply knowledge from different disciplines (including mathematical and scientific knowledge) it is analytical reasoning that provides both the direction of thinking and provides the glue that ties the work together. The reasoning runs from control of simple operational skills with propositional logic to larger units of these processes. Such larger units involve samples of inductive reasoning, deductive reasoning, and critical thinking that employ rules of inference, hypothesis testing strategies, and checks for common misconceptions and misapplications of logical reasoning. Such

applications of logic are applied in both goal-directed and more open problem-solving situations to problems from cross-curricular work involving combinatorial, proportional, probabilistic, and spatial aspects.

82. Analytical reasoning starts in the early grades as students begin to classify objects and make comparisons between objects as they develop language. Later the production of the child's language is guided by applications of this logic as students begin to structure their thoughts using connectives such as "and," "or," and "not." Gradually the logic of "If..., then..." statements begins to formally emerge. With them come understanding of the more formal uses of reasoning schema known as modus ponens, modus tollens, law of syllogism, analogical reasoning, use of the contrapositive, and other approaches to structuring and evaluating arguments.

83. This growth of analytical reasoning tools and skills is accompanied by a greater metacognitive awareness of the uses of analytical reasoning by students and their teachers. This growth is shown by the broader realm of representations that students use in characterising problems. Students must develop both ways of structuring their thoughts and actions within disciplinary based work and, at the same time, begin to monitor their actions in doing so. This shift in the level of metacognitive awareness of reasoning processes and representational skills in school parallels the development of students' ability to use these processes across disciplinary walls. The more students become aware of their analytical reasoning and its application, the more able to avoid misconceptions, prejudices, and mistakes arising from content issues.

84. Problem-solving in cross-curricular settings calls for skills that focus one's competencies in complex settings drawing on varied aspects of one's total knowledge base. Such problems are more real world in that they draw on two or more disciplines. They are embedded in rich contextual settings, settings that often involve emotional aspects for those involved in the problem-solving. These emotional aspects add to the difficulty levels, as well as to the complexity levels, in such problems. Such problems often involve aspects that can trigger specific reasoning or conceptual miscues that plague students' growth as they try to apply their knowledge in wider domains.

85. To ascertain the true scope of students problem-solving competencies in cross-curricular settings, PISA must assess students' competencies in settings that focus on the solution of complex tasks similar to those that may occur in real life. Such tasks are necessary in order to be able to test the possible effects of schooling on the development of analytical reasoning skills. These items must include tasks of greater complexity, greater novelty, and contexts where prior knowledge may stand in the way of drawing valid conclusions.

86. Recommendations for change in education world wide have assumed that the knowledge acquired in school can improve students' everyday thinking by making students more capable of applying known schemata in novel situations, preparing them to deal with more complex situations, and immersing them in situations that require restructuring existing knowledge (as in some gestalt problem – solving or as in some of the conceptual change literature) to form new conceptions or ways of approaching problems. The proposed PISA framework outlines a way of assessing such goals.

# Possible framework for the assessment of problem-solving in PISA

87. The framework for assessing problem-solving in a cross-curricular setting consists of a matrix with columns representing the directiveness of the problem posed and the rows representing the type of reasoning required to solve the problem:

	Goal directed	Open ended
Inductive reasoning		
Deductive reasoning		
Critical reasoning		

88. The goal directiveness is determined on the basis of setting a very deterministic situation for the students and directing to find that solution. An open-ended problem is one where students have more freedom to consider possible outcomes and select a strategy for comparing and contrasting among them.

89. Items measuring goal-directed inductive reasoning would be items that propose a specific goal, but only provide students with information about cases from which they can abstract a pattern and propose a generalisation as a solution based on the cases. An open-ended inductive reasoning problem might be an item that provides students with data and asks them to determine a particular outcome. The problem may specify a specific line of attack or method for analysing the data as well.

90. Items measuring goal-directed deductive thinking might have students consider a law, rule, or generalisation and a set of evidence and then build an argument based on the evidence and the nature of the principle given. Students would be given a specific goal and generally be either given the generalisation or directed toward it by one or another means of specification.

91. A goal-directed critical-reasoning problem might present students a series of data sources and a reading or a listing of information. Students would have to combine various aspects of inductive and deductive reasoning, along with organisation of intermediate findings to structure a final specific outcome as solution. Such problems require a number of logical decisions and generally involve a reformulation of information and multiple representations of given information in moving to the solution.

