FRAMEWORK FOR DIAGNOSTIC ASSESSMENT OF MATHEMATICS

Edited by
Benő Csapó • Mária Szendrei

NEMZETI TANKÖNYVKIADÓ

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Edited by

Benő Csapó
Institute of Education, University of Szeged

Mária Szendrei
Department of Algebra and Number Theory, University of Szeged

Nemzeti Tankönyvkiadó
Budapest

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Authors:
Benő Csapó, Csaba Csíkos, Katalin Gábri, Józsefné Lajos, Ágnes Makara,
Terezinha Nunes, Julianna Szendrei,
Mária Szendrei, Judit Szitányi, Lieven Verschaffel, Erzsébet Zsinkó

The chapters were reviewed by
József Kosztolányi and Ödön Vancsó

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Detailed Framework for Diagnostic Assessment of Mathematics

Csaba Csíkos
Institute of Education, University of Szeged

Katalin Gábri
Educational Authority

Józsefné Lajos
Educational Authority

Ágnes Makara
Department of Mathematics, Faculty of Primary and Preschool Teacher Training,
Eötvös Loránd University

Julianna Szendrei
Department of Mathematics, Faculty of Primary and Preschool Teacher Training,
Eötvös Loránd University

Judit Szitányi
Department of Mathematics, Faculty of Primary and Preschool Teacher Training,
Eötvös Loránd University

Erzsébet Zsinkó
Department of Mathematics, Faculty of Primary and Preschool Teacher Training,
Eötvös Loránd University

The structure of the detailed assessment frameworks of mathematics is based on the theoretical background explained in the introductory chapters. In this chapter a three-level structure is presented according to the following scheme. The primary structure of the chapter is determined by three dimensions of learning mathematics. Within this chapter, first the psychological principles are highlighted showing that only such mathematics teachning can be successful which adjusts to the natural processes of cognitive development and improves reasoning. The second part of this chapter describes mathematical knowledge according to its application, and the third part is built according to a pure mathematical disciplinary approach. In the case of mathematics, the three dimensions of knowledge are mutually intertwined, and – as emphasized in previous chapters – distinguishing them serves the purpose of detailed diagnostic assessment. Certainly, the three dimensions appear in teaching in an integrated way, almost unnoticed and the problems of different dimensions are manifested parallel during the assessment.

The second aspect of the structural division is the school years. Due to the big differences between the pupils the age intervals can only be approximate, while by assigning the frameworks to several age groups the principle of interdependence and development is emphasized. The third basis of structuring is determined by the different fields of mathematical science. Since the developmental processes arch several grades, these contents appear at different levels in every grade.

It follows from the above-described structure that this chapter is divided into 36 parts. To every age group 12-12 units are belonging; the different fields of mathematics are represented by 9-9 sub-chapters and also 12-12 sub-units belong to the three knowledge dimensions. The theoretical chapters describing the different knowledge dimensions (the first three chapters of the present volume) contain the criteria of selecting the age categories and knowledge areas. It comes from the nature of the development processes that the focal point of development in certain areas is earlier, in other comes areas later. Therefore the following 36 parts are not equally proportionate or detailed. The further clarification of the details is however only possible after conducting surveys and possessing empirical data.

Diagnostic Assessment of Mathematical Skills

Detailed Assessment Frameworks of Grades 1-2

Numbers, Operations, Algebra

During the development process, in the lower grades we get from the well planned concrete activities, from the reality experienced by the learners to the more abstract formulations by drawings, words, signs and symbols through visual and audio-visual representations of real life. The correct harmonization of reality, concept and symbol (sign), their bringing in compliance with each other is the result of a lot of activities. The development of the system of skills indicating the competent use of whole numbers begins already in the preschool age. The fact that it is clear for the learner beginning the school that the bigger quantity is represented by bigger number is an indicator (among others) that the learner is at good development level concerning whole numbers as elements of mathematical reasoning.

A typical preschool exercise:

Draw more circles on the right side than you can see in the left side frame.

In the first grade we go further in the questions, instructions:

1. *Draw three circles more on the right side than you can see on the left side.*
2. *Describe, by the language of arithmetics, what you see on the figure. (*Solution*: 3+3+3=9; 3+6=9; etc.)*

In the second grade the mathematical content of the questions is extended further:

1. *Add more circles on the right side so that you get a total of 18 circles on the drawing.*
2. *Write additions, subtractions in connection with the figure. (*Solution: *18 – 3 = 15; 3 + 3 + 12 = 18; 15 – 3 = 12; etc.)*

3. *Surround the circles with red colour so that the same number of circles chould be within every enclosure. (*Solution*: 1×18 circles or 2×9 circles or 3×6 circles or 6×3 circles or 9×2 circles or 18×1 circles)*

The common experiences and collective mathematical activities create a kind of shared reference basis for the class/group. The richer and more mobile this reference basis is the more sure it is that the same image, sequences of actions, memories, ideas will be evoked in every pupil by the questions, statements and other formulations.

Numbers

Children coming from the kindergarten have memories about comparing objects and pictures, about studying characteristics, looking for relations and about their efforts to formulate relations, in accordance with their developmental level. The well prepared and diverse activities continue in the school, the content elements of concepts are made understandable. In this way the pupils understand and use appropriately the relations of more-less (for example, by one to one mapping), same quantity (for example, by putting into pairs, which pairing is the method of mastering this relation), smaller-bigger, longer-shorter or higher-lower (for example, by comparative measurements), etc. The relation symbols (>; <; = symbols) are given names related to the children's environment, fairy-tale world (for example, the mouth of the fox opens to that direction because he sees more chicken there), but in certain cases the "relation symbol" name is also used. (The early introduction of mathematical expressions has to be treated carefully, because due to this they may be imprinted incorrectly (for example, with narrower content) and this may be disadvantageous, leading to a lack of understanding later).

The sequences of observations, comparisons enable the pupils to make identifications, to recognize and name the important characteristics contributing to differentiation, to make abstractions gradually (for example, making them recognizing the differences between two drawings of small dogs draws the attention not only to the physical contours (for example, pulling off or lifting up the dog's ear), but also to the differences expressing emotional/mood status (for example, the dog is sitting quietly or muscles taut and face angry, mouth open). The observation, discussion, conscious observation of the differences and changes project the visual representation of operations and is a kind of preparation of operational symbols.

Activities like reading of specific images, figures, drawings by properly selected movements (for example, standing up, sitting down, using different hand positions during making sequences), saying verses by syllables (for example, picking up an element by a counting-out rhyme), making sounds (for example, throb, knock, clap or any intoned tune) represent a kind of „counting". For example:

Let ♣ mark a clap, ♥ mark a foot stamp.
"Read" the picture below according to the symbols.

♣♣♣♥♥♣♣♣♥♥♣♣♣♥♥♣♣♣♥♥

Find out the different readings by using movements, sounds.

Counting can be made in different ways in the case of the same picture (number). Many people formulate this in the following way: "a number has different names". This means that the number can be expressed by its decomposed members, by different characters. The purpose of the listed activities is to enable the pupil to make calculations reliably in the learned number ranges, to imprint names, nominations to memory, to recall and use them.

Operations

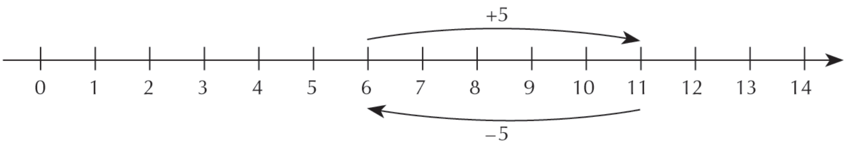
Mathematical operations with whole numbers represent the typical field of appearance and assessment of the phenomenon called additive reasoning in the system of mathematical skills. Literally the word additive refers to an addition, but in the wider sense of the word it also includes knowledge elements of comparing quantities, numerosities. These knowledge elements make us understand that by taking away from a given quantity and by adding the same quantity we get to the initial quantity.

In the process of activities aiming at the formation, deepening of number concept we prepare the mathematical concept of addition (+) and subtraction (–): by reading of the different numbers, sums (for example, 5 walnuts and 2 apples are equal to 3 apples and 4 walnuts) and differences: for example, a picture shows that of 5 boys 1 has not eaten the food that is 4 boys of the 5 have eaten their food. The 5 – 4 is the difference form of 1.

From a content point of view adding (supplementing to a certain amount (for example, $3 + \triangle = 7$)) and partitioning (dividing the whole into two or more parts (for example, 8 = △ + △)) are basically related to addition, but as to their mathematical background they represent the solution of open sentences. Partition allows the production of a number in many different ways, but a number can be produced both by adding and by taking away (for example, number 4 can be produced from 1, 2, 3 by adding and from 5, 6, etc. by taking away). The experiences collected during the various displays, pictorial and text situations typically formulated in words prepare the algorithm of making operations. By the time children can write down the numbers the understanding of operational signs (symbols), their safe use is well founded in the learned range of numbers. In the first two grades we mainly lay the grounds for addition, subtractions and gradually deepen them (in grade 2 extended to number range up to 100), and we develop the need for self-checking.

Outstanding role is given to the interpretation of operations by means of the number line.

For example:



Moving along the line number in two directions connects the operation and its reverse. Arrows showing to the right represent additions, those showing to the left represent subtraction. They illustrate well that 11 is by 5 bigger than 6 and 6 is by 5 smaller than 11.

The conceptual characteristics of multiplication (addition of equal addends), partition into equal parts (for example, by visualization, marking (for example, introduction of 20/4), division (visualization, marking (for example, 20 : 4)), division with remainder (with display, indication of remainder) are prepared through a sequences of activities.

In the course of studying the characteristics of, and relations among operations we mainly make the 1st grade students discover the interchangeability and grouping of the members of addition and look for relations between addition and subtraction. In the 2nd grade we also observe the relations between the changing of addends and the change of the result, the relations between multiplication and division, and we also observe the interchangeability of the multipliers during manipulative activities.

Algebra

The algebraic symbols and procedures composed a special module in the field of Numbers, number systems in the disciplinary division of mathematics. The abstraction needed to the handling of symbols presumes the conversion operation in the Piagetian sense, representing the basic element of mathematical thinking as element of additive and multiplicative reasoning.

Relations, Functions

The subject of relations and functions plays an outstanding role in the development of cognitive abilities. Inductive reasoning (sequences of numbers, number and word analogies) belonging to the Relations, functions topic can be mentioned as an element of multiplicative reasoning. Similarly, the interpretation of proportion as a function also appears during the development of proportional reasoning.

In connection with the development of the counting ability children shall be able to continue declining and increasing sequences of numbers in the set of natural numbers up to hundred. They have to find the rule for sequences where the difference between successive numbers forms a simple arithmetic sequence.

Continue the sequence by adding two members. What is the rule?

1 4 7 10 13 ___ ___

The learners should be able to follow and continue the periodically repeating movements, rhythms. In the case of number sequences they have to recognize if it is a declining or increasing, or periodical sequence.

Continue the sequence by adding two members.

1 3 5 3 1 3 ___ ___

How would you continue the following sequence? Find at least two rules.

2 4 6 ___ ___

The exercises where correlations have to be found between the elements of number sequences, or other sequences (of objects, other elements), or tables also represent the application of multiplicative reasoning. These problems improve both the inductive and deductive reasoning abilities of pupils. It is important to discuss, interpret the many different ways of formulation of rules both from the point of view of development of skills and the assessment of the solutions.

Look at the following sequences of flowers and answer the questions.

a) Draw the next member of the sequence.
b) What rule was used in the preparation of this sequence?
c) If you continued the drawing what do you think the 12th, 15th and 20th members of the sequences would be?

Word problems or parts of them contain ideas the collective discussion of which is educative, thus we should by all means talk about them (for example, the text can be about environment protection, friendship, selfless help, sharing our snack with the fellows, conditions of civilized coexistence, it can be based on family, holiday, geographical, historical, artistic subjects).

Regular dealing with word problems develops accurate, clear and intelligent communication of learners, strengthens the competence of understanding and creating texts, problem solving thinking, creativity, initiating disputes based on reasoning, the need for control and self-control.

By the end of the second grade the students should be able to state the rules of sequences and to continue the sequences to determine the rule based on the difference between the members of the number sequences

Continue the sequence. What can be the rule?

1 3 6 10 15 ___ ___

In the case of most number sequences there is an obvious rule which can be found with the least cognitive effort. One of the elements of inductive reasoning is that the pupil should be able to recognize the "economic" solution from information theoretical point of view, which can thus be called the obvious or the most intelligent solution.

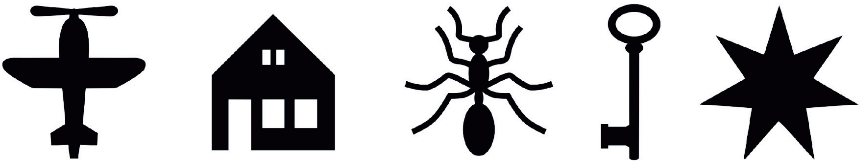
On the other hand it comes from the requirement of developing divergent reasoning that besides improving inductive reasoning all such rules which the learner is able to justify rationally must be accepted as a solution. In the case of the above problem for example the difference between the numbers always increases by one, that is the following member will be by 6 bigger than 15. We also have to accept the simplifying rule-making which does not use the information content of the sequences, but in these cases we have to show during the class that there is "more" in the sequences than for example the following two possible simplifying rules: (1) simple, monotonous sequences where the next member is bigger than the previous one. After formulating this rule we have to accept any two natural numbers which ensure the monotony of the sequences. (2) It often happens among small school children that they consider a sequence of numbers periodical, although this was not the aim of the author of the problem. In this case 15 would be followed by 1 and 3. Thus during the setting of problems we either give a priori the rule of continuation of the sequences (or we should at least refer to the type of the rule to be determined) or rule-making will be inseparable from the continuation of the sequences.

Geometry

Within the system of mathematical abilities, we highlight two components which are closely linked to geometrical contents. One of the actively tested fields of the research on intelligence is spatial reasoning, that is the ability of people to turn plane and spatial forms in mind and to make operations with them like for example rotations interpreted as geometric transformation. On the other hand, proportional reasoning interpreted as part of multiplicative reasoning can be linked to measurement, one of the subsections of geometry. Problems can be set both in the area and volume calculations and in the conversion of units which essentially indicate the developmental level of proportional reasoning. This latter ability is not yet explicitly mentioned in the frameworks fro grades 1-2, in the above we wanted to mention two abilities which are typical for geometrical contents. In this age group the following contents belong to spatial thinking.

The observation of the countless patterns created by transformations (including patterns to be found in the nature, in folk art, in the built environment, in the different human works) prepares the mathematical interpretation of symmetries, repetitions, rhythms, periodicities. The activities promote that *the pupil be able to recognize symmetries, at experimental level (manipulative and pictorial). They should be able to differentiate between the mirror image and the shifted image on the basis of the total view.*

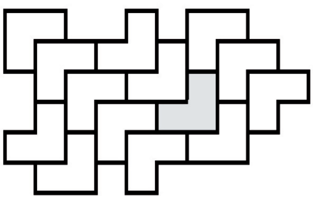
Copy the following illustrations on a transparent paper.



Check which of the illustrations can be folded in a way that the two parts cover each other completely?

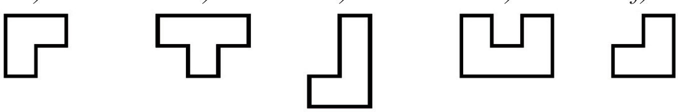
Solution: forms 1., 3., 5 can be folded according to the condition.
Typical exercise for testing spatial abilities:

Colour with graphite pencil the sheets which stand in the same way as the grey-coloured sheet.



Circle the letter of the sheet which can continue the above parquet building.
~~**Cross out**~~ the letter which cannot be used.

b) *c)* *d)* *e)* *f)*



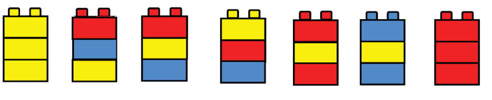
Combinatorics, Probability Calculation, Statistics

Operations of combinative ability can partly be linked to the elements of combinatorics, a content domain of mathematics. By revealing the psychological constructs enabling the mathematical phenomena of permutation, variation and combination we arrive at several other operations (for example, finding all sub-sets of a given set, generation of Cartesian product of sets) which typically do not belong to the combinatorics domain in school mathematics education. Among the mathematical reasoning elements, however, these latter are also manifestations of multiplicative reasoning while from psychological point of view they are part of combinatorial reasoning.

In general, by the end of grade 2 we do not get to the building up of independent system of combinatorial abilities, since this would indicate reasoning in some kind of structure, which in turn requires high level mathematical abstraction skill. Therefore the assessment of different components of combinatorial reasoning is feasible in the case of tasks containing small sets.

