



FRAMEWORK FOR DIAGNOSTIC ASSESSMENT OF MATHEMATICS

*Edited by
Benő Csapó • Mária Szendrei*



NEMZETI TANKÖNYVKIADÓ



Framework for Diagnostic Assessment of Mathematics

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*Nessuna umana investigazione si può dimandare vera scienza,
se essa non passa per le matematiche dimostrazioni.*

*No human investigation can be called real science if it cannot be
demonstrated mathematically.*

Leonardo da Vinci

Contents

Intruduction (<i>Benő Csapó</i> and <i>Mária Szendrei</i>)	9
1 <i>Terezinha Nunes</i> and <i>Benő Csapó</i> : Developing and Assessing Mathematical Reasoning	17
2 <i>Csaba Csikos</i> and <i>Lieven Verschaffel</i> : Mathematical Literacy and the Application of Mathematical Knowledge	57
3 <i>Julianna Szendrei</i> and <i>Mária Szendrei</i> : Scientific and Curriculum Aspects of Teaching and Assessing Mathematics	95
4 <i>Csaba Csikos</i> and <i>Benő Csapó</i> : Diagnostic Assessment Frameworks for Mathematics: Theoretical Background and Practical Issues	137
5 <i>Csaba Csikos</i> , <i>Katalin Gábri</i> , <i>Józsefné Lajos</i> , <i>Ágnes Makara</i> , <i>Julianna Szendrei</i> , <i>Judit Szitányi</i> and <i>Erzsébet Zsinkó</i> : Detailed Framework for Diagnostic Assessment of Mathematics	163
About the Contributors	317

Introduction

As suggested by the quote from Leonardo chosen as the motto for the present volume, mathematics plays a special role in the development of science. It has similarly special significance in formal education. It is the oldest of sciences, and even its early achievements continue to be part of the school curriculum today. It is one of the first fields of knowledge to be arranged into a school subject, and still tends to be the subject assigned the highest number of school periods. In the Hungarian education system mathematics is the only subject taught throughout the twelve grades of schooling. Children start preparing for formal mathematics education even before they start school and it remains a core subject in all science and engineering degree programs as well as in a substantial share of social science degree programs in higher education.

The study of mathematics has always been intertwined with the development of thinking and the acquisition of the ability of abstraction and logical reasoning. Mathematics also plays a role in solving everyday problems and the ability to use mathematical knowledge is an indispensable skill in several jobs. It is this special significance that has earned mathematics a permanent place among the assessment domains of the large-scale international comparative surveys, the results of which are taken into account when the development potential of participating countries is estimated. In Hungary, besides reading, the annual comprehensive educational assessment program also covers mathematics, and it has been naturally included together with reading and science in the project undertaking to develop a diagnostic assessment system.

Over the decades around the turn of the Millennium, research in education sciences and psychology has produced results that – if integrated and transferred into practice – may bring about a major turn in the improvement of the efficiency of education. The project providing the framework for the present volume occupies the intersection of three major research trends.

One of the key factors in the development of education systems is the availability of increasingly frequent, accurate and detailed feedback mechanisms at the different levels of decision making. In this respect, the most spectacular change in the past few decades was brought about by the development that the large-scale international surveys became regular events. The international comparative data enables us to identify the system-wide

attributes of public education and the results of the consecutive assessment cycles provide feedback on the effects of any interventions. The methodological solutions of the international assessment programs have assisted the development of national assessment systems, and in several countries, including Hungary, an annual assessment program has been implemented providing primarily institutional level feedback. Through an analysis of their own survey data, institutions can improve their internal processes and activities, and as the results are made public, this may act as an incentive to seek ways of improvement. The experiences of countries where a system of this sort has been in place for a relatively long time show, however, that placing this sort of pressure on schools has the effect of improved efficiency only within certain limits, and too much pressure may lead to various distortions. Methods and tools directly assisting the work of teachers are indispensable for further improvement in performance. In this respect, the next stage of the development of evaluation can only be reached through the construction of systems suitable for providing frequent and detailed student-level feedback.

Traditional paper-based tests are not suitable for sufficiently frequent student assessment. For this reason, in the past teachers did not have access to measurement tools directly assisting learning through following student progress and signaling possible delays in good time. The second key factor we highlight is therefore the explosive advancement of information and communication technologies, which offer novel solutions in every area of life. The availability of these technologies in education allows the simple implementation of tasks that were previously impracticable, such as frequent educational assessments providing diagnostic feedback. Computers were put in the service of education effectively as soon as the first large electronic computers appeared; educational computer software was already available several decades ago. The use of information technology in education was, however, often motivated by the technology itself, i.e., the reasoning was that now that these tools were available, they might as well be used in education. Online diagnostic assessment reached this conclusion coming from the opposite direction, as an appropriate technology was sought for the implementation of an educational task of crucial importance. In our case, info-communication technology is a system component that has no substitute and that expands the range of possibilities for educational assessment.