92. Items measuring open-ended inductive reasoning might present students with a listing of cases and ask the student to consider the cases and discuss what possible related outcomes might be. In particular, such problems might have an embedded pattern or generalisation in the data, but it would remain for students to discern the generalisation and develop it.

93. Items measuring open-ended deductive reasoning might provide students with a folder of data and a listing of laws relative to the situation and ask students to develop the materials in the folder. Such an item might present students with a dilemma and leave students to argue one side or another. The products might be assessed with respect to the quality of the argument that was constructed and the logical links evident in the structure of the students' reasoning.

94. An open-ended critical-reasoning problem might present students with a complex set of data and directions to potential outcomes. Students would be left with the task of bringing order out of chaos and developing some structure and organisation, including some generalisations, to develop understanding of the situation and knowledge of the principles that might be embedded in the data. As in previous cases, the organisation and discussion would be based on the structure developed through a combination of inductive and deductive reasoning on the part of the student.

95. Items meeting any of these descriptions can be embedded in more complex project-like tasks (such as those of Klieme). One project, or theme based assessment block, can include a number of items, possibly of different formats, that test a variety of the cells of the model proposed. Some may be inductive, others deductive. Some may measure goal directed aspects of topics within the theme, while others allow the students to impose their own structures on the situation. As one examines any of these problems relative to the problems found in the curriculum area based assessments, they would see that the content has shifted from familiar to unfamiliar, the context from school based to real world, and the complexity from relatively straightforward to novel and challenging.

#### Goal-directed versus open-ended problem-solving

96. The contrasting of goal-directed with open-ended problem-solving would see differences in the items along the dimensions of:

- the end-state is specified and the problem-space is bounded,

- the rubric is considerably tighter in expected outcome format,
- the means-ends relationships are more obvious and specified,
- the possibility of ordering being specified, and
- the evaluation strategy for problem being more evident.

97. The analysis of student work in open-ended settings may reflect students being able to set goal as a metagoal, that is, to what degree can they reflect on the setting and maintenance of a self-set and self-regulated goal in problem-solving.

# Inductive problem-solving

98. In assessing the quality of student reasoning in inductive settings, the assessment should provide students with ample opportunities to show their command of argument through similarity and analogy, through pattern recognition, and through rule induction. In the latter case, one would be looking to see whether the student looked for confirmatory and exclusionary evidence. Evidence should be collected to see the degree to which students have some flexibility in applying the simple operational skills associated with inductive problem-solving methods. Situations here might include comparison buying or trip planning.

# Deductive problem-solving

99. In assessing the nature and quality of student reasoning in deductive settings, the assessment should provide students with ample opportunities to show their command of argument through applications of reasoning schemes built on prepositional logic, especially those where the logic must be applied to contextual based information. In particular, are students capable of handing the standard argument structures and reasoning through syllogisms and analogies. Situations here might include interpretation of zoning requirements, structuring of proofs, and restructuring of common language into more formal arguments.

#### *Critical/complex reasoning*

100. In assessing the quality of student competencies in critical reasoning, the assessment should provide students with complex logical decision making situations. These are situations involving larger units of processes. They, most likely, are drawn from the social sciences, but involving mathematical or scientific aspects. To what degree can students critically evaluate information that they are given (checking for falsification, using hypothesis testing, etc.), exhibit aspects of proving ability (using processes to establish the validity of general statements, relate the relevance of the information to tasks at hand, etc.) The information load in these problems is heavy and multiple comparisons or contrasts are required. These situations require careful organisation and reordering of information, as well as careful interpretations to avoid forming misconceptions. Situations here might include issues of health, risk, environment, cost-benefit, or population.

# Complex problem solving

101. The PISA problem-solving assessments should investigate analytical reasoning in a variety of situations, contexts, and combinations of disciplinary requirements so that the novelty, complexity and counter-intuitiveness of all facets can be manipulate and examined with care.

#### *Future international options*

102. In addition to the core portions of the cross-curricular problem-solving assessment, countries participating the PISA assessment should consider the possibilities of two international options being considered for future cross-curricular problem-solving assessments. These options involve assessments of collaborative problem-solving and computer-delivery project assessment designed along the lines of the work of Klieme and his co-workers.