In the following the building up of combinatorics is presented through some problems in the foundation stage (grade 1 and 2):

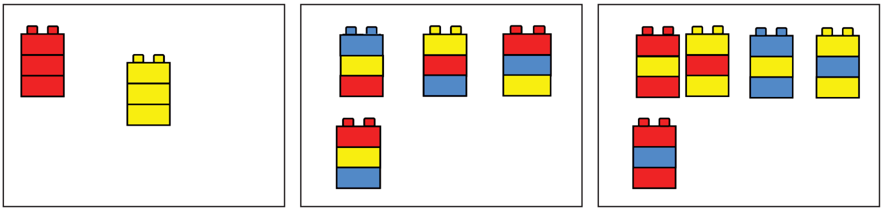
I have built three-level towers of red, yellow and blue Lego elements.
What else could I have built? Draw the other towers.



In this problem the difficulty is in keeping some characteristics constant while others may change. Does a solution fulfill the conditions (three-level, made of red, blue, yellow colours)? Are there any newly built towers which can already be found among the formerly built ones? Assessing children's knowledge it is import to know who and by how many objects extended the set, who were able to create objects different from the existing ones and from those of their mates.

We can make the task more difficult by formulating the problem in a different way:

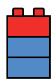
I have built towers of red, yellow and blue Lego elements. Then I arranged them in three groups:



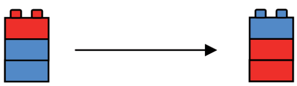
What else could I have built? Put new towers at the correct places.

In the problem above the criteria of systematization is shown by the drawing and not by the text. Finding the criteria is an important element of the problem (one, two or three coloured towers). In this arrangement however, the transparency of the whole system is questioned. It is also a question whether other criteria can be found to the solution.

The second group shows that the elements below each other were created by "reversing" the towers. This strategy works very well here. But it cannot be carried forward to the third group, since here some typical characteristics were left out of the row of problems thus the eventual absence cannot be discovered. It is possible that somebody detects some kind of regularity in the arrangement of elements in the third group, namely that the elements are inverses of each other. In this system however the finding of all the elements cannot be guaranteed, since the drawing does not give an example of the following type:



Thus in the problem different strategies shall be used when finding the one, two or three colour elements. It is possible that for somebody exactly the solution strategy gives the basis of the criteria system and puts the above element into the second group, since

of this tower:  this tower was made by reversion.

By presenting the above problem we wanted to illustrate the diversity of combinatorial reasoning, the direct consequence of which is that in grade 1 and 2 in the assessment process we have to be content if students find some other elements fitting into the given system of criteria.

Detailed Assessment Frameworks of Grades 3-4

Numbers, Operations, Algebra

The correct representation of whole and rational numbers is of key importance in the development of the number concept. There are abilities belonging to additive reasoning which lead to the representation of rational numbers. In people's thinking rational numbers are mental representations of the relations between the numerator and the denominator. With the help of division into parts, we prepare already in the preschool age the empirical basis for learning fractions.

By dividing the whole into equal parts, the notion of unit fraction is developed with the help of different quantities (length, mass, volume, area, angle), then by uniting several unit fractions, fraction numbers with small denominators are produced. During this work the children are performing double direction activities. By cutting, tearing, folding, colouring and fitting the parts they produce the multiple of unit fractions, or they name the produced fraction parts in comparison with the whole. They compare fractions produced from different quantities, put them in order according to their size and look for the equal parts.

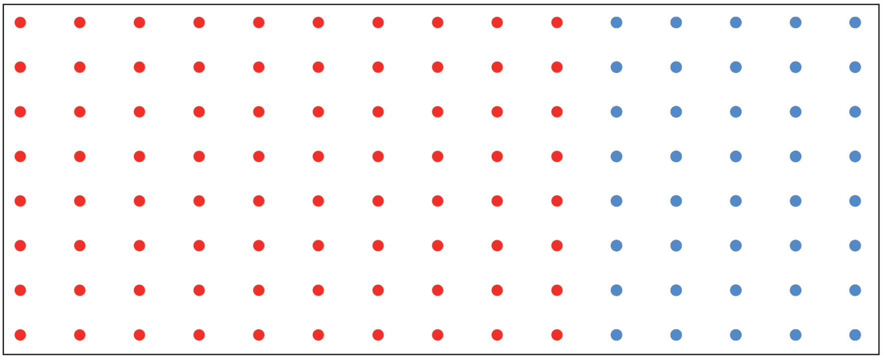
Additive reasoning includes abilities which enable for learning the characteristics of arithmetic operations. The children continuously obtain experiences about the operational characteristics of addition. The computation procedures make possible that the pupils safely give answers to problems which require operations with actual numbers or their comparison.

For example:

The Szabó family made a four day excursion. On the first day they travelled 380 km, on the second day 270 km when they arrived at their destination. On the way back they took the same route. After travelling 400 km they arrived at the night accommodation place. How many kilometers did they have to go on the fourth day?

Working with different object, numbers and word problems offer possibilities for practicing the role of parentheses in connecting into one number and in the multiplication of the sum by members.

For example:
The drawing shows an orchard. The red circles represent apple trees, the blue ones plum trees. How many fruit trees are there in the garden?



The operational properties are consciously used during multiplication in writing.

For example:
Which multiplication is correct?

a)	*b)*	*c)*
$263 \cdot 27$	$263 \cdot 27$	$263 \cdot 27$
1841	1841	1841
526	526	5260
2367	18636	7101

Division as a written algorithm is the most difficult operation. With the help of tools the children learn to divide by one-digit number in grade 4.

During computing operations the different types of control methods which they learn during the acquaintance with the procedure provide safety to the children. Estimation, multiplication, partitioning and the use of pocket calculator are among the methods of checking.

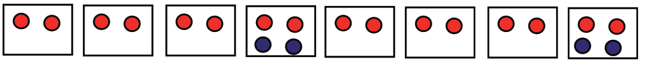
In general, in the fourth grade we offer opportunities for the children to look for different solutions and to compare them. In this way the ability to recognize the existing relations between the models can be developed. The fact that the data of different models are identical, that there is connection between representations and operations are recognized consciously by the children. The teaching of the different ways of solutions and their appreciative use is the guarantee that the children will be able to activate, if necessary modify according to the type of the problem these solution methods in new situations, in case of changed conditions. In this way the knowledge of children can be easily developed. Getting acquainted with, and, comparing different solutions children can judge the usefulness and beauty of different solutions.

Below is an example of solving a problem in several different ways:

The top of a high hill can be reached by a lift. In some lifts two people are travelling at the same time, in others four people. A company of 20 people was carried up by 8 cabins. In how many two and four seat lifts did they travel?

Solution 1: *with activity, using tools*

Children place 8 sheets of paper in front of them, which represent the cabins, they prepare 20 discs, representing the travellers. They put the discs on the papers so that two or four discs were on every sheet.



The answers to the questions are given on the basis of the picture they get: 6 pcs of 2 seats and 2 pcs of four seat cabins were taking the 20 member to the hill.

Solution 2: *trial and error method, using a table*

Number of two seat cabins	1	2	3	4	5	6
Number of four seat cabins	7	6	5	4	3	2
No. of travellers in the two seat cabins	2	4	6	8	10	12
No. of traveller in the four seat cabins	28	24	20	16	12	8
Total number of travellers	30	28	26	24	22	**20**

From this solution more information can be obtained, we get answer for questions which were not formulated by the original problem. For example, how can 30 persons travel up to the hill in eight cabins?

Solution 3: *using open sentence*

Mark the number of used two seat cabins: $\square$

Consequently, the number of used four seat cabins is: $8 - \square$

Number of travellers in the two seat cabins: $\square \cdot 2$

Number of travellers in the four seat cabins: $(8 - \square) \cdot 4$

Total number of travellers: $\square \cdot 2 + (8 - \square) \cdot 4 = 20$

From this it can be determined that the number of two seat cabins is 6.

(Children use the planned trial and error method to produce this result.)

The number of used four seat cabins is 2.

The three completely different ways of solution of the problem above shows that we cannot expect from the children to solve the problems on the basis of only one scheme, we should not insist on following strictly determined steps. That's why it is more preferable to evaluate the selection of the correct model and the solution of the problem within the model.

In these grades we begin the preparation of concepts, procedures which need to be further developed later without making the children consciously aware of what is happening. The organized collection of experiences is only the beginning of a long process (for example, arriving from the fraction to the whole). The mathematical knowledge of the pupils develops in the higher grades in accordance with the curriculum, therefore it is undue to expect children to give precise definition of the concepts they use.

Relations, Functions

In grades 3-4 the pupils can prepare simple graphs and are able to read their data. They are able to look for mathematical models to a given situation with texts, pictures and to match them with data. If necessary they use other mathematical models (sequences, tables, simple drawings, graphs) in the solution of word problems.

Learners can recognize simple correlations, express them by examples, basic generalizations. The relations can be recognized, correlations can be read from figures, tables.

In these grades, the acquired knowledge, skills and abilities can be evaluated by means of tasks formulated by simple instructions. Here we mainly ask the pupil to perform an acquired, practiced step or sequences of steps.

It may happen that we do not use mathematical symbols for the description of the problem, but rather drawings, figures and we often expect from the children the steps to be made not in "mathematical" form, but in drawing or by some kind of illustration what's more, in the everyday life we can expect some kind of activity. Through some examples, we present below how many different types of problems can help to practice inductive rule generation and to follow the rules.

Continue the drawing in a way it has been started:

¤ *Δ* *Δ* ♥ # ¤ *Δ* *Δ* ♥ # ¤ *Δ* …………………

Add the missing numbers in the "number snake".

Continue the sequences below with 3 elements on the basis of the given rule: the difference between the elements always increases by the same.
1 3 6 …………………………………………………….
Look for a rule yourself and continue the sequences on this basis.

What symbol can be found in the square marked by (5;C)? …….

D		☺		⌂		⌂	
C	⌂		☼		☼		☼
B		☼		⌂		☼	
A	☼		☺		☺		⌂
	1	*2*	*3*	*4*	*5*	*6*	*7*

Colour the quadratic lattice below according to the following instruction.
yellow: (3;f) (4;e) (4;g) (5;g)
red: (2;f) (3;e) (3;g) (4;h) (5;e) (5;g) (6;f)
green: (3;c) (4;b) (4;c) (4;d) (5;c)
brown: (1;a) (2;a) (3;a) (4;a) (5;a) (6;a)

h						
g						
f						
e						
d						
c						
b						
a						
	1	*2*	*3*	*4*	*5*	*6*

What regularity do you find among the index numbers of brown squares?

In case of proportionality there are many possibile ways for selecting a problem. Every conversion of measurement unit, buying, uniform motion, work, enlargement, etc. are eligible for the formulation of simple routine problems.

How much does 6 kg potatoes cost if 4 kg costs 312 HUF?

Zsófi travelled the 27 km long bicycle route in one and half hour with constant speed. How far did she get in 10 minutes?

Children measure the length of their classroom by steps. Csaba could take 18 steps from one wall to the other, but Julcsi takes 24. Who has the larger step?

Grandma prepared pastries for the children, in all 32 pcs. She made croissants and pretzels. How many of each?

	5	6	7	10					
	27								

On Monday Zoli received a piggy-bank and 200 Forint. He put the money into the piggy-bank, plus he put a fivo-forint coin and a tan-forint coin into it every evening. On which day did he have 320 HUF in the piggy-bank?

Among the word problems the ones describing the events of real life, some kind of motion, changes are of great importance. We most often describe change of temperature, growth, movement. The pupils have to recognize these changes, sometimes illustrate them, and look for relations, correlations, and regularities. The following sequences of problems illustrate the very varied possibilities of recognition of relations and of following the rules.

When Panni was born her mother was 25 years old. How old is her mother today if Panni is 9 years old? How old will be Panni, when her mother is 50? When will the two be 99 years old together? Make a chart about the age of the two persons and based on the chart formulate other statements.

The distance between two cities is 190 km. Trains depart from each city every morning at 8 o'clock towards the other. One of the trains takes 50 km in an hour, the other 45 km. Make a drawing about their movement and find out when they will meet.

In a reservoir there is 4800 hl of water. A pump is lifting out 8 hl water per minute and 2 hl water is added to the tank via a pipeline system. When will the tank become empty?

Péter is making a puzzle. He has to make drawing according to an instruction starting from a certain point of a squared paper. The arrows show the direction, the numbers show the number of steps. What did Peter draw if he followed the instruction correctly?

8↑ 5→ 2↓ 3← 1↓ 2→ 2↓ 2← 3↓ 2←

Geometry

Through the knowledge elements of spatial abilities the learners will be able to create line patterns, spread patterns, parquet patterns, colouring, drawing with templates or on net.

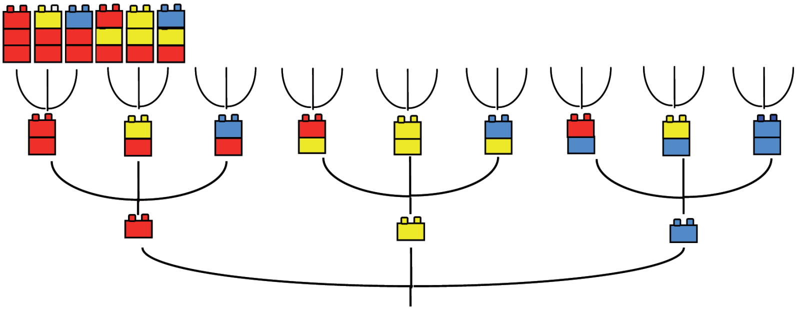
In the field of measurement there appears the requirement for the conversion of measurement units. The pupils should only know the conversion of units in cases to which – in principle – they can connect realistic experiences. Thus the technique (and together with this the safety) of mechanical computation can be taken over by proportional reasoning rooted in real life experiences.

Combinatorics, Probability Calculation, Statistics

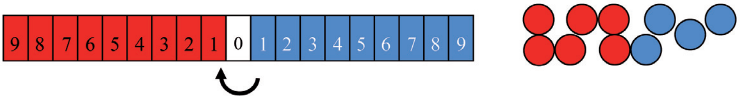
In these grades priority is given to the development of systematization skills in the combinatorics and probability content domains. For example, during the lessons we can give the task to the children to build three level towers and to try to build towers as varied as possible. They should look for all the options. During the lesson the teacher asks the children to observe and collect all the ideas based on which they can say: all the possible towers were made. The pursuit of completeness does not necessarily develop in the children by itself not even after a longer time. From the side of the teacher problem proposal or giving support may also be required: Is there any other, or there are only so many and no more? How can a small child realize whether he/she was able to find all possible options and if not what is missing? An important and good opportunity for this is that they somehow arrange in front of them the built up towers in a “beautiful” way.

Some of them pay attention to the colour of the lowest element of the tower and put aside those which they started to build in red, they separate the blue and yellow base towers. In this case they may realize that the same number of towers should be in all the three groups and this can be a starting point to the determination of shortage, perhaps to the finding of the missing building. It is commonly said it is because of “symmetry” that the same types of towers can be found in all the three groups. This concept means that there is no explanation why the building can be continued in many different ways if we put one colour below or if we put another one.

The advantage of this arrangement is that it can be continued: whatever the starting colour was, three different colours can be put in the middle and whatever the first two were the building can always be finished by the third elements in three different colours. This type of system building can be illustrated by a diagram looking like a tree (this is how it is called: "tree-diagram"):

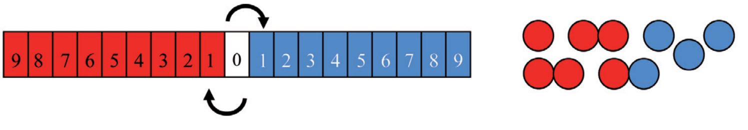


In the course of the improvement of probabilistic reasoning a lot of different games are practiced, for example, games with discs. The game is played in pairs. The members of the pairs select a side on the table and move a figure starting from 0 (white field). They can step one to the right if after throwing up 10 discs there are more red than blue discs and they can move one to the left is there are more blue discs than red ones. (If the number of the discs is equal they do not step.)



In our example one step to the right is allowed. The winner is the player on the side of whom the figure is standing let's say after 20 throws. (If it actually is at position 0, it is a tie). The game is simple and the probability sense suggests that the blue side is as good a choice as the red side. When they compare their experiences on class level, they will find the same.

On another occasion the children play with two figures and 10 discs so that "A" can move if the number of red discs is even, while "B" will step if the number of blue discs is even. Both players move their own figures.



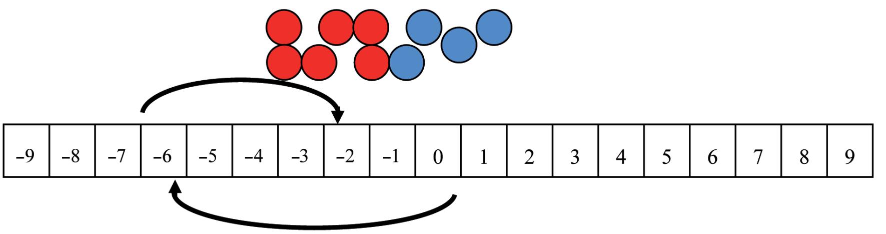
In our example both players step one. They have to complete a number of games so that they could make the following conclusion: the game will always be a draw, since either both players can step or neither of them. With this problem it is worth however to "make a joke" with the children, since in this way they can acquire the idea that 10 can only be divided into sums both members of which are either even or odd.