The third factor, one which is closest to the concerns of this volume, is the

cognitive revolution in psychology, which had an impact in several areas at the end of the last century and gave a new impetus to research efforts in connection with school learning and teaching. It has led to the emergence of new and more differentiated conceptions of knowledge, which have made it possible to define educational objectives more precisely and to develop scientifically based standards. This process also paved the way to a more detailed characterization of student development processes.

With the recognition of the crucial role of early childhood development, the focus of attention shifted to the initial stage of schooling, especially to the encouragement of language development and to the fostering of reasoning skills. Several studies have provided evidence that the acquisition of basic skills is indispensable for in-depth understanding of the subject matter taught at schools, which is in turn essential for students to be able to apply their knowledge to new contexts rather than just reproduce exactly what they have been taught. If the required foundations are not constructed, serious difficulties will arise at later stages of learning and the failures suffered during the first years of education will delimit students' attitudes towards education for the rest of their lives.

School mathematics plays an outstanding role in the development of cognitive abilities. In comparison with other subjects, it presupposes relatively little prior knowledge, thus its education can start at a very young age, in early childhood. Learning mathematics provides opportunities for students to recognize regularities, to weigh different options and to construct models. Very early on in mathematics education students can be encouraged to question what is believed to be true and to look for causes and proofs. Mathematics provides unique opportunities for understanding the significance of verification and proof. We have access to an enormous body of unstructured information and data. Mathematics can improve the skills needed for classifying data and information and for drawing the correct conclusions. There is a growing need for an ability to recognize and verify relationships, which is an issue that should be addressed in education. Science and technology advance at an enormous rate and factual knowledge may rapidly become out-of-date. Reasoning and problem solving skills, in contrast, never become obsolete and are needed in a growing number of areas in life. An important task of mathematics education right from the first grade of school is the development of reasoning and problem-solving skills, any deficiencies in which cannot be compensated for at later stages.

In line with the above trends, the Centre for Research in Learning and Instruction at the University of Szeged launched the project “Developing Diagnostic Assessments” in which the frameworks for diagnostic assessments in the domains of reading, mathematics and science have been developed. The current volume presents the results of our research in the domain of mathematics. Based on these results, assessment instruments, item banks of several hundred tasks covering the first six grades of school is constructed as a part of an online testing system. This system – the implementation of which is a lengthy process involving several hierarchically organized steps – will be suited to providing regular and frequent student level feedback on the various dimensions of changes in knowledge.

Diagnostic tests first of all give an indication of individual students’ state of development relative to various reference points. As in the case of system-wide surveys, the population average may act as a natural standard of comparison: The performance of a given student relative to his or her peers’ performance is an important piece of information. Online diagnostic tests, however, go even further; the system keeps a record of the students’ results allowing their progress and the evolution of their knowledge to be monitored over time.

The tools of measurement are based on content frameworks resting on scientific foundations, which are outlined in three volumes of parallel structure. The present volume discusses the frameworks for the assessment of mathematics while the two companion volumes are devoted to reading and science. The development work for the three domains proceeded in parallel and the same broad theoretical framework and conceptual system were used for the development of the detailed contents of their assessment. Besides having an identical structure, the three volumes also contain some identical sections in their introduction and in Chapter 4.

The work reported in this volume draws on the experiences of several decades’ research on educational assessment at the University of Szeged and on the achievements of the University of Szeged and Hungarian Academy of Sciences’ Research Group on the Development of Competencies, with special reference (a) to the results of studies related to the structure and organization of knowledge, educational evaluation, measurement theory, conceptual development, the development of reasoning skills, problem-solving and the assessment of school readiness, and (b) to the technologies developed for test item writing and test development. Constructing theoretical

foundations for diagnostic assessments is, however, a complex task requiring extensive collaborative effort in the scientific community. Accordingly, the development of the frameworks has been a local and international cooperative enterprise involving researchers in the fields that are to be assessed. The opening chapter of each volume has been prepared with the contribution of a prominent specialist in the relevant field; thus our work rests upon scientific knowledge on the cutting edge of international research. The details of the frameworks have been developed by researchers and teachers and other professionals with practical experience in curriculum development and test construction.

The frameworks are based on a three-dimensional conception of knowledge in line with a tradition characterizing the entire history of organized education. The wish to educate the intellect, to cultivate thinking and general cognitive abilities is an age-old ambition. Modern education also sets several goals applying to the learners themselves as individuals. In order to attain these objectives we must first of all be guided by the achievements of scientific fields concerned with the human being and the developing child. This dimension can therefore draw on the results of developmental psychology, the psychology of learning and, more recently, on the achievements of cognitive neuroscience. With respect to the domain of mathematics, the core issue in this dimension is the development of mathematical thinking and skills.