#### Collaborative problem-solving

103. The collaborative problem-solving option could consist of a separate block of items which students would complete in groups of three. Items in such blocks could build off of items in one of the other blocks of the regular cross-curricular assessment. This would allow for a comparison of students' work in individual settings with their work in group settings. Such assessment blocks would have to allow time for idea generation and formulation and for the development of group roles on the part of the students involved.

104. The *Pacesetter* programmes of the College Board Such have working models of such assessments of group problem-solving. Many recommendations for student competence in problem-solving and in education in general have called for the development of problem-solving competencies in an environment that values social learning. If so, it must also be assessed. Given the relation of this to country specific goals for students and the ease with which it can be pulled off in one country versus another, this is left to be developed as an international option within the PISA cross cultural problem-solving assessment.

#### Computer based delivery

105. This alternative would allow for the administration of computer delivered project work along the lines described by Klieme (paper to Network A). Such assessments provide a vivid display of students' problem-solving competencies in a dynamic environment. They also provide for an examination of the ways in which students order and conduct their work in complex settings in a way that no paper-and-pencil based assessment can provide.

#### Features of a problem-solving assessment

106. The assessment of problem-solving in PISA's must constantly be focused back on PISA's definition of problem-solving. That is problem-solving that builds on school education programmes, but extends the problem-solving competencies and knowledge developed in school to consider rich and novel contexts that take the student beyond situations they have experienced in the classroom. These situations should also challenge students to address problems that extend, in either goal-directed or open-ended ways, to settings that involve information and concepts from at least two different disciplines. This is the type of learning that students should carry into their lives and chosen fields of endeavour. Students who can solve problems involving a number of issues will be well prepared to address the problem-solving situations they will encounter in life.

107. Other issues are important in measuring students' problem-solving skills and developing assessments that do so. Some of these are discussed in the following sections.

#### Accessibility/equity

108. The test should be accessible to *all* students participating in educational programmes in the participating countries. Making an item accessible means that the item can be understood and addressed

by 15-year-old students regardless of the curriculum in which they are enrolled. Items should be developed in a fashion that presents them in a representational mode (graph, table, words, symbols, pictures,...) that is easily interpretable by *all* students.

109. Further, it is assumed that care will be take to see that other standard forms of bias are avoided in the design and construction of the test items. For example, excessive technical vocabulary, difficult reading level/vocabulary, and items calling for specific personal life experiences should be avoided.

#### Information sources

110. Items calling for specific information sources, i.e. specific data, should have that information provided as a part of the assessment. The provision of such information could be done within the item or as an accompanying data-bank book. Part of problem-solving is knowing where and how to look up information at appropriate times. If such information is required, it must be provided as part of the assessment package for all students, either within the blocks of items themselves, or in a data book that all students would have as part of their assessment package.

# Technology

111. An assessment of problem-solving is not an assessment of students' ability to perform calculations. As a result, all students participating in the PISA problem-solving assessment should be allowed to use any hand calculators they routinely use in their regular classroom learning environments. The decision of whether to use calculators should rest with the individual student based on their chosen approaches to the items. No item should be constructed so that its solution is solely dependent on calculator usage or of a length that students not using a calculator would be disadvantaged in performing any calculations required.

#### Manipulatives/formula sheets

112. Items requiring any manipulatives (geometric pieces, counters, spinners,...), rulers, or protractors, maps, formula booklets, or other materials to be handled in the course of solving a problem must be designed so that those materials can be provided to each student during the assessment sessions. Such materials should be designed to be inserted in the assessment booklets for student accessibility. No special materials should be required that demand extra handling by teachers or others administering the assessment.

#### Assessment administration conditions

113. Where possible, the assessments should be administered in students' regular classroom settings. Such administrations help in promoting an academic atmosphere for the assessment and help assure that regular materials (calculators, pencil-sharpeners,...) are all available as they would be on a regular school day. Further, the assessment should be designed to be administered in a single school day in two settings, separated by a short break. This will alleviate the necessity of scheduling make-up administrations for students missing one of the administration periods should the assessment be spread over two or more days.

#### Items and rubrics

#### Principles for item selection

114. Throughout the design process and development of the final blocks of items, it is important that, to the extent possible, all items should be:

- reflective of the cross-cultural attributes discussed earlier;
- broad, rather than narrow, in scope and reflect real-life contexts;
- free of double-negatives, excessive technical vocabulary, and other
- accessible to 15-year-olds in terms of mathematics content and difficulty; and,
- reasonable in light of the views of the general public.