If we change the number of discs to 9 now, we again play a game where the probabilities are the same.

Further observations can be made by the generalization of the problem. For example they play with different even or odd number of discs. When developing students' reasoning, it is much more motivating for the children to obtain experiences about the division of sums into even or odd numbers by playing a game, than by doing mechanical operations.

For another didactic purpose, the pairs again play with 10 discs. One figure starts from 0, but here the table is replaced by a number line. After throwing the discs the players shall make the same number of steps to negative direction as the number of red discs dropped on the table and to positive direction according to the number of blue discs.

For example I threw this:



I step six to the negative direction and then from the arrival point I step four to the positive direction. I could have first stepped four into the blue direction and then six in the red direction. (Shall I finally arrive at the same place? Is commutativity working in the case of the negative numbers too?)

Now they throw ten times in a row in a way that the figure always steps further from the point where it stopped after the previous throw. Before starting the game the children should make a guess where the figure will most often arrive at from the following options after 10 throws: –6, –3, –1, 1, 3, 8. Shall it arrive at point 8? Or at point –3? Before starting the game everything is possible. Our idea about probability suggests that the many throws somehow compensate each other and the guess should be somewhere around 0. Yes, but now 0 is not among the possible guesses, thus 1 or –1 or perhaps 3 or –3 can also be good.

After some games the teacher asks the children where the different pairs arrived, for example the following notes can be made: –2, –8, –2,– 4, 0, 0, 6, 6, 4, 8, 2, 2

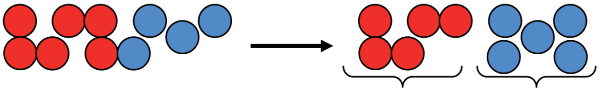
Can it be *accidental* that all of them arrived at even number?

A new round may confirm the guess and the search for explanations can be started.

We can collect the possible throws and the possibilities for the length of a step:

10 r = –10	10 b = 10
9 r + 1 b =–8	9 b + 1 r = 8
8 r + 2 b = –6	8 b + 2 r = 6
7 r + 3 b = –4	7 b + 3 r = 4
6 r + 4 b = –2	6 b + 4 r = 2
5 r + 5 b = 0	

Or they simple look at what is happening if one blue disc is changed to red:



Another statement which the children can discover themselves and can feel much closer than by simply getting the teacher's word: "If I reduce the minuend by one and increase the subtrahend by one the difference will be reduced by two".

Thus whatever we throw with 10 discs we will always arrive at an even point after the first throw. And during the further throws we will always step even numbers. During these steps the children get experiences about activities required to the interpretation of the opposites of positive numbers, about the addition of positive and negative numbers and about the fact that the relation about the parity of the sum will remain valid in the circle of negative numbers, too. Children can get more realistic experiences about an *impossible event* with regard to probability concept, than by getting such an extremely obvious example that the sum of numbers thrown by two cubes can never be 13.

Detailed Assessment Frameworks of Grades 5-6

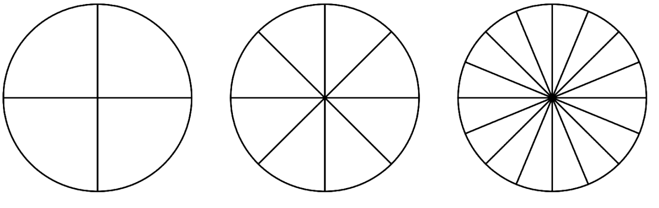
Numbers, Operations, Algebra

In grades 5-6 whole numbers (both positive and negative) up to arbitrarily high absolute values turn up in the school, that is together with keeping the empirical basis of numerosities typical of the earlier grades the representation of "big" numbers should also be developed. From a mathematical point of view the device of this is the normal form of numbers, from a psychological point of view the element of additive reasoning. In the comparison of the size of the numbers the interchangeability of relations "smaller than" and "bigger than" appears as elements of additive reasoning.

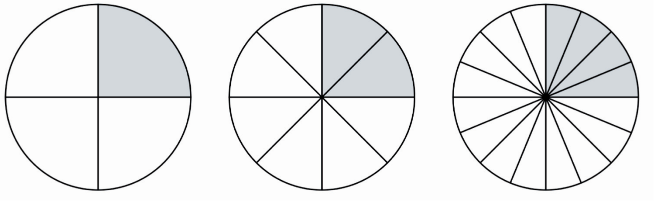
In the sphere of numbers connectable to the empirical basis the varied and purposeful forms of activities will certainly continue in grades 5-6, too: working with objects, cutting, decomposition, making, filling in of place value tables, reading numbers from them, writing down verbally pronounced numbers, representations, reading numerals, comparisons on number line, etc. The diversified experiences help for example the deepening of the concept of fraction, decimal number, negative number, the varied representation of the same values (for example with additions, simplifications) and the representation of the same values in different forms (for example decimal number form of a fraction and vice versa). Only the concepts and contents which were experienced in many different ways will be long lasting, easily usable, and can be recalled.

In the case of the fractions it is important to show (with a lot of folding, cutting, putting out of same cubes, using varied units, by drawing, etc.) that we can divide a unit into equal parts in many different ways, thus a given fraction value can be represented in many different ways.

On the figure below we have divided three circles of the same diameter to 4, 8, 16 equal sectors. Colour one quarter of the circles.

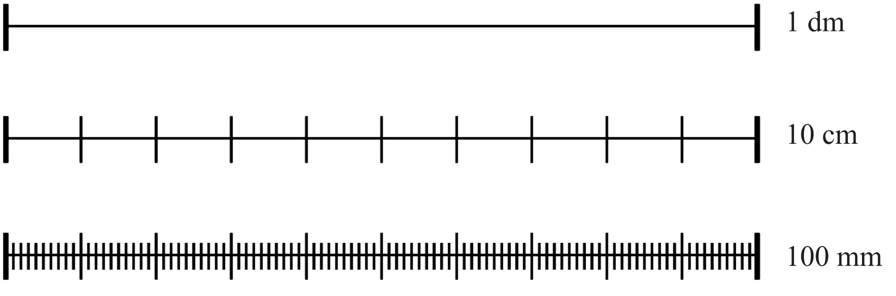


Solution:



The figure clearly shows that 1/4=2/8=4/16. Should these identical circles represent alike cakes, the child eating 1/4 cake would eat the same quantity as children eating 2/8 or 4/16. The only difference is that one of them would get 1, the other 2 equal, but smaller pieces, while the third one 4 equal, but even smaller pieces of cakes.

Mark one third of each of the three line segments. Describe the received quantity in terms of the unit indicated at the end of the line segment. Compare the quantities.



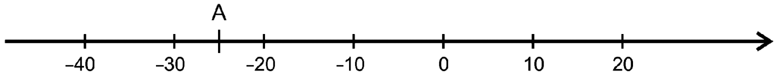
Solution: *By copying them to a transparent paper, with folding we can also see that 1/5 decimeter is exactly 2 cm (2/10 decimeter) and exactly 20 millimeter (20/100 decimeter), thus it is true that 1/5=2/10=20/100.*

Making a lot of similar tasks will give the basis to the understanding of the extension and simplification of fractions and will explain why changes during application are necessary (looking for common denominator).

The representation of wholes and fractions on the number line illustrates well the understanding of the numbers' relations with each other, their increasing and decreasing order.

Answering questions related to the number line deepens the understanding of the number concept and the concept of operations, too.

Answer the following questions.



Which number is smaller 20 or –40?
Which number belongs to point A on the number line?
What is the distance between –10 and 10?
Put the absolute value of the numbers 1,5; –17,8; 0; 65; –197 in increasing order.
Put the numbers –325; 3,25; 32,5; 0 and 0,325 in increasing order numbers.

The pupils should become capable of representing the learned numbers on the number line, to determine precisely or approximately the number belonging to a given point on the number line or to compare the numbers according to their size.

In addition to performing the verbal and written operations in the appropriate order with the correct results we also make efforts in the first two grades of the upper grades that the children learn methods, procedures making computations simpler, faster (for example, by using operational properties, parentheses). This also confirms the deepening of concepts, the increasing of awareness of operational algorithms.

By the end of grade 6 the pupils get acquainted with the basic operations in the set of rational numbers.

We only allow the use of pocket calculators during the lessons if the children possess the basic computation algorithms and are able to give adequately correct estimation of the final result. We generally do not allow the use of calculator in the paper-pencil tests. One of the main reasons for this is that by this we provide unequal technical conditions (plus the problem of the use of technical tools which look like a calculator but have much more sophisticated functions).

The pocket calculators with different "knowledge" serve the interests of our learners if these tools do not take over too early the steps, operational elements needed to the development of students' reasoning. The problem solution model is born in head, the calculator can be a tool of implementation. For example, when we teach how to solve equations, children are working in head and in writing, since we want to make them understand and to teach them the algorithm of solution. In the case of more difficult word problems the challenge is the setting up of the mathematical model and the calculator, or its equation solving program can perhaps be used if the model already exists. If for example we would like to check the correctness of an estimated or computed result by fast replacement, the use of calculator can also be justified. Knowing the actual conditions we can make a good decision about when and why we let the children use the calculators, computers. The use or the refusal of use should always be supported by rational pedagogical reasoning.

The application of highly developed information technological environment requires the improvement of good estimation skill. If for technical reasons the machines are not working, the good estimation skill gives a kind of security (for example, in the calculation of the amount to be paid /or to be claimed back).

New elements of understanding word problems

In grades 5-6 the continuously growing knowledge (operations covering the rational numbers, order of operations, knowledge of proportions (direct and inverse) and calculation of percentage) make possible the introduction of more complex word problems. The more demanding implementation of solutions (writing down, aesthetical aspects) is formulated as a requirement, it is realized that the rounding rules can be overwritten by real life (for example, if we need 56,3 m of a wire fence which can be bought in meters, we have to buy 57 meter, if based on the actual calculation of the surface we need 37,2 pcs of tiles to covering, we buy minimum 38 pcs and some additional), the estimation skill and the need for checking, self-checking is developing.

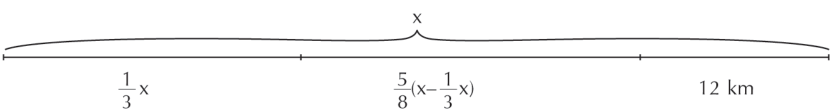
In these two grades the word problems mainly serve the development mathematical reasoning (for example, solution of simple first-order equations and inequalities by means of deductive reasoning processes), the improvement of proportional thinking (for example, conversion of standard units, direct and inverse proportions, simpler percentage calculation tasks), of problem solving skill (recognition of problem, identification of problem and solution) and the development of knowledgeable-analyzing reading.

During the development the consecutive steps of solving word problems are continuously recognized by the pupils (good understanding, interpretation of the text, clear separation of the conditions and the question, recognition of data (including the unnecessary data, too), recognition, stating, displaying, writing down of relations, links read from the text, preparation of solution plan(s), putting down the estimation of the result, calculation of the result (with written and verbal operations), its determination, checking, comparison with the estimated value and real life, preparation of an answer in words), the need for searching for different solutions is developing.

The pupil has to be able to solve simple equations with optionally selected method, to solve simpler word problems by deduction, proportional problems, to represent the solutions on number line. Of the solution methods mention should be made - besides deductions - of the methods using drawing, figures, segments, number line. In many cases these drawings, figures show if the learner understood the problem, the task. Some kind of actual representation of texts by drawings, figures can give a lot of information to the teacher about the current level of the slowly developing abstract reasoning of the pupil.

Edit and Dani went on an excursion. On the first day they made one third of the planned route, on the second day 5/8 of the remaining distance, thus they had to walk 12 km on the third day in order to get to the destination. How long was the route of the whole tour?

Solution in segments*: x marks the length of the whole tour.*



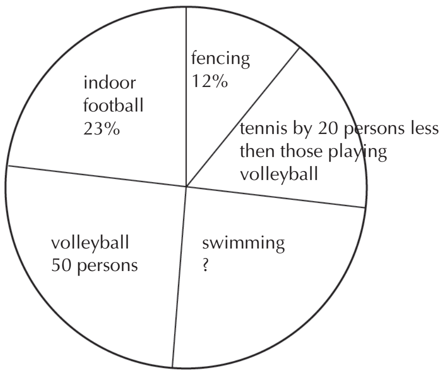
12 km is 3/8 part of the 2/3 of the total route
4 km is 1/8 part of the 2/3 of the total route
8 · 4 km = 32 km is 2/3 of the whole route
Length of the whole route: (16 + 32) = 48 km
Checking can be made by the calculation of the parts and by their summing.

Find a connection between the following quantities.
a) The price and the height of the Christmas tree
b) Travel time and speed of the car (let the route length be 20 kilometer)
c) Number of slices of a birthday cake and the size of the slices (we cut equal slices)
d) Quantity and price of green peas
e) Side and perimeter of a square
f) Price of the ice cream and number of balls
Solution: *Discovery, formulation of the correct correlations between quantities.*

Answers which can be expected from pupils can be for example:
a) In the case of the same type of Christmas tree we pay more for the taller tree, than for the shorter.
b) If a car goes twice as fast then it will take half the time to cover the 20 km.
c) The more equal slices I cut of the cake, the smaller the slices will be.
d) The price paid for the green peas changes in direct proportion with its quantity.
e) The lateral face and perimeter of the square change in direct proportion.
f) The number of ice cream balls and its price change proportionally.

48% of the monthly family income goes for the payment of different credits, invoices. In this month the family covers its living (food, clothing, repairs, entertainment, etc.) from the remaining 104 thousand Forint. How much is the family income in this month?
Solution: *The remaining money (100 – 48)% that is 104 thousand Forint is 52% of the monthly family income.*
1% of the family income is 2 thousand Forint, thus the total income is 100×2 thousand Forint, that is 200 thousand Forint.
Checking of the problem: 48% of 200 thousand Forint is 96 thousand Forint, this together with the 104 thousand Forint is exactly 200 thousand Forint.

200 sportswomen and sportsmen disclosed which their favourite sport wss. We will show this on the diagram below. What percentage of them selected swimming as the favourite sport?



Solution*:*	*100%*	*200*	*sportsmen*
	1%	*2*	*sportsmen*
	23%	*46*	*sportsmen (indoor football)*
	12%	*24*	*sportsmen (fencing)*
		50	*(volleyball players)*
		30	*(tennis players)*

Totally: 46+24+50+30=150 sportsmen
Swimming is the favourite sport of 200–150=50 sportsmen
50 is exactly the quarter of 200 that is 25%.
Swimming is the favourite sport of 25% of the interviewed sportsmen.
Checking can be made for example by adding the partial sums.

Requirements of constructing text for word problems

At the beginning of the upper grades the extended mathematical knowledge contributes to the description of mathematical models by symbols. In spite of this even at these grades there is still a need for reading texts, information, instruction, questions from activities, working with objects, pictures, figures, drawings. If the texts constructed by the students for the computation problems, open sentences are faulty, it is worth to show mathematical expression matching well to the problematic text and compare it with the initially given mathematical model. The presentation of differences, deviations helps the pupil to understand where he made a mistake. If somebody cannot (does not dare) to start the formulation of a text to a model, the teacher should begin it encouraging the learner to continue and finish the text. If this does not help the teacher should tell several simple adequate texts so that the learner understands clearly what his/her task is.

As a result of appropriate development, children become able to create more and more complex and better formulated texts to a given mathematical model. In general the texts relate to the applications within mathematics, to the everyday real life surrounding the children, but we should direct the attention to texts relating to the natural sciences, too. Models produced by using special correlations (formulas) taken from this field (for example, relations between route-time-speed, measurement data, use of graphs) give good basis for the implementation.

Nora had 1200 Forint. She spent 3/5 of it. Put questions to the text.

Solution: *a) How many did Nora spend?*
b) How much money was left to her?
c) What portion of the 1200 Forint was left?
d) What percentage of the money did she spend?
etc.

Create a text to the following computation problem.
2(300+100) = 800

Solution (for example): *I had 300 Forint saved, I received an additional 100 Forint from my grandpa. My father doubled my money for my birthday. How many Forints do I have?*

Create text to the following open sentence.
2(1kg + 3kg) = x kg

Solution: *Kati was sent to the shop twice by her mother and both time she had to buy 1 kg sugar and 3 kg of potatoes. How many kg of food did she take home after the two shopping trips?*

Create text to the following open sentence.
2 (30 + x) = 200

Solution: *One side of a land of rectangular shape is 30 m, its perimeter is 200 m. What is the size of the other side?*

Create word problem to the following relation.
a×b = 50, (a and b are positive whole numbers)

Solution: *The area of a rectangle is 50 units. What are the sizes of its sides?*

It is advisable to make the children calculate the length of the lateral faces, since there are several possible solutions here. 50 is divided into the product of two factors in all possible ways: 1×50; 2×25; 5×10. By interchanging the factors we do not get a solution different from the above, a new rectangular. Thus the lateral faces are 1 unit and 50 units long, or 2 units and 25 units long, or 5 units and 10 units long.