Another set of objectives is related to the usability of the knowledge acquired at school. The dictum “*Non scholae sed vitae discimus.*” is perhaps more topical today than every before, since our modern social environment is changing far too rapidly for public education to be able to keep pace with it. As revealed by previous research, the transfer of knowledge to novel contexts is not an automatic process; special teaching methods are called for in order to improve the skills of knowledge application. For this reason, it is essential that the question of the application of knowledge should appear as an independent dimension in the frameworks of diagnostic assessments. This constitutes a different system of goals, for which we must define what is expected of students that will enable them to apply their knowledge in different school contexts and in contexts outside of the school.

The third important issue is the question of which elements of the knowledge accumulated by the sciences and the arts should be selected as contents to be imparted at the school. Not only because the above objectives cannot

be attained without content knowledge but also because it is an important goal of its own right that students should become familiar with the given domain of culture, the knowledge generated by mathematics and science and organized according to the internal values of a given scientific discipline. Mathematics is not only a tool for the development of reasoning and practical problem-solving skills but also an autonomous discipline of science with its own internal logic and factual content, which students should acquire adhering to the field's organizing principles and structure. Although the first grades of primary education focus on students' personal development and on the development of skills, neither efforts to improve cognitive abilities nor efforts to prepare children for practical problem solving can be successful in the absence of meaningful acquisition of scientific knowledge.

The above goals have been competing with each other over the past few decades with one or another coming into fashion or gaining dominance at different times at the expense of the others. For the purposes of this project, we assume that while education strives to achieve these objectives in an integrated way, they should be treated as distinct dimensions in diagnostic assessments. The surveys must be able to show if there is insufficient progress in one or another of these dimensions.

The first three chapters of this volume summarize the theoretical background and research evidence related to the three dimensions mentioned above. In Chapter 1, Terezinha Nunes and Benő Csapó provide an overview of psychological issues related to the development, fostering and assessment of mathematical thinking. This chapter discusses the natural process of the development of reasoning using numbers and quantities, which may be encouraged and enhanced by efficient mathematics instruction. In Chapter 2, Csaba Csíkos and Lieven Verschaffel summarize research results related to mathematical literacy and the application of mathematical knowledge. Chapter 3 by Julianna Szendrei and Mária Szendrei discusses the organization of mathematics as a scientific discipline, what aspects of this knowledge are appropriate for teaching, what aspects are generally taught at schools, and what kind of content the science of mathematics offers for the task of developing mathematical thinking and for practical applications. Each chapter relies on extensive research literature and the detailed bibliographies can assist further research efforts. In Chapter 4, Csaba Csíkos and Benő Csapó discuss theoretical issues and practical solutions in the development of frameworks and outline the basic principles

guiding the construction of the detailed contents of diagnostic assessments. This chapter serves as a link between the theoretical chapters and the detailed content descriptions.

Chapter 5, which is the longest chapter making up half of the entire volume, contains the detailed frameworks of diagnostic assessment. The purpose of this chapter is to provide a basis for the development of measurement tools, the test items. The contents of assessment are grouped according to the three dimensions mentioned above. For the purposes of diagnostic assessment, the first six grades of schooling are considered to constitute a continuous education process. The results of the assessments therefore place students according to their current level of development along scales spanning all six grades. The content specifications of assessment questions could also essentially form a single continuous unit. However, in an effort to allow greater transparency and to follow the traditions of educational standards, this process has been divided into three stages, each of which covers approximately two years. For the three dimensions, therefore, a total of nine content blocks are described, each of which includes four main areas of mathematics.

In their present state, the frameworks detailed in this volume should be seen as the first step in a long-term development process. They specify what is reasonable to measure and what the major dimensions of assessment are, given the present state of our knowledge. As the domains covered develop at a very rapid rate, however, the latest findings of science should be incorporated from time to time. The content specifications can be constantly updated on the basis of our experiences of item bank development and an analysis of the data provided by the diagnostic assessment in the future. Our theoretical models can also be revised through an evaluation of the test items and an analysis of relationships emerging from the data. In a few years' time we will be in a position to look at the relationship between the various areas of early development and later performance allowing us to establish the predictive and diagnostic validity of test items, which can be a further important source of information for the revision of theoretical frameworks.

In the preparation of this volume *Csaba Csikos* played a prominent role. In addition to co-authoring three of the chapters, he also led the research team developing the detailed description of the contents of the assessment. Besides the authors of the chapters, several colleagues have contributed to

the completion of this volume, whose support is gratefully acknowledged here. Special thanks are also due to the team responsible for the management and organization of the project, *Katalin Molnár*, *Judit Kléner* and *Diána Túri*. The development and final presentation of the content of the volume have benefited greatly from the comments of the reviewers of earlier versions. We would like to take this opportunity to thank *József Kosztolányi* and *Ödön Vancsó* for their valuable criticism and suggestions.

Benő Csapó and Mária Szendrei