#### *Item types and examples*

115. In previous large scale assessments of problem-solving the majority of items used have been multiple-choice, true-false, or short response items. These items were used in the name of reliability and because of their low-costs to score and the ease with which such forms could be administered. However, to adequately ascertain a student's ability to reason, problem solve, and communicate the results of such activities, more extensive records of a student's work are needed. Hence it is proposed that a variety of item formats will be employed in developing the PISA problem-solving assessment.

#### Multiple-choice

116. Multiple-choice items are appropriate for quickly and inexpensively determining whether students have mastered certain skills, knowledge, or information gathering abilities. Well-designed items can measure student knowledge well beyond simple conceptual and procedural knowledge. They can be designed to reach beyond the ability of students to "plug-in" alternatives or eliminate choices to determine the correct answer. However, these items are somewhat limited in their ability to ascertain the breadth and depth of a student's knowledge for many contexts.

Short student constructed responses

117. Short student constructed response items are items designed to require students to construct their own responses to items. This allows for examiners to ascertain what students can produce from their own understanding of the item and to display the heuristics they have used in approaching the item. SCR items either require students to give brief answers representing either a numerical result or the correct name or classification for a group of objects, draw an example of a given concept, or, perhaps, write a brief explanation for a given result. In general, these items should:

- Require about 2 to 3 minutes of work;
- Ask the student to show integration of information or concepts, along with the way in which these lead to a solution to the problem proposed;
- Tap multiple areas of understanding and require their connection in the response displayed by the student;

- Require the student to justify their answer;
- Use where the situation requires multiple steps to a solution and has several different components;
- State explicitly what the student needs to do in responding.

Extended student constructed responses

118. Extended Student Constructed Response Items are items that require students to consider a situation that demands more than a numerical response or a short verbal communication. These items require students to carefully consider a situation within, or across, content areas within a discipline(s); understand what is required to solve the problem; choose a plan of attack; carry out the plan; and interpret the solution in terms of the original situation. These items require students to provide more complete evidence of their work or to show that they have used more complex thought processes in solving a problem. In either case they are expected to clearly communicate their decision-making processes in the context of the problem (e.g. through writing, pictures, diagrams, or well-ordered steps).

#### **Rubrics**

119. Holistic rubrics for evaluating the student responses to items should be constructed about a general framework that values the stages of problem-solving laid out by Polya in his famous book on problem-solving (Polya, 1945). Such rubrics would be noted by giving special attention to student work achieving the various levels of restating the problem and noting assumptions, designing a method of approaching the problem, solving the problem, and reflecting on the solution—perhaps testing it or modifying it for special cases.

- 120. For regular student constructed response items, the rubric should provide for rating levels of:
  - \* No response.
  - 0. Incorrect or irrelevant remarks.
  - 1. The response contains evidence of an understanding of the problem at a conceptual level evidenced by the logical approach taken. However, on the whole, the response is not well developed. Although there may be serious logical errors or flaws in the reasoning, the response does contain some correct work. Examples provided are incorrect or inappropriate.
  - 2. The response demonstrates a complete understanding of the problem, is correct, and the methods of solution are clear, appropriate, and fully developed. The response is logically sound, clearly written, and contains no errors beyond ones that may be a result of miscopying from elsewhere in the student's work. Examples are well chosen and fully developed.
- 121. For extended student constructed response items
  - \* No response.
  - 0. Incorrect or irrelevant remarks.
  - 1. The response indicates a minimal understanding of the problem posed but does not suggest a feasible approach to a solution. Although there may or may not be some correct work signifying a logical approach, the response is incomplete, contains major errors of reasoning, or reveals other serious flaws. Examples are absent.
  - 2. The response contains evidence of an understanding of the problem at a conceptual level evidenced by the logical approach taken. However, on the whole, the response is not well

developed. Although there may be serious logical errors or flaws in the reasoning, the response does contain some correct work. Examples provided are incorrect or inappropriate.

- 3. The response demonstrates a clear understanding of the problem and provides an acceptable approach. The response is generally well developed and coherent but contains minor weaknesses in the development. Examples are provided, but not fully developed.
- 4. The response demonstrates a complete understanding of the problem, is correct, and the methods of solution are clear, appropriate, and fully developed. The response is logically sound, clearly written, and contains no errors beyond ones that may be a result of miscopying from elsewhere in the student's work. Examples are well chosen and fully developed.