Relations, Functions

Relying on the solution of previously solved tasks on proportional reasoning, students learn the concept, definition of direct proportion. They will be able to recognize direct proportions in the practical problems, and also during learning science topics in the school. They can solve with certainty simple proportional problems of everyday life by means of deductive implications.

During the studying of relations between variables the learners gain experiences about the recognizing of inverse proportionality, about the determination of their matched value pairs.

The proportional implications improve the perception of correlations of the learners, their abilities for making conclusions. The learner will be able to recognize relations, correlations in simple examples. In the case of the simplest linear correlations which occurred often before children are able to add the missing elements, to present the data in tables. They have to meet with non-linear relations, too, what's more it is advisable to check the same thing from several points of view.

In this age phase of the development of inductive reasoning the learners are able to determine the missing elements, or in case of known elements to formulate the rule. They can continue a sequence according to a given rule and to induce a rule from some elements. They can also describe the recognized rule by a formula.

In this school phase students' location determination skill is improving. They are able to find points according to the given properties on a number line, to represent number intervals, to demonstrate data described by terms like smaller, bigger, at least, maximum, or to read from a figure. They know the Cartesian coordinate system and the related terms (axes, origin, index, coordinates, and quadrant). They can represent given points in the coordinate system and read coordinates of points.

They are able to prepare diagrams to relations given in tables and to give the table elements on the basis of the diagram. They recognize the linear function and can represent it on the basis of its points. They can recognize, write down, and represent relations in the simple examples taken from everyday life.

They can solve simple percentage calculation problems using direct proportionality, proportional deduction (for example, shopping, savings, agenda). In the course of practicing these tasks in parallel with the discovery and use of the necessary algorithms they learn the basic terms of percentage calculation: basis, interest rate.

Initially the problems formulated by mathematical symbols can be used for the presentation of the acquired knowledge, skills, and abilities. Through them the mathematical structure of the problem is transmitted without any "disturbing factor", in most cases we refer to the operations, algorithms which should be used during solution, and in many cases mathematical symbols can be found in the text of the problem.

Calculate 15% of 120.

Prepare a number line with corresponding scale. Indicate numbers with the following properties. $-3 \leq x < 9$ *and x whole number*

Indicate in the coordinate system the points A(–2;1), B(3;1), C(4;3) and D(–1;3). Connect them in alphabetical order. What is the name of the produced plane figure?

Draw points in the coordinate system the second index number of which is bigger than the first.

What is the connection between the data of the following table?

Time passed (hour)	*1*	*2*	*3*	*4*
Route travelled (km)	*4*	*8*	*12*	*16*

Find a rule to the data of the following table. Based on the rule add the missing data.

x	*8*	*4*	*2*		*0*
y	*4*	*8*		*1*	

The last three tasks is an example that learners can be asked to solve problems on this very simple level of application, where several correct answers, solutions can be given. By giving these types of tasks we can prepare the studying of more complex, problem-type, authentic tasks. Certainly, this aspect can only appear in the course of teaching, during assessment one has to refer to the possibility of several solutions.

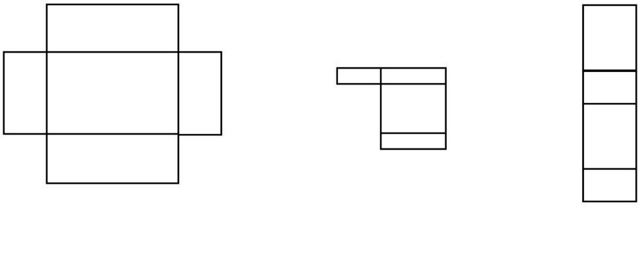
Geometry

In addition to the two abilities (spatial and proportional) playing a role in the former grades, due to the concept enrichment in grades 5-6 it is possible to create several different tasks to the geometrical contents which allow the diagnosis of the development level of inductive, deductive and systematizing abilities.

Typical problem for the testing of spatial skills:

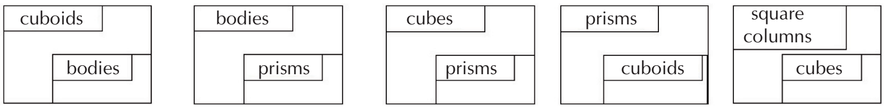
Add the following figures so that all of them be a net of a cuboid.

a) *b)* *c)*



Example of a problem which evaluates the systematizing skill on a geometrical content:

Are the nominations written in the figures below on the right place? **Circle** the letter mark where they are on the correct place and **~~cross out~~** where they are not.



a) *b)* *c)* *d)* *e)*

Finally, we present an example, where several different mathematical abilities can be used during the solitions, thus for example the elements of deductive and combinatorial abilities.

The rounded value of the volume of three same size bottles is 2 liters. The value of the volume of one bottle given in dl is whole number. **Answer** the following questions.

a) At most how many deciliter could the total volume of the three bottles be?
b) At least how many deciliter could the total volume of the three bottles be?
c) At most how many deciliters could the colume of one bottle be?
d) At least how many deciliter could the volume of one bottle be?
e) **Give** all possible volumes of a bottle in dl.

Combinatorics, Probability Calculation, Statistics

In the field of combinatorics, probability calculation and statistics the development of basic skills and the deepening of content knowledge of the subject are relevant objectives in this age group, too. In addition to the possibility of developing combinative and correlative abilities embedded in the content there will be an opportunity for the mathematically correct foundation of data handling and data presentation and of the probability event based on theory of sets. In the system of mathematical reasoning the ability for correlative reasoning can be interpreted as a form of multiplicative reasoning. Here the recognition of relations between data sequences and the formulation of the problem is the task where the correlation is not only not linear, but in general cannot be described by a simple formula (even so in many cases the relation is not deterministic). In the world of mathematical phenomena the development and assessment fields of correlative reasoning belong to the world of the statistical phenomena. The formulation of correlative relations like for example, „The more vertices a polygon has, the more diagonals it has" or „the third power of bigger numbers is also bigger" can be regarded less valuable. Thus the correlative reasoning can primarily be improved by experiencing statistical phenomena.

Diagnostic Assessment of the Application of Mathematical Knowledge

Detailed Assessment Frameworks of Grades 1-2

Numbers, Operations, Algebra

In the lower school age groups the word problems have dual functions. On the one hand they are used for mastering arithmetic operations, on the other they develop the problem-solving skills. In both cases it is typical that the text emulates the experiences of everyday life and the cases of children's world of fantasy making possible for the children to imagine or to model the story. At the beginning we cannot expect in grades 1-2 the conscious use of the solution steps of word problems, the teacher's help is needed by giving honts, formulating simple questions.

In the early phase the word problems describe activities, stories the playing or imitations of which lead to the solution. The problems became realistic when the everyday observations, visual and other images stored in the memory get an active role in the solution of the problem and the learner creates a mathematical model by using them during the solution of the tasks.

Look at the picture below carefully and tell a short tale, story about it. Also make up number problems about the picture.

The guideline to the solution of these types of problems usually contains the identification of mathematical terms and symbols, nevertheless the correct model creation reconcilable with real experiences will be decisive.

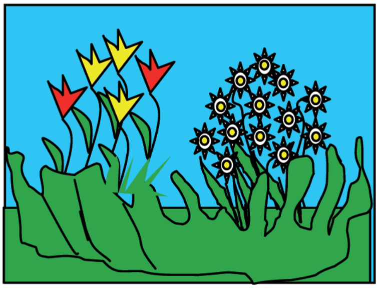
It is clear that the same problem can be a routine word problem in upper grades and can be regarded a realistic problem in lower grades. Most probably the following example belongs to the realistic category for the majority of learners of grades 1-2, while it is a simple routine task for the learners of upper grades.

Every child gets three plums after lunch. How many plums will be put on the table if 6 children are having lunch?

Six pupils in the class play the children sitting at the dinner table. Every child gets 3 plums. Children will determine how many plums they have got together.

It is easier to interpret a text if it is about a specific picture or situation. The text formulated about a picture can be an example of the inverse direction activities, where the task of the child is to make a picture to the text. The problems can be made realistic by making the children describe – in connection with pictures – their experiences, create questions which can be answered on the basis of the picture.

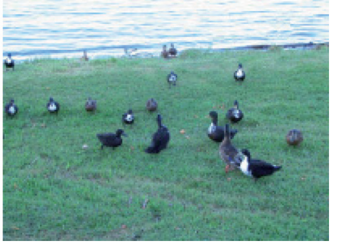
For example



In the garden tulips and white daffodils are flowering. How many tulips are flowering if 2 tulips are red and 3 tulips are yellow?
How many more daffodils are there than tulips?

It is good if the translation of the word problems into number problems or open sentences is preceded by the representation of picture pairs showing the changes well. The reading about the picture pair, the connection of the text and the picture pair shows the recognition of the relationship between the given and the missing data. Picture pairs recalling real situations make possible the creation of real problems.

For example:
Describel what happened between the two shots if the photos were taken in the order shown.

What happened in the reverse order?


Word problems given by telling a story become realistic for the children, if they can be represented by manipulation with objects or by drawing. At first the tools and drawings are realistic, they show what the story is about. Later we can expect from the children the interpretation of simpler drawings, more abstract figures. This process at the same time show how an authentic task provoking activities turns into a routine word problem during the development.

Mother sewed 6 buttons on Évi's coat, 2 less than on Peti's coat. How many buttons were needed on the two coats combined?

Level 1: Putting real buttons on the drawing of two coats.
Level 2: Instead of buttons, putting of discs under the children's names.
Level 3: Drawing circles or dots corresponding to the number of buttons after the initial of the children's name.

Other examples of realistic problems building on the children's experiences:

All of us will put on gloves for the walk today. How many pairs do we have to prepare if 5 boys and 4 girls are going out?

Discussion of the terms contained in example (all, pair, 5, boy, 4, girl) contributes to the preparation of the mathematical model.

How many nights do we sleep from Monday morning till Sunday evening?

A lot of significantly different mental models can be prepared to this problem, including the mental number line, the drawing of calendar.

The children will be able to formulate questions and to create problems on the basis of examples of word problems interpreted and solved by activity.

Tomi has 15 toy cars. His younger brother, Dani has 7.
Ask questions.

Children can make several questions.
– *How many cars do the two children have altogether?*
– *How many more cars does Tomi have than Dani?*
– *How many more cars does Dani have to collect to have the same number of cars as Tomi?*
– *How many cars should Tomi give to Dani so that the brothers have the same number of cars?*

The above activities prepare the connection of word problems to mathematical models. First the expression by numbers, symbols and operations of the relations formulated in words is made by collective activity. The collective model creation can be followed by independent activity, where we expect the connection of the simple word problem to the number problem or to the open sentence.

For example:

Which open sentence matches the text? Connect the open sentence corresponding to the problem.

Marci went fishing to the lake.	$8 + 5 = \square$
He threw back 8 of the caught fish.	$8 - 5 = \square$
He returned home with 5 fish.	$\square - 8 = 5$
How many fish did Marci catch?	$\square - 5 = 8$
	$\square + 5 = 8$

Open sentences 1, 3 and 4 are all rational models of the word problem.

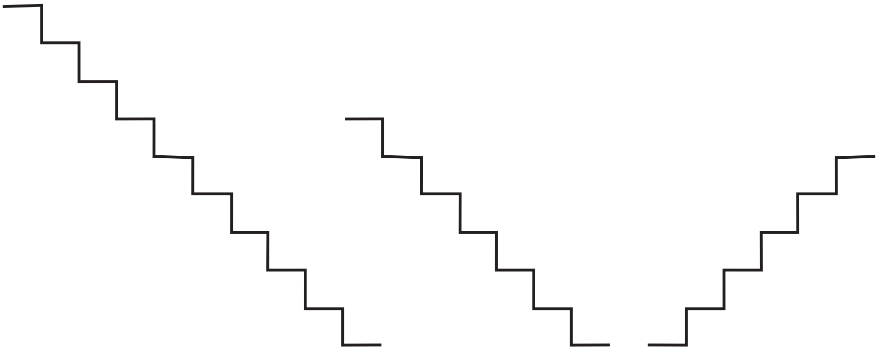
The mutual relations between the texts of examples and the determination of the operation needed to the solution are promoted by problems which require the pairing of text and number problem or open sentence and contain a mathematical model which does not fit to any of the word problems. In this case we can ask for making text to the number problem or open sentence. We can expect and require that the verbally formulated word problems contain real data, connect to the everyday life or real experiences of the children.

The above activities prepare the recognition of solution steps of the word problems. The appropriately gathered and written down data collected from the information of word problems formulated in colloquial language, the description of the relations between them or their representation by activities, the correct estimation of the answer to be given to the question indicate the mathematical model leading to the solution. The creation of the model is the most difficult step of the problem solution. The solution within the model is followed by connecting the solution to the original problem. The children, by comparing the found solution with the text data, with the preliminary estimation and reality evaluate the reality of the solution, too.

In the first years of schooling, children get acquainted with numbers in the course of real problem settings. They make observations, comparisons and measurements. They recognize the sensible properties of objects, persons, things, and select them based on their common and different characteristics. During their activities they gain experiences about the properties, relations of the numbers.

For example, they become able to find solution to the following problem by evoking their experiences about walking on steps:

Which staircase could you walk through in a way that you always skip one stair? Circle the number of stairs, which can be stepped on this way, and ~~cross~~ the number which not.



9 *6* *5*

The presentation of authentic problems creates real, lifelike problem situations for the learners. In the course of this they process problems about which they can have personal, real experiences. We can also present new situations which are regarded by the children authentic based on the stories heard from others. In many cases the problems – as in real life – have several possible solutions. The solution depends on the conditions influencing the event and on the conditions which are prevailing in the given situation. In early school age we cannot expect from the children the taking into account of all the conditions and the recognition of the possible situations. We will be satisfied with the presentation of a possible solution to the problem.

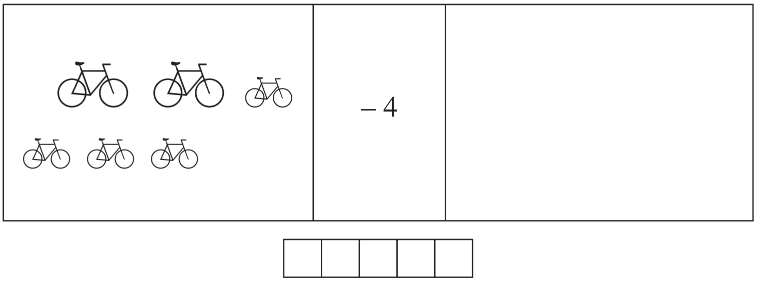
Marci and his younger sister Zsófi go to bed at 8 o'clock in the evening. They have to get up at 6 o'clock in the morning, since the school is far from their home. How many hours can the children sleep?

We can help the solution of the task by showing a clock which strikes every hour. Set the clock to 8 o'clock and the children should close their eyes. While time is passing (now speeded up) the clock is moving. Children can open their eyes when the clock shows 6 o'clock. The teacher makes a clang at even time intervals. During the game, by the speeding up the time, children experience in this new situation what happens to them every day. They see the example, how long lasting events can be played and made repeated several times. Based on their experiences they can state that the characters of the story can sleep maximum 10 hours.

Greater imagination is needed in cases when for the illustration of the problem we use objects which are not real, but still touchable, movable symbolic objects. It is important that these objects be first selected by the children or perhaps the teacher should offer different options. Since one of the main features of authenticity is that the simulation of a problem situation *realistic for the learner* can be made by the definition and solution of the problem.

The next step after making illustrations by objects can be the illustration of problems by pictures, drawings. At first we can connect word problems to photos of personal experiences. Based on the photos the children recall the real events, formulate their experiences, tell what they have lived through, and talk about their observations. Based on their memories they can supplement by data the story told by the teacher, or can put questions themselves. These conversations can contribute to their being able to make stories about photos on their own later.

For example:
Prepare the second picture. Describe what could happen. Describe it in arithmetical language.



This picture can recall the experiences of the children who regularly go for cycling with their parents, brothers and sisters. If they do not have bicycle their wording may express their wishes. They perhaps have seen cyclists on the streets or visited a bicycle shop. Their experiences collected from real life may have an effect on their stories.

For example, they can tell stories like: In a six member family everybody has a bike. On the week-end four of them went for a bicycle tour. How many bikes were left at home?

The problem solution can be made easier if it is really connected to the own experiences of the learner. If we complete the problem by a question which is about the learner, the problem becomes specific and realistic. After that the small child solves the problem about himself/herself, it is easier for he/she to image a situation related to other persons. The problem becomes in this way realistic, natural for the learner.

Marci collects toy cars, Évi collects plush toy figures. Neither of them collected 20 toys. How many toys do they have if Évi has 5 more plush figures than the number of Marci's cars?
How many cars do you have? How many plush toys? Which do you have more and by how many?

Begin the solution of this problem by the collection of data brought from home. Now the children experience how many different number pairs can be given as an answer to the question and perhaps there will be a child in the classroom who has by 5 more plush toys than cars. The number pairs collected in a table format show an example of the purposeful solution of the original problem.

In the next example we have selected the word problem not in order to experience the operational properties, but that the children could see during the problem solution the two types of computation options.

Do you consume 4 liters of milk in one week?

The children begin the solution of the problem by data collection. Every learner can know how many deciliters his/her own home cup in which he/she drinks milk, cacao or other milky liquid is. Here they can discuss how many things are made of milk and children can speak about what others usually eat for breakfast and supper. We can let the children decide about the way of counting. During the discussion it may become clear that from the daily milk consumption we can predict the weekly milk consumption, or we can add to the milk quantity consumed in the morning the quantity consumed in the evening. In this way the unit conversion is made necessary by a real-life problem.

The daily activity of children, their environment and the nature offer a lot of possibilities for the formulation of authentic word problems for small school children. They can collect data about their everyday activities (For example: When do they get up?, When do they go to bed?, Do they have extra classes?, How much sports do they do?...), they can sort the collected data, compare them, formulate questions and can change them. We can also put questions the answering of which requires data completion. The collection of the missing data can be left to the learners, but we can offer options, can make proposals for this.

Data which cannot be completed on the basis of observations, experiences or by measurements require creativity by the learners. The missing data can provoke estimation, or the solution of the problem according to the condition. At the beginning we can accept from the children a formulation like: "in my opinion…". Later they can find several solutions acceptable by them: "May be…, it can also be that…". Ideas collected in groups or frontally can give all possible solutions of the problem.

When doing independent work we can encourage the learners to look for more solutions, or by specifying one or more conditions we can ask them to determine the data specified by the condition.

16 people are sitting altogether at 3 eight-seat dinner tables in the dining room. How many people can have lunch at each table? Look for several possible solutions.

Table 1	8		2	6					
Table 2	6	8			4		0		
Table 3		2	6			7			

Relations, Functions

As in the case of other content areas of mathematics the criteria of a problem's being realistic in the case of relations and functions is also that the learner be able to imagine the content (mostly based on everyday experiences) of the problem.

The basic characteristics of the realistic problems are that they mainly promote the inductive and correlative reasoning in the scope of reasoning skills. Relations observed in the everyday life and working in the fantasy world are created on the basis of finite number of cases, then the produced rule or relation will be valid for the infinite wide circle of the world of phenomena. Compared to the authentic problems the difference is that the problem directs the search for relations and rules and we do not expect that the learner initiate it.

In the realistic problems related to sequences the formal characteristics of the task remain, but the content will be modified that reasoning in the horizontal mathematization starts from the real experiences and from the internal cognitions and the learner tries to find mathematical model to them. In the case of sequences for example the following problems can be regarded realistic by the majority of learners:

Continue the sequence with two members. What can the rule be?

(A) Monday Wednesday Friday Sunday Tuesday ___ ___

(B) January 1 March 3 May 5 July 7 ___ ___

(C) Anna Ágnes Beáta Antal Ábel Barnabás Anita Ágota Bernadett Attila ___ ___

Another field of this topic can be found in the relations between data pairs, that the building up of mental mathematical models is possible by the transformation of the content of the problem with keeping the problem format unchanged. The solution of the following problems requires from the learner to imagine the things contained in them and to construct a mathematical model which can be used in the case of the specific problem. In the case of describing relations between relatives drawing a family tree or any type of tree diagram can make a mathematical model. The visual images of the habitations of animals can be used in the solution by the formulation in words of the analogical relation.

Fill in the chart below.

Father	Younger brother	Great grandpa	Grandpa	
Mother	Younger sister	Great grandma		aunt

Bird	Dog	Man	Squirrel	
Nest	Doghouse	House		stable

The most important general characteristic of the authentic problems is that a kind of problem situation is realized which is connected to the learner's activity and where the learner can act as an active participant. In many cases a kind of "reverse problem setting" can take place, which means that the main point is that in a given problem space the learner has to create the problem himself, or should analyze in what conditions a problem in mathematical sense can be created.

In the case of sequences the basic principle can be that the children recognize patterns, regularities in a given problem space (definition system) and formulate the relations. They should look for examples and counter examples. In this way the authentic problems of relation and functions in addition to the inductive and correlative reasoning are excellent means of development of systematization skill.

In authentic problem situations children with special educational needs should be conducted with more explicit instructions, since without this the contexts and frequent intransparency of the problem make for them focusing on the mathematical characteristics of effects difficult.

In the case of sequences we encourage the learners through authentic problems to search for sequences themselves based on a certain criteria in a well-defined problem space. In the following example the name of the learners define a problem space.

Write on the blackboard the various given names in the class. How can they be sorted? Write down the sorted names.

The solutions can be much diversified. The alphabetical order seems evident, but the length of the name can also be a criteria, or such a refined idea can be used as the sorting of learners' name on the basis of their birth dates. It may happen in every case that the sequence of names will not be strictly monotonous. In this case it is advisable to express the parity relation by the writing of the names one under the other in case of names otherwise sorted in monotonous order.

How could you put the 12 months into order? Find out as many orders as you can.

In the case of the data pairs it is also a possible solution that we draft a two-dimensional data population and the primary task of the learners is to find some criteria based on which the things belong together. It is important to select a basic problem which is natural and relevant for the learners. Such problem areas are for example: school timetable, definitions in connection with meals, relations between relatives, dressing, holidays.

What rule did we use to start to fill in the chart below? Continue according to the rule.

mathematics	Reading	Singing		sports
4	4	2	2	

It is possible that the weekly number of classes is indicated in the table, but may be it is somebody's marks, or how much he/she likes the subjects.

Geometry

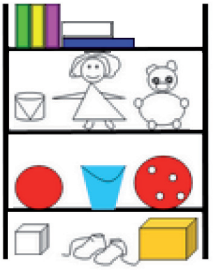
The field of geometry – due to its characteristics – is excellent for the mathematical modelling of things known from everyday life. Geometry is mainly dealing with mathematical characteristics of shapes which can be represented visually, thus it is extremely good for the connection of the visual ideas and mathematical system of definitions. Of the four sub-domains of geometry we first deal with orientation, accentuating by this how many evident possibilities this area offers for the use of realistic texts.

Orientation

It is the purpose of the first grade to lay down the basis of spatial and plane orientation skills using sense perceptions, following directions, changes of directions by movement, comprehension and use of terms referring to determination of places (for example, below, above, next to, between, right, left). In the second grade the formulation of own movements, following routes in reality and on model table, its realization, description of a travelled route, getting to given places, going through given routes in reverse order, impact of change of direction are required. Compared to the expectations of first grade the searching for places characterized by two plane data (direction, distance, vicinity) makes the problems much more difficult.

The shelves shown on the picture are in the Nóri's room. She told us what she had put on the shelves. Write in the missing words.

The shoes are .... the bucket.
The small bucket is .... the two balls.
The bigger box is .... the ball with dots.
The teddy bear is on the .... side of the doll.
The drum is at the .... the hand of the doll.
On the shelf .... the doll there are books.



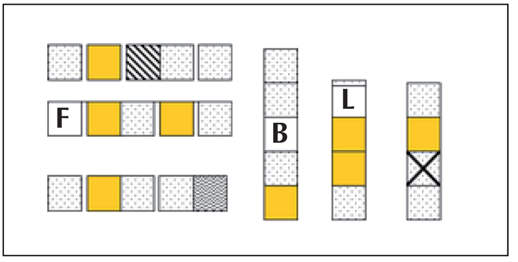
Going-over the route by following the route written by words and by passing by given points.

The drawing is part of a city map. X marks the starting point. Indicate the route.

Look for the house where grandma lives.

- *You leave the house marked by X.*
- *You first go to the Library (L).*
- *After this you go to the Bakery's (B) street along the shorter route.*
- *At the Bakery you buy 5 croissants.*
- *Leaving the Bakery you turn to the right and walk till the end of the street.*
- *You walk around the house whose roof is monochrome.*
- *You turn into the street where there is a house with a wavy pattern on its roof at the corner.*
- *You go to the Flower(F) shop and buy a bouquet of tulips.*
- *Walking around the house of the flower shop you are in the street where grandma lives.*

- *Grandma's house is next to the house with striped roof. But its roof is not monochrome.*



Examples where the task is the comprehension of verbal or written information, the following of directions and changes of directions reflect real situations and cases which are relevant to the learners. The solution can be manipulative or image level. In the case of paper-and-pencil and computer tests obviously the visual problem formulation is an option.

We have hidden a treasure box in the class-room. You will find it, just follow the instructions.

- *Start from the door of the class-room.*
- *Stand opposite to the window.*
- *Take 3 steps ahead.*
- *Turn to the left.*
- *Take 2 steps.*
- *Turn to the right.*
- *Take 2 steps ahead.*
- *The treasure box is at your left leg.*

Work in pairs. Tell your mate the route which leads from your home to the school. Make a draft map. Draw some known places on the map. Your mate should mark the route told by you on the map. Check his/her work.

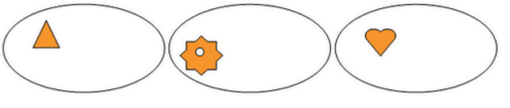
Constructions

In order to develop the abilities required for observing, the comparison of shapes (identification, differentiation, recognition of the shape on the basis of the total view and of one-one accentuated geometrical property) begins in grade 1 and continues in grade 2; also continues the recognition of the part and whole, expression of observations by selection, their formulation by own words, the continuation of the started selection on the basis of the interpretation of properties expressed by words and interpretation of relations. Learners become able to interpret the properties, relations expressed by words. The separation of plane and spatial forms on the basis of their characteristics and their categorization on manipulative and image level, followed by explanations are requirements.

The learners are able to build spatial geometrical objects on the basis of a model. They are able to produce shapes by activities: from mosaic, paper folding, threading of straws, free hand drawing, and later in the second grade by folding right angles, rectangles, squares of paper, copying onto transparent paper, drawing on square sheet, on other nets. Here the creation of forms on the basis of a specified simple condition, as well as the collection, identification, differentiation of works (recognition, naming of some characteristics of polygons: vertexes, number of lateral faces, equality of lateral faces, convexity) are already requirements. All these activities are able to improve creativity, systematization and combinatorial skills. The creation of spatial constructions and plane works with given specificities, and the checking of characteristics contribute to the development of making deductive and inductive inferences.

Example for the sorting, categorization by activities of plane forms on the basis of the observed geometrical characteristics:

Cakes are put on plates in the cake-shop. They began to sort them as shown on the picture.

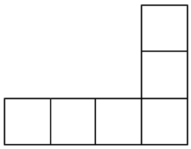


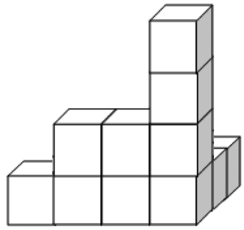
Where will the rest of the cakes be put?
Draw the cakes on the plate where they belong.

Recognition of sensible characteristics of bodies, selection on the basis of identities and differences:

Recognition of bodies on the basis of picture, making of floor plan:

Lali made a house of small white cubes.
Write in the floor plan how he built.
How many small cubes did he use to the house?



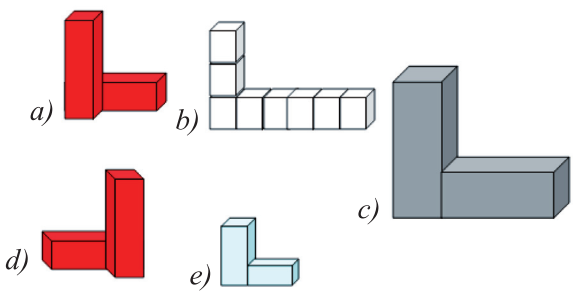


Transformations

The collection of experiences by flat mirrors, the discovery of the symmetry of plane shapes and spatial objects begin already in preschool age, then as a continuation in grades 1-2 the production of mirror shapes and simple mirror image by motion, display, cutting, using copy paper, rotation, or reflection around the axis and by using plane mirror are requirements. Here again the observation (identification, differentiation) comes into the foreground. The monitoring and reformulation of the transformation procedure is important.

It is required to distinguish between the mirror image and the shifted image on the basis of the general view.

Miklós made these houses of identical cubes. Look at the picture. Answer the questions. After the questions write the letter symbol of the house.
Which house is the highest?........
To which house did he use the most building blocks? …….
House A is the mirror image of which house?........
Which two houses are of the same shape?



Distinguishing of the mirror image and the shifted image on the basis of the general view.

Emma has received a new pullover. She liked it very much.

She put it on and went for a walk. She looked at herself in every shop window and in every puddle.

Of the pictures in the second row which one could Emma see in the glass of the shop window?

We suggest using the following example as a task based on practical, playful activity:

Build a house which has a door using the building blocks.

Build the mirror image of each house as well.

You can use a mirror as help.

Measurement

Measurements appear in grades 1-2 connected to the development of number concept. In this connection the main role is given to abilities enabling for comparison and distinction, to the observation, recognition, ordering of correlations: in grade 1 the requirement is the comparison, comparative measurement of different quantities, and the solution of practical problems. After this in grade 2 getting acquainted with standard units (m, dm, cm, kg, dkg; l, dl, hour, minute, day, week, month, year) and the use of their names and symbols belong to the requirements.

Students should observe the relations between quantities, units and measurement index numbers. They use their measurement experiences in estimations and formulate them by their own words.

Father and mother bought a new carpet into the living room. Father and little Gabi walked through the nice, soft carpet hand in hand. Who do you think made more steps, Father or Gabi?

Individual and group project work based on the active, conscious learner's activity give excellent opportunity for the geometrical implementation of authentic problems. In one of the groups of the authentic measurement tasks the learners have to give estimation in situations which are relevant to them. Also measurements made with occasional units, the use of standard units of measurements also belong here - supposing that the problem is not only realistic for the learners, but also relevant.

Estimate how many steps are needed along the length and the width of your classroom.

Choose the shortest child of your class. He/she should measure the width of the room by his/her steps. Measure the length of the room by the steps of your teacher.

What did you find?

Measure the width and the length of the room by a meter rod.

What did you receive? Explain the measurement results.

The following description shows the possibility of a project task:

In your surroundings, search for the symmetrical decorating elements (clothes, furniture, Easter eggs, toys, buildings, trees, flowers, butterflies, churches, pattern decorating eaves, etc.) observe them carefully, analyze the details, record them on drawings, photos, write their histories. Present the results of your research by lecture, exhibition (for example, on posters), by building them (for example, of plasticine (modelling clay), building blocks, gypsum), by making video, etc. The presentation can be made individually or in groups.

Combinatorics, Probability Calculation, Statistics

The foundation of combinatorial reasoning and probability approach is in most cases implemented by games or playful experiments in the school. The child can build into the solution of the problem his/her experiences gained during the game.

For example they play with three red-blue discs[1]. Before every throw they have to guess if the statement will be true or false. The person who makes more correct tips will win.

a) There will be at least two red.

Guess										
Throw										

No. of good guesses: …………

b) There will be at least two blue.

Guess										
Throw										

No. of good guesses: …………

c) There will be at least two of the same colour.

Guess										
Throw										

No. of good guesses: …………

d) Both colours will occur.

Guess										
Throw										

No. of good guesses: …………

1 Such disks have a diameter of about 2 cm, and one of their side is red, and the other side is blue.

In this activity the most important thing for the child is to win the game, therefore he/she tries to use his/her former experiences during the game. From the changing of the tips the teacher can see how the probability approach is improving. For example the fact that there will be at least two disks with the same colour is a sure event. This however will be evident only after making some throws.

During the experimentation we would like to know to what extent the experiences collected by the activities build into the children's thinking.

Therefore a possible version of the above activity formulated by measurement can be the following:

We throw with three discs. Put an X on the right place.

	Sure	Impossible	Probable	Possible
There will be at least wo red ones				
There will be at least two blue ones				
There will be at least two of the same colour				
Both colours will occur				
There will be more red ones than blue ones				
There will be the same number of red ones as blue ones				

In the lower grades during the formation of combinative reasoning and of probability approach we can mention problems which belong not only to this subject. It would be misleading to think that when the primary aim of the school lesson is the improvement of probabilistic reasoning, we only throw dices, rattle coins or pick colour balls out of bags during the whole class. The development of probability approach can be realized in the class-room also by the raising of problems which have relevance to also other fields of mathematics.

Children have to poke on a 0-99 number table blindfolded. Before the starting of the game they have to make a tip if the number can be written as the product of two numbers smaller than 10. (Number 1 is excluded here.)

This game can be played for example when they have to practice the multiplication table. Since they have learned the multiplication tables for a long time earlier – 100 cases separately – there is a great chance to think that there are more numbers in the table which can be found in the little multiplication table than which cannot.

In order to take into account which numbers can be written as the product of two numbers smaller than 10, they cover for example by self-adhesive tapes the fields marked by yellow.

0	1	2	3	2 · 2	5	2 · 3	7	2 · 4	3 · 3
2 · 5	11	6 · 2	13	7 · 2	3 · 5	4 · 4	17	3 · 6	19
10 · 2	3 · 7	22	23	3 · 8	5 · 5	26	3 · 9	4 · 7	29
10 · 3	31	4 · 8	33	34	7 · 5	6 · 6	37	38	39
4 · 10	41	6 · 7	43	44	9 · 5	46	47	6 · 8	7 · 7
5 · 10	51	52	53	9 · 6	55	7 · 8	57	58	59
6 · 10	61	62	9 · 7	8 · 8	65	66	67	68	69
7 · 10	71	9 · 8	73	74	75	76	77	78	79
8 · 10	9 · 9	82	83	84	85	86	87	88	89
9 · 10	91	92	93	94	95	96	97	98	99

It can be surprising how few cases can be found in the little multiplication table, therefore there is not much chance that we poke the desired number (we poke numbers which can be written as a product where both numbers are bigger than 1 in only 36 cases of 100).

In this game children can gain experiences about the commutativity of multiplication and can look for numbers which can be written down in many different ways as a product. The observation that what happen more often in more different ways is more probable can change their probability approach.

They repeat the same activity later, but this time they are looking for numbers which can be written as a product. (For example 33=11 · 3) In this way there is a much greater chance that we poke a number which can be written as a product. A game like this offers the first opportunity for the children to gain experiences about prime numbers. It is not the teacher who mentions the problem, but there is a strong urge in the child to follow up the problem. The systematic search for complex numbers can be the basis of the procedures aiming at the screening of prime numbers (for example Sieve of Eratosthenes).

In the school the children poked blindfolded at a table with numbers ordered from 0 to 99. The winner was who poked a number which could be written as the product of two numbers smaller than 10, but bigger than 1. Colour in the winning fields.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

What do you think is the more likely outcome of a game? Underline the correct answer.

Winning is more likely *Loosing is more likely*

Reasons for my answer: ....
....
....

Based on the reasons we can have a view about the development level of the child's probability approach. Based on the answer it becomes clear whether the child feels the fact that what can happen in different ways is more probable.

Detailed Assessment Frameworks of Grades 3-4

Numbers, Operations, Algebra

The practical use of mathematical knowledge about numbers, operations and in algebra is an extremely important field of the mathematical reasoning. It is also the task of teaching mathematics to show the indispensable role of the subject in other disciplines and in the everyday life. The examples taken from other subjects and from the practical life prove the usefulness of mathematics for the children. We can provoke the children's interest and curiosity towards mathematics by diversified problem settings. Therefore the selection of the subjects of problems requires discretion. It is not only the mathematical model which determines the difficulty of the problem solution. The same mathematical problem can be difficult to varying degrees for the children if we present them in different context. Therefore in the analysis of the problem solution we also take into account what is difficult for the learner. The selected model, the drawing made by the learner can give information about the comprehension, about the recognition or misunderstanding of the relation formulated by the text. By making a proposal or giving an instruction for the use of a certain model we can promote or just the opposite we can make difficult the solution of the problems. In this case we want to check not only the understanding of the problem, but also the problem solution with the selected model. We can expect from the learner the successful problem solution with the use of an optional or given model if we paid enough attention to this with the diversified solution of problems, with their comparison, with the discussion of the advantages and disadvantages of the selected solution method.

For example:
In the flower shop one daffodil costs 60 Ft, one tulip costs 80 Ft. We bought the same quantity of the two sorts and paid 420 Ft. How many flowers did we buy of the two sorts?

Visual representations promote understanding, the revealing of relations and correlations which are indispensable parts of problem solution. Therefore the improvement of the children's model making ability is very important. Different models can help the recognition of the contexts, the tools can be for example, demonstrating by means of manipulating objects, drawings, open sentences, tables, representation by segments, number line.

The solution of the first task by using tokens is evident. For example children draw a daffodil and a tulip and put the corresponding sums on the drawings. They do this until they get to 420 Ft.

Children having better abstraction skill can solve the problem with the help of a table. For example they can make a table like this:

Number of tulips and daffodils	1-1	2-2	3-3
Price of daffodils	60 Ft	120 Ft	180 Ft
Price of tulips	80 Ft	160 Ft	240 Ft
Amount to be paid	140 Ft	280 Ft	420 Ft

In this solution from two known data we arrived at the amount specified by the problem with systematic trying, evenly increasing the amount to be paid. In the meantime we calculated data which are not necessary to answering the original problem. In the previous table the evenly increasing sequences can be recognized, the new data obtained give new information.

For example:

- The value in row 2 and column 6 of the above table is an answer for what?
- What can we learn from the data in column 8 of the last row?
- What does the sum of numbers is row 2 column 3 and in row 3 column 2 mean?

…

The children can also choose open sentences to the solution of the original problem.

They can say: 1 daffodil and 1 tulip costs together $60 + 80 = 140$ Forint. We do not know how many pieces we will buy, therefore we shall mark this by: □

We pay as many times 140 Ft as many tulips and daffodils we ask and this costs 420 Forint. We can describe this with the following operation: $140 \cdot \square = 420$

The solution of the open sentence can be looked for by estimation, by trying estimation then by its correction, for example in the following steps:

The value of 140 rounded to hundreds is 100, and of 420 it is 400. We have to take 100 4 times so that we get 400. Testing shows that $140 \cdot 4 > 420$, therefore we have to try with a number smaller than 4. Trying number 3 we find that the equality is true $140 \cdot 3 = 420$.

In the solution we intentionally wanted to answer the question. We did not get other information, we cannot formulate new questions which can be answered easily. Every new question requires new open sentence and its solution.

In a housing estate there are identical ten-story houses. In every house, i.e. high-rise apartment block, there are 6 apartments to the left and 8 apartments to the right of the staircase on every floor above the ground. On the ground floor, there are shops. In this housing estate there are 420 apartments in total. How many houses are there in the housing estate?

The problem can be well solved by making a drawing, by creating a tasks consisting of numerals only, or by an open sentence. Certainly, we expect simplified drawing from the children by indicated the most necessary data. For example:

They can think in different ways. For example: In this house there are totally 140 apartments of the 420, on the left side of the staircase 60, and on the right side 80. The rest of the apartments (420 – 140 = 280) are in the other houses. There are also 140 apartments in the second house, the remaining apartments can be found in the third house: 280 – 140 = 140.

In this solution we moved from the known data towards the solution. In the different steps we got answer to the questions how many apartments were in the housing estate, if by 1 or 2 fewer houses were built.

Also starting from the total number of apartments children arrive at the solution through the following steps: if every house has 10 stories, and there are the same number of apartments on every level, on one level there is one tenth of that amount, that is: 420/10=42 apartments. In each house there is 6+8=14 apartments on a level, therefore the number of houses is equal to the division of 42 by 14. The result of operation 42:14=3 means the number of houses. In this case we arrived at the solution through two number problems and we obtained one plus information, namely that there are 42 apartments per levels in the housing estate.

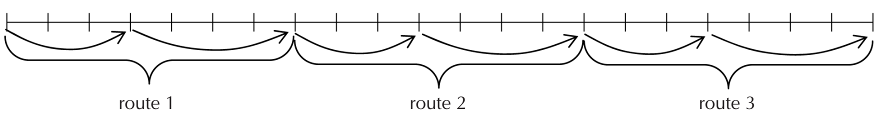
Children can use the open sentence method to the solution of this problem, too: in a house there are 6+8 apartments on a level, on the ten levels there are 10 times more, that is (6+8)·10=140 apartments. Mark the number of houses by □. □ houses multiplied by □ times 140, that is there are 140·□ = 420 apartments. The finding of the solution can be made in the same way as described in the previous problem.

A bus driver travels between two cities. He covers the distance from city X to city Y in 60 minutes, and gets from Y to X in 80 minutes. How many turns did the bus driver take on the day when he drove 7 hours?

The solution of the previous problem is followed by the discussion why the travel time is longer into one direction than into the other. Children can find the answer for the question based on their experiences. For example:

- Bus goes on a longer route from Y to X.
- There are a lot of slopes when bus goes from Y to X.
- In one direction the bus works as express service, in the other it stops on many places.
- In one direction it goes on motorway in the other on highway.

The story can be illustrated by a time line where the 7 hours are divided into for example, 20 minutes intervals. Children can indicate the time passed on the line. For example:



This figure can also give new information. Children can put and answer questions themselves. For example we can get questions like:

- Where was the bus driver after 200 minutes driving?
- Where was the bus driver when he said: “I have already driven 3 hours.”
- How many times had he already driven when he departed from Y city to X city?
- During the day when had the bus driver the chance to have a rest?
  …

The above figure shows well the mathematical models which can be the aids to the solution of the problem with real content. A sequence of changing difference can be red about the arrows: 60, 140, 200, 280, 340, 420…

Braces include two arrows and illustrate the mathematical content of problems like one arrow instead of two. Based on this we can read the members of an evenly increasing sequence: 140, 280, 420…

These numbers get meaning by the fact that we relate them to the problem and we say which number informs us about what.

The time line shows continuity well, with the help of it we can have an approximate view about the staying place of the bus driver.

Cities A and B are 420 kms apart from each other. Two cars depart from each city towards the other one at the same time. The car starting from city A makes 60 km per hour, the one starting from B makes 80 km per hour. When and where shall they meet?

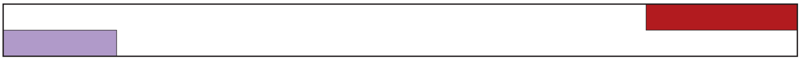
The problem can be well solved by a 42 cm paper strip and by the lilac and claret rods of the Cuisenaire-type[2] rod set.

Children can represent the routes travelled per hours with the help of these tools.

What information can be read from the display?

Children follow the way of cars in head.

They imagine what route did one car made during 1 hour and where did it get and see well what distance is left between the two cars. They can find intelligent explanation for both number problems of 420 – 60 – 80 or 420 – (60 + 80).



They can also easily read from the picture how long distance is left for each car from the total route. They can even find answer for question like where were the cars half an hour before.

2 The colors assigned to the rods are different in Hungary from the original Cuisenaire standard. Here lilac refers to the 6, claret refers to 8.

If they travel the whole route by the cars they can again get a lot of information.

On the one hand they can see how the cars get farther from each other after the meeting. It can be followed well that the car leaving city A makes the whole route in 7 hours, while the car starting from B needs only somewhat more than 5 hours. The smarter children can even calculate during exactly how many hours the car gets from B to A.

The last two problems contain quantitative data, but their solution is more difficult for the children. Therefore in grade 3 we spend more time on the discussion of word problems about motions, as a model of which we can use illustration by segments, in addition to colour rods and paper strips.

It is clear from the above-presented solutions that in addition to the solutions other information can also be read from the displays and illustrations the great advantage of which is that they strengthen the detection of the relations between mathematics and real life.

For learners of grades 3-4 we often set problems which they can often meet in their everyday real life situations. To the solution of these problems they need to apply the acquired mathematical knowledge, to mobilize their various skills. Children have the feeling that the everyday problems telling real stories are close to them, since they can have the impression that the moments of their own life, their activities are livened. These stories make possible that the children live through these situations with empathy and they obtain knowledge which can be used and easily activated in the everyday life. The methods of differentiated problem settings offer possibilities for active learning, for the discovery of relations and make the learners to activate their reasoning.

The real problems taken from the children's life and environment help them to recognize the model role of mathematics in the solution of the problems raised by real life or by different disciplines. Activities requiring the measurement of quantities and problems about shopping contribute to this.

Children can also often get the task to go for shopping. Many problems can belong to this activity. The problems may be connected to the payment of the purchased goods, to the delivery of goods and the estimation of the weight of several goods

During a big shopping trip we have put a lot of things into the shopping trolley.

Items:		*and their prices:*
1 carton of milk in paper box	1liter	93 Ft
One and half kilogram of meat	1 kg	768 Ft
4 boxes of eggs	1 box	12 Ft
400 grams of cheese	1 kg	720 Ft
2 boxes of 250 gram – chestnut purée	1 box	174 Ft
3 dl of cream		105 Ft
3 kg of washing powder (detergent)		1300 Ft
4 kg of apples	1 kg	150 Ft
2 kg of mandarins	1 kg	280 Ft

a) Going to the cash-desk, we are pondering whether the 8000 Ft we have in cash will be enough or we should pay by card. What do you think?
b) We have put everything into two bags except for the milk, the washing powder, the apples and the eggs. How could we distribute the items into two bags that if their weight is nearly the same? What would you put into the first and what into the second bag?

The solution of the problem improves various abilities. First, there is a need for the estimation about real-life data and also for the measurement of quantities (for example, how heavy one box of eggs is?). Some data are missing or are unknown to the children, these data have to be added. For example: How many liters of milk are there in one carton? How many eggs are there in a box? When adding the data the children see that the solution of the problem is not clear, since we can get eggs in many different packages. Thus the amount to be paid depends on how many eggs we bought. This however does not influence the weight of the bags, since we do not put eggs to the bag.

In the everyday life we often get into decision-making situations. In general there are different options for the solution of a problem and it depends on our choice how we solve the problem. Our choice can be influenced by a lot of factors, the solution depends on different conditions. Therefore we need to bring the children in situations on the mathematical lessons where they have to think over the possible conditions and in case of meeting several conditions, they will select the most realistic solution.

Hopefully reading is part of the children's everyday life. In addition to the discussion of the reading experiences it is also a possibility that they find ideas for the solution of the technical problems. This can be the sorting of books, their placing on a given shelf, borrowing from the library, or the scheduling in time of the reading of a book.

For example:

Andris likes to read very much. He reads every evening one hour before sleeping. One of his favourite books is Kele from István Fekete. He borrowed it for the third time from the library but has to give it back in one week.

a) He already read half through the 270-page book but had not arrived at its two third yet. At least how many pages of the book does he have to read every day so that he could finish the book in one week?

b) Andris made a note about the opening hours of the library.

Monday	*10:00–12:00 and 15:00–16:30*
Tuesday:	*14:30–18:30*
Wednesday:	*11:00–17:15*
Thursday:	*9:30–11:30 and 15:15–18:00*
Friday	*10:00–13:30*

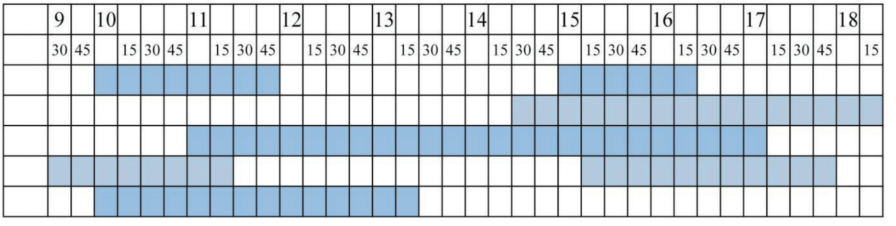
Because of his school schedule and sports program Andris can go to the library early afternoon, between half past one and 2, or after 5 o'clock in the evening. On which days can he return the books to the library?

The first part of the problem tells us that we have less than 135 pages left, but we still have more than 90 pages to read in the book. If somebody wants to read this during a given period of time more specific data is needed. Thus we can only think over how much one can read during one day if 134, 133, …, 91 pages are left from the book. We can also consider that the problem still does not have 44 solutions since Andris every day reads the same quantity, thus if the number of pages read every day increases by 1 page, the number of pages red will increase by 7 pages during 1 week. Thus it is worth to collect the possible solutions of the problem in a table:

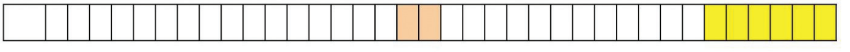
Number of unread pages	91	92–98	99–105	106–112	113–119	120–129	130–134
One should read this much during 1 day	13	14	15	16	17	18	19

One should also consider in case of how many pages is it necessary for Andris to read the same quantity of pages on every day of the week and in which case are less pages left for the last day.

The second question mentioned in the problem requires the comparison of time intervals and the determination of their common part. The representation of the opening hours of the library on a time strip gives great help to the children.



We can also represent on a time strip how many times Andris can go to the library and we can cut this strip out and pull it along the table.



By moving the strip the children can easily read the possible solutions.

The correct choice of different tools makes the activity of "mathematization" easier for the children. It is an important and difficult task to develop the children's ability to translate the problems formulated in everyday language to the language of mathematics. The translation is supported by tools and carefully selected pictures. The tools proposed for the activity are similar to the real-life objects (for example, measurement, invoicing of specific objects, use of tokens), in other cases pictures, figures (for example, representation by segments), or models leading to abstraction appear (for example, colour rods, table). We should not urge making solutions without using tools, it is important that we often use the reasoning by illustration. Before using algorithm we should expect from the children for example, the estimation of operations, in this way they can check the reliability of their calculation, they can recognize the mistakes made.

Problems containing natural, geographical, climate data or connected to excursions, travels, sports, healthy way of life prove that mathematics is a useful means in solving problems of other disciplines and of other fields of life.

The problems should set realistic situations which the children meet day by day, thus it will be easy for them to imagine the situation. When selecting a topic it is not the mathematical problem to which we are looking for a nearly realistic situation, rather we formulate real problems which often happen in everyday life and by thinking about these problems we help the children to get easier orientation in the everyday life.

We can pose problems where we expect from the children the collection of the required data.

For example:

Collect data about yourself.

a) How many times does your heart beat in 1 minute?

b) How many times do you take a breath in 1 minute?

Calculate.

c) How many times does your heart beat in 1 hour?

d) How many times do you take a breath in 1 hour?

The children have a simple problem in front of them which they can solve in one step. There can be big differences between different solutions of a problem, since the collected data can change based on the children's measurement results. The comparison of the solution can screen the incorrect measurement results in this way it leads to the use of realistic data.

The solution of the problems independently, in pairs or in teams allows the review of problems different from the customary ones and the consideration of real life situations, the recognition of solution methods, collection of ideas, methods, and it contributes to the improvement of creativity which essential to the problem solution. During the team activities the children learn the rules of co-existence in a natural way, they experience the good feeling of helping somebody. They have a lot of opportunities for expressing their opinions, to let others know their ideas. They are trained to respect the opinions of others and to being tolerant to their fellows through the confrontation, discussion of views. They learn how to accept the imperfections, eventual limits of their own and of others. The correction of mistakes, giving opinions and persuading others about their goodness of the own ideas can be implemented by reasoning, using rational, acceptable arguments. We have to ask the children to check the solution and to explain why they selected the given method so that the children take responsibility for their work supported by facts and be able to evaluate their activity in a realistic way.

Relations, Functions

Based on the requirements of grade 1-2 similar problems and requirements can be set by the end of grades 3-4. The recognition of more complex rules in the case of sequences is a requirement and we can suppose greater skill in the re-coding of mathematical features of everyday objects and phenomena. For example the transformation into numbers of facts in connection with time can become a routine, because for example the names of the days of the week and the fact about which day of the week we are talking about is already present as a factual knowledge element at this age and it is not necessary to use a strategy similar to the practice of counting starting from Monday.

The children can meet recursive sequences of numbers already in grades 1-2, too (for example, with sequences where the next member is the sum of the previous two members), there are however many possibilities for the formulation of the everyday problems where recursive sequences appear. For example in how many ways can a 2/4 musical cadence be filled by quarter and eighth notes? Then: in how many ways can a ¾ musical cadence be filled by quarter and eighth notes?

The next problem is the text version of the classical Fibonacci sequence, using a drawable wording, recalling the world of tales instead of the unnatural growth of rabbit population:

When the oldest tree of Fairyland was planted, the tree had only one branch. One year later the tree still had only one branch, but then in each consecutive year a new branch grew. How many branches had the tree in (a) two years, (b) three years, (c) four years, (d) eight years?

What can be the rule in the following table? Mark the symbol of connection that is true for the table and cross out the one that is not true.

△	Wheat	House		treasure
☐	b	h	f	

a) ☐ = of the letters of △ we leave out the ones that are not initials
b) ☐ consonant
c) ☐ = initial of △
d) ☐ letter

In this example all the four options are true for the table.

These types of examples – although they are less customary – measure the high level components of cognition which are connected to the falsification principle.

What can be the rule in the table below? [This exercise is specific to the Hungarian language.]

☉	lent	mellett		mögött
☐	lefelé	mellé	alá	

(The first row concerns directions and the second row concerns positions of the according directions.)

From content point of view the problem is grammatical, but it illustrates the mathematical laws of the grammar. This strengthens the relations between mathematical modelling and the knowledge obtained in the everyday life and this is a specific objective of mathematical education which has the aim to develop mathematical cognition and to make the mathematical knowledge transferable.

What can be the rule in the table below?(Source: Az általános iskolai nevelés és oktatás terve, /Concept of elementary school education and training/ 1981, Edition 2, p. 278)

□	Horse	Bear	Cow	Hen
✷	Colt	Cub	Calf	Chicken

Formulate the solution by an open sentence, too: the young of □ is ✷.

There are a lot of means for understanding binary relations. In a lot of school subjects the closed matching problems are regular problem types, when relations have to be found between the elements of two sets and it may happen that several elements of the set can be matched with the elements of the other set. Questions in connection with the agenda, eating, and clothing make possible the making of data pairs, where the order and the relations between the prefix and suffix are also important in their connection.

We can define authentic problems by the use of requirements and problem types of grades 1-2, but by moving in a wider number circle. In addition to data pairs correlations recognized in data triads can also be expected.

Problems requiring selection according to two aspects can be found among tasks for developing the systematization ability. The systematization of a given number of things by the projection of two aspects on each other can be solved already in grades 3-4 mainly by manipulative and pictorial tasks, the content of which is well-known to the learners from the everyday life. The two-way classification is at the same time a means of improvement of correlative reasoning, since in the two-dimensional system produced by the projection on each other of the two aspects the eventual correlation between the two aspects becomes evident.

In the case of similar problems the cooperation of the learners in heterogeneous groups formed according to students' ability levels can be proposed from instructional methodological point of view so that they could learn the ideas of each other.

This is especially important in the case of authentic problems that do not have only one, well defined solution, but in many cases the solution itself is the reasoning process in the course of which mathematical models develop and change.

The principle of inverse proportionality also appears in grades 3-4 mainly in problems based on the experiences, trials of the learners.

During the class excursion the children wanted to travel in a little carriage pulled by a pony. The owner of the pony said that they have to pay 1200 Ft for a quarter of an hour ride, irrespective of the number of passengers.
What other questions did the children put before they rented the coach?
Write the price per chind of the ride into the following table if they travel single, or in two, three, or four of them together.

Number of participants	1 pupil	2 pupils	3 pupils	4 pupils
Fee/participant	1200			

The other possible occurrence of inverse proportionality is the computation of area. (Certainly we do not think of area computation described by a formula.)

Anna would like to arrange 24 identical paper boxes in her room. If she puts them one on top of the other the column will be high, if she puts them one next to the other they will occupy a lot of space on the carpet. What kind of arrangement would you suggest? How many boxes should we put next to each other and how high should Anna put the boxes? Make a drawing then a chart.

No. of boxes next to each other	1	24	2		
No. of boxes on top of each other	24	1			

The problems where two features which can be determined by numbers are correlated not in deterministic way, but a tendency is outlined are good for the development of correlative reasoning. In the following example the learners can also use their own data.

The health assistants measured the height and weight of the pupils of the class. Some data can be found in the following table. Two pieces of data were however erased by somebody by mistake. What data can be put into the empty places?

Height (cm)	135	142	127		140
weight (kg)	31	36	28	40	

The actual data can be taken from a relatively wide interval, but the explicit formulation of the correlation between the data rows is much more important, which can be the description of positive correlation in the language of children. Even more important is however the cognition, that the specific value cannot be determined on the basis of correlation.

Geometry

Constructions

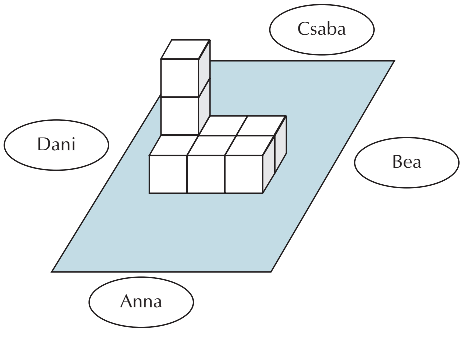
In grade 3 creative thinking comes into the foreground together with the development of vision of forms and spaces. During the creation of geometrical objects text comprehension (by verbal expression of geometrical features), observation skill and students' memory develop. Abstraction is improved by observation, and by description of the properties of specific forms.

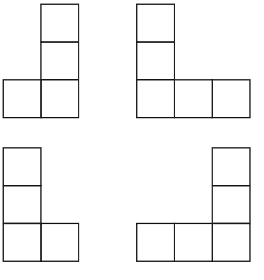
By the end of grade 4 the construction of objects is enriched by taking account of specified conditions and by checking them. The understanding of the relations between parts and whole, the analysis, formulation of observations, and the basic use of the acquired mathematical language becomes evident. Combinatorial reasoning is developed by the creation of works. The aim is to reach completeness and to build up the system of creations.

For the upper grades the building of objects after models or the production of plane shapes by activity according to given conditions are necessary prerequisite knowledge items. Further requirements: The recognition of geometrical properties, the selection of shapes, their sorting on the basis of recognized features. Recognition and taking into account of edges, vertexes, faces in case of simple bodies, recognition and taking into account of sides, vertexes in case of simple polygons. Recognition of cuboids, cube, and rectangle, square based on total view in different positions of bodies and plane forms. Listing the learned properties of rectangle, square, cuboids, and cube – with the help of a model shape or drawing.

Task for the improvement of spatial vision, of the observation of relations between part and whole:

Four children have built a solid of four small cubes. They sat around it and made drawings from four different sides about what they saw. What do you think the children could see. Write their names on the correct places.

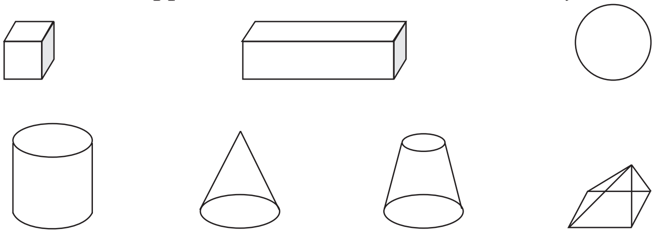




Example for the recognition of forms, for the recognition of the properties of shapes and for the determination of the trueness of statements:

Play in pairs.

Select solids you can see in the picture. Pick a card from a deck of ten *cards, taking turns. Supplement the statement on the card so that you pick up a solid for which it is true. (You can play the game according to different rules, too: Supplement the statements so that they become false.)*



Make cards with the following sentences. Only one sentence should be written on one card.

All of its faces are of the same form and size.
All of its faces are square.
All of its faces are rectangle.
It has 12 edges.
The number of its vertices is 5.
The side faces are quadrilaterals.
It also has round shape face.
It only has curved surface.
It has both plane and curved surfaces.
It has triangle shape face, too.

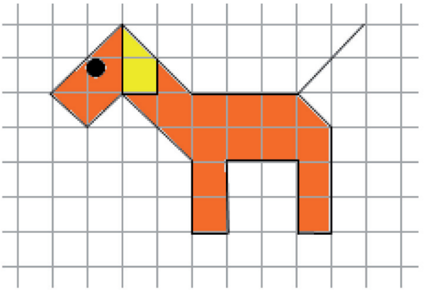
Transformations

In grades 3-4 in addition to the identification of different geometrical objects, the recognition and making aware of changes and invariability lay the basis of generalization in the field of transformations. The recognition of rhythm, periodicity, the observation and following of symmetries aim at the improvement of observation skills. It is important to formulate statements about the observed shapes, or to determine the trueness of given statements.

The development conditions needed to the progress require the recognition of "similar" and "congruent" relations, laying the basis of visual definition of similarity and congruency, implementation of two-dimensional congruent transformations (translation, reflection around the axis, rotation) with the help of copying paper, differentiation of mirror image and translated image in the case of more complex forms.

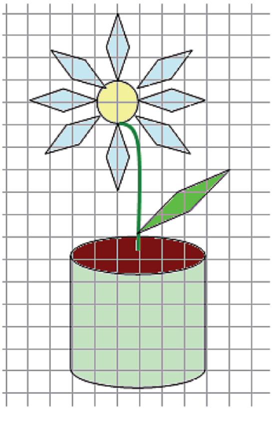
An example of similarity when producing an enlarged image:

Grandma is embroidering a dog on Danika's blanket. She has found a pattern in the "Skilled hands" journal, but its size is too small to the blanket. Copy the pattern into your exercise-book. Enlarge it by doubling the units of lendth in both directions.



In addition to the recognition of the "similar" and "congruent" relations an important element of this topic is the creation of mirror image around the axis with the help of copying paper, or the production of enlarged picture using a quadratic grid as examples of plane congruent transformations.

You can make a nice gift for Mother's Day.
Take a strong paper box.
Enlarge the drawing to its double,
and glue it on the top of the box.
It will be a good sewing box.



Orientation

In grades 3-4, the further development of spatial orientation, the understanding of information, elaboration of geometrical information by words or symbols, the development of ability to determine places are all considered means for ability development. Similarly, remembering directions, dimensions, vicinity.

The further development requires that the learners be able to orientate in their own environment (street, house number, floor, door, direction, distance), interpret or prepare simple draft maps with the approximate indication of direction and dimension, and the precise definition of vicinity. They should be able to orientate on line, in plane, in space with the help of one, two or three data.

Below please find the following example of a task built on authentic activities:

> *Work in pairs. Make a draft map about the environment of your school. Mark the places of the school, the shops and of your home. Indicate on the map if there are parking places, railway station, sports ground, library, cinema, theatre around. Give a route description what your pair has to follow. He/she has to tell where he/she has got to. Then he/she will give you a route plan to follow. Where did you get?*

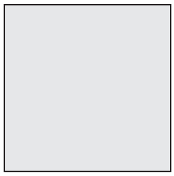
Measurement

In grades 3-4 the collection of experiences in the field of recognition, differentiation of quantitative characteristics, detection of differences is continued. The task is to improve skills necessary for giving estimations, to express the extent of precision in practical measurements, to make simple quantitative deductions. It is important to build relations between mathematics and real life. The practical measurements contribute to orientation in quantitative characteristics of the world.

In order to lay the basis of the developments in the consecutive years the learners should be able to make measurements and use the occasional and standard measurement units. Based on practical measurements they are able to understand the relation between unit and index numbers, to make conversions with the learned units of measurement in connection with practical measurements and to determine the perimeter and area of a rectangle (square) by measurement and computation.

Example for the construction of a rectangle, and determination of its area:

Uncle János has paved the sidewalk with square shaped flagstones. With the remaining 36 stones, he wants to pave the ractangle shaped place in front of the kennel. He is trying out how to place them on a piece of paper. Make a drawing of all the possible solutions.
In reality the length of one side of the square-shaped flagstones is 1 dm:



How many square decimeters of area could he cover with the remaining blocks?

Additional ideas of examples which are activity-focused and are directly related to the everyday experiences of the learners:
The wind has slammed the window and unfortunately, has broken the glass. The housekeeper measured that a 1253 mm long and 1245 mm high glass sheet should be cut into the frame.
a) Measure and cut out a paper sheet of this size.
(You can glue together several pieces to create this sheet.)
b) Give the dimensions with centimeter precision.
c) How many centimeters is the total length of framing strips around the glass sheet?
d) By how many pieces of 1 cm side length square could you cover the glass sheet?

Combinatorics, Probability Calculation, Statistics

In grades 3-4 the main emphasis is placed on the more conscious way of gathering and interpretation of data. By the end of grade 4 children are able to arrange data in a sequence or table, they can represent them of diagrams, they can read data about diagrams, sequences, tables, graphs and they can find data representing a whole data set (for example, the middle one according to size; the biggest, smallest data and their distance; the most frequent data). They can calculate the mean value of the data. This topic gives a lot of opportunities for the solution of realistic tasks if we carefully select the data.

We often examine the setting and solution of realistic problems with the help of the ability to analyse tables. For example:

Gabi, Béla, Pista and Jutka are very good friends. They like to play cards, therefore they play at least once per month. They play a game where there is always a first, second, third and fourth place. They write down the results after every round and at the end they announce the winner at the end of the year. The following table shows the results of this year's tournament.

	1st place	2nd place	3rd place	4th place
Gabi	12	24	23	17
Jutka	18	22	21	15
Béla	24	13	13	26
Pista	22	17	19	18

During the interpretation of the table the following questions emerge: How many games were played this year? How can this be counted? About how many rounds are played on each occasion? Who won the most games?, etc.

Another problem can be that if we select the winner on the basis of other aspects, another competitor will be the winner. Children are able to look for rational criteria based on which it can be determined who can be regarded the winner.

The explanation of the students, the dispute gives a hint that the statistical data set can be interpreted and explained in several ways, since

- in Béla's view he is the winner, since he has won most of the games.
- Jutka has the opinion that although she has not won so much, but had very few last places.
- Gabi is of the opinion that she herself has won very few games but was on the second place many times what is very difficult to make. And she has less last places than the boys.
- Pista feels that he is at least better than Béla, because although he has less first places, but less last places, too.

The solution of the problem can be for example that they give 4 scores for every winning, three scores for the second place, and two scores for the third and one for the fourth places. It is possible that they think that more scores can be given for the winner. For example 5 scores for the win, three for the second place, one for the third and nothing for the fourth place. Would both ways of calculation produce the same result, the same winner? In addition to the orientation in the table an important advantage of the activity is the improvement of computation skill.

A little more simplified example of the above problem can be:

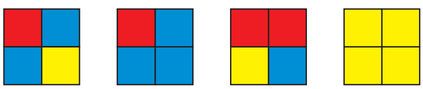
The school football championship is over. The teams got 2 points for a win and 1 for a tie. The results of the matches were written in the following table:

	3.a	**3.b**	**3.c**	**4.a**	**4.b**
3.a		3:0	2:1	1:3	1:1
3.b			0:0	0:2	2:1
3.c				4:3	1:3
4.a					2:2
4.b					

Determine the number of points each team collected.
3.a: … …points
3.b: … …points
4.a: … …points
On which match did they score the most goals?
How many matches ended in a tie?

Children meet a lot of authentic problems during the learning of combinatorics and probability calculation. Built on their everyday experiences a lot of problems can be set which are practical, relevant to them and are intransparent as to the problem solution process.

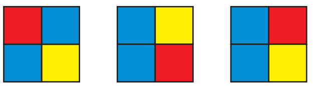
A good occasion for this is for example the creation of a set of toys by a team work.



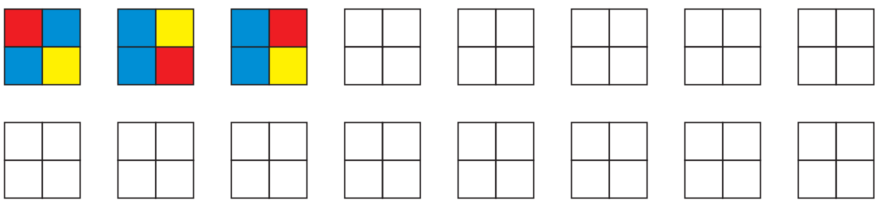
We draw the middle lines of squares and paint them using red, yellow and blue colours. The shared task of the children is to create all possible different elements. Since the papers can be rotated we can agree that we regard the sheets which can be translated into each other by rotating around the centre point of the square. In this case both the organization and division of the labour pose a combinatorics problem. Children regard the prepared set as their own and this makes the activity authentic.

The version formulated in the course of assessing the above-written activity can be the following:

We make the following puzzle using painted squares. We used three colours.

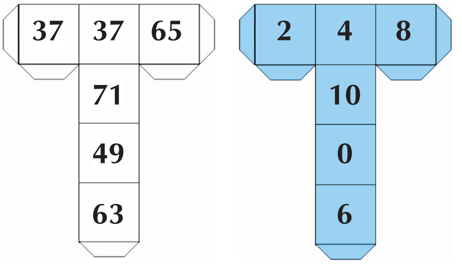


Next, we separated the group where all three colours were found. What other elements can be found in this group? Colour.



The development targets of the probability approach contain that gathering, observing and processing of data in grades 5-6 should be made more and more without the teacher's help. This contributes to the development of systematization ability and makes possible the observation of frequencies, too. The statistical observations offer a lot of opportunities for the insertion of authentic tasks.

In another case they throw simultaneously with the two dices shown on the picture below. Before the starting of the test they formulate some guesses (for example, whether the even or odd sum will be appearing more often) and after the putting down of some cases they compare their experiences and the guesses.



The questions separating the sure and impossible events are still very important during the measurement. We can ask the following when throwing up the above dices:

I threw these numbered dice, then I claimed things about the product of the numbers thrown. Write next to the statement if in your opinion it is true (T), false (F) or can be true, but it is not sure (C).
a) ended in 4 .....
b) smaller than 6 ....
c) odd....
d) 491...
e) smaller than 711
f) ....

There are good opportunities for describing authentic problems when children have to plan the rules of a game themselves.
For example:
Jancsi and Peter take turns throwing a regular dice five times. They agree that Jancsi scores one point if the result is 2, 3, 4, 5, or 6. Peter gets some points otherwise. After throwing five times the winner is the one who could collect more points. How many points should Peter get when the dice shows 1, if we want the game to be fair?

As an expectable solution children will propose 5 or 6 scores to be given for throwing 1. The teacher does not have to take a position about the final, correct solution. The problem follows the idea of the so-called problem-based learning that is the learners make mathematical activity on an intransparent problem, while the teacher becomes the facilitator and moderator of students' reasoning stepping down from the role of the owner and distributor of mathematical truths.

Detailed Assessment Frameworks of Grades 5-6

Numbers, Operations, Algebra

Compared to the former grades the word problems can bring a lot of novelties in the assessment of students' knowledge. In the extended number circle quantities not connected to practical experiences but known from the media or from the school material (for example, historical years, geographical quantities) can be included in the problems. In addition to this the problems to be solved in several steps also gain greater space. The majority of steps consist not necessarily of more arithmetical operations to be performed one after the other (although this also creates a lot of difficulties), but of the sequence of the conscious decisions appearing in the different phases of the problem solution process. Certain steps become especially important in the realistic problems. The understanding of the text of the problem and the selection of the correct mathematical model are in general of greater importance than in the test problems. Also of outstanding importance is the interpretation in general, control step of the problem solution, which does not mean here that we perform the completed mathematical operations again, or compute them with their inverse, but that we test the matching with the problem's text and the conformity with real life.

In our introductory chapter about the application of mathematical knowledge we presented several examples of the realistic arithmetical word problems. Of these prototype problems other realistic word problems can be generated.

In the apple garden of Uncle Jancsi the fruit trees are in 8 rows and 12 apple trees can be found in each row. At his son's suggestion he treats the trunk of the trees at the edge of the garden by chemicals to keep the roving deer away from the trees. How many fruit trees will not be treated by chemicals?

It is proposed to prepare a draft drawing to the solution of the problem that is we connect the things in the problem's wording to a geometrical model.

It is printed on a cinema ticket that "LEFT, row 17, seat 15". How many seats can be found in the cinema?

From the point of view of intransparency this open-ended problem can even be placed among the authentic problems. Several different estimations can be given as a solution, which can be formulated as inequalities with mathematical symbols.

280 pupils are transported to the Children's Day celebration in 44-seat buses. How many buses should be ordered by the headmaster of the school?

International experiences were collected about the problems where some kind of "trick" is hidden. Probably the majority of children can compute correctly the division with remainder the result of which is 6 and the remainder is 16. Many children will however give an answer of 6 or they may also give the answer „6, 16 is left ". The realistic answer here will be 7, to which we use the implicit information that obviously they will order the least possible buses.

Because of the data not contained in the text of the problem or due to the factors typically not regarded mathematical the learners often feel themselves cheated when they solve problems, like for example:

The best result of Jancsi in 100 meter run is 17 seconds. How long would it take for him to run 1 km?

Our proposal is that these types of "tricky" problems have a place in the school lessons, especially in order to avoid the over automatization of the usual problem solving strategies, they can however hardly be used for diagnostic assessment purposes because we can only reveal using other fine tests if somebody answers 170 seconds to the above problem because of being uninformed, or because of a lack of courage.

It is an important step to the better understanding of the word problems if we often expect from the learners of this age to find out word problems by themselves for a given mathematical structure. This is an extremely difficult task. It may be a difficult work to create a text even to one basic arithmetic operation. But if the children are allowed to create texts to the problems this allows the putting next to each other and the comparison of the routine word problems and the realistic problems.

If for example 20 liter water should be divided equally in 8 vessels, formulating this as a routine word problem the end result of 2,5 liter comes easily. We can ask the learner what other elements can be put into the problem so that the numbers remain unchanged. The breaking of the chocolate can be solved, but it can also evolve among the ideas for example that a class of 20 persons slept in rooms during the excursion where there were 4-4 pieces of bunk-beds. How many rooms had to be rented? …And where will the class teacher sleep? Among the many different ideas there will be some where with the unchanged numerical and unchanged division task the whole part of the division result is a number by one bigger than the whole part, or the solution of the problem will be exactly the division remainder.

Interesting type of realistic word problems are the ones which can basically be solved not by arithmetical operation, but by logical inferences (certainly the arithmetical operations will also get a role in certain steps).

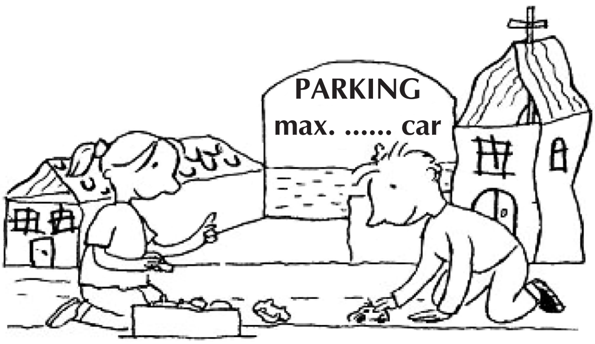
There is a bus stop in front of Eva's house from where the bus departs towards the school every 10 minutes between 6 and 9 o'clock a.m. The journey takes 15 minutes. Éva must be at the school by 7:45. By what time should she be at the bus stop in order not to be late for the school?

The can be different tasks found in the international literature which belong to the authentic category and which are appropriate for children belonging to the age group of grade 5-6. In an experiment made with 10-12 year old children there were several problems which stimulated the learners for activities as a result of which they can find the correct mathematical model to the realistic problem – often as a result of group work work.

In a well-known Flemish development program Verschaffel et al. applied for example the following example:

Pete and Annie build a miniature town with cardboard. The space between the church and the town hall seems the perfect location for a big parking lot. The available space has the format of a square with a side of 50 cm and is surrounded by walls except for its street side. Pete has already made a cardboard square of the appropriate size. What will be the maximum capacity of their parking lot?

1. *Fill in the maximum capacity of the parking lot on the banner.*
2. *Draw on the cardboard square how you can best divide the parking lot in parking spaces.*
3. *Explain how you came to your plan for the parking lot.*



All the typical characteristics of the authentic problems listed in our theoretical introductory chapter are met in the present problem:

- The picture belongs to the detailed presentation of the problem situation. Besides this a narrative story is outlined which together with the picture helps that the children feel the problem of their own that is they compare it with their previous experiences.
- The described situation has to be formulated by a genuine mathematical model. Based on the drawing and of the specified (and searched) data several different geometrical models can presumably be made.
- The learners have to obtain the other missing data. They can collect the missing data by field measurement, or by conversations.
- The complete problem is divided into several sub-problems: the setting of the different sub-problems, the checking of the attainment of the sub-aims is the duties of the learners.

In another very well-known intervention program Kramarski and Mevarech created the famous "pizza-task". In this authentic task the prices of three pizza restaurants are given: the pizza's diameter is given in centimeter (regarding the need to take into account the area of the circle the task is suggested to be used rather from the 7th grade) and the prices of the different pizza supplements are very varied. The student's task is to find the best buy which also proves the above described characteristics of the task: verbal emulation of real situation, model-making, it should be decided which numbers are significant and which not, the task can be divided into sub-tasks, the solution process can be divided into sub-purposes.

The task mentioned in the previous part where we calculated the time intervals of the bus going to the school can be changed into an authentic task if the children look for the appropriate mathematical description according to their own, realistic, experienced travel habits.

The authentic tasks have a special role in assessing mathematical knowledge. We have seen that in the case of realistic tasks it not only the question whether "the end result will be found". The authentic tasks do not have an end result in the sense as the routine tasks have. But there is a solution process which is based on the comprehension of the text, on cooperative learning, on mathematical model making and the decision about the lack of data or their redundancy bring the learners into decision-making situation. By the end of the lower grades the sensitivity to problems, the conscious knowledge about and control of the phases of the problem-solving process can develop instead of the often rooted mathematical beliefs (for example, which task has a correct solution).

Similar to the routine word problems and to the realistic word problems in general the authentic word problems also offer possibilities for the use of a "reverse" problem-solving strategy: the creation of the problem situation and the text to a given mathematical structure. In an intervention program we have used with success already with 4th graders for example the task where they had to create a text to the division of 100:8 where the solution of the problem first should be division without remainder, in the second case division with remainder, in the third case the remainder, in the fourth case a whole number by one bigger than the whole part received by the division with remainder. By setting a problem of this type we clearly evaluate also creativity and verbal abilities which are not so much inherently connected to mathematics. This however cannot be criticized if we make it clear that we are diagnosing the application of mathematical knowledge in authentic problem situation.

Relations, Functions

The most important characteristic of the realistic problem types is that experiences of everyday life, in certain cases the specific knowledge get relevant role in the solution of the problems. In the case of certain tasks, it can be presumed that the correct solution requires the active utilization of everyday knowledge and experiences at least on one point of the problem solution (in the planning, implementation, or control phase). All this does not mean that the problem describes an everyday situation for the learner, the situation can be known, but a little strange to him/her, belonging to the world of the "adults". Such are for example situations related to the household, baking-cooking, travelling, shopping, saving.

What do expressions 1,5%, 2,8%, 3,6% mean on a milk carton?

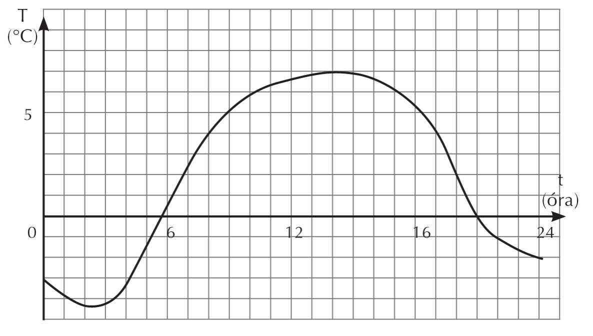
The proportion of margarine and farmer's cheese needed to make a scone is 4:5. How much farmer's cheese do we have to use if we add 20 dkg margarine to the pastry?

Is there direct or inverse proportion between the following pairs of quantities?

- *the length of the side of a square and its area*
- *the length of growing wheat and growing period*
- *sides of a square of 120 cm^2*
- *time needed to cover a given distance*
- *the mass and price of the fruit purchased*

A 24 cm high candle burns down to the bottom in 4 hours. In how many minutes after the lighting of the candle will it shorten to 16 cm?

The diagram below shows temperature values measured on a winter day.



When was the coldest? What was the highest temperature? In which period did the temperature decrease?

Csaba went for an excursion. During the first 3 hours he walked at a steady 4 km/h speed then he had half an hour of rest. After the rest he continued walking 2 hours at a 3 km/h speed when he arrived at his destina-

tion. He had a rest for an hour and a half, and returned home at a 3 km/h speed without rest.
Represent Csaba's movement in a coordinate system. Answer the questions on the basis of the figure: How many kilometers did Csaba make? How long did the excursion last? At how many kilometer distance was he from the point of departure at the end of the 9th hour? …………

It can be a practical characteristic in the case of the authentic problems that the solution of the task supposes the initiation, problem setting by the learner. It is by all means necessary that the learner translates the problem into his/her own language, feel it as his/her own in certain respect and be able to imagine the given situation. In many cases this simplifies the mathematical content of the problem and the key to the problem solution is the completion of this transformation, the finding of the correct solution model.

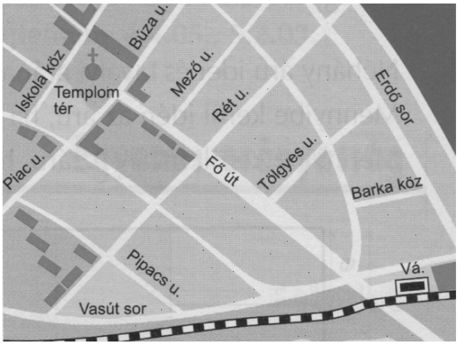
About 65% of the mass of a human body is water. How many kgs of water is in the body of a man of 80 kg? And in your body?

Is it really true that the discount on the ice-cream is more than 25%?

In the class the boys and girls are taking part in a steeplechase in separate groups. The boys made the 2 kilometers in one and half an hour, the girls made seven during 2 hours. Which team has won the speed competition on the 18 km distance?

At the parent-teacher meeting your mother would like to sit exactly on your place. Prepare a description, a "map" for her so that she could for sure find your place.

You can see a detailed map in the figure. 1 cm on the map is 20,000 times bigger in reality. How far is the church from the railway station?



In the more complex, unusual tasks we can direct students' thinking by questions and sub-tasks. During the development this method serves for the more detailed analysis of certain problems, for the recognition of relations, links, for the finding of different solutions. When evaluating students' work, the divergent solutions would make a problem, therefore it is worth to somehow determine the path of thinking.

At a telephone company we can phone two minutes for 80 Ft. If the conversation lasts longer they bill another 80 Ft and so on for every commenced 2 minutes. How does the fee of the conversation depend on the length of the call? Make a table and plot the cost per minute on a diagram.
Calculate the price of a 7 minute conversation. And that of a 12 minute call? Make a table and a diagram showing the total costs of the conversation.
Another company uses billing on a second basis. During a phone call we pay 1 Ft after every second passed. This company also charges fee for the calls, 30 Forint for each call.
The services of which company are cheaper?

Geometry

Constructions

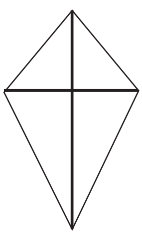
An example task on the properties of rectangles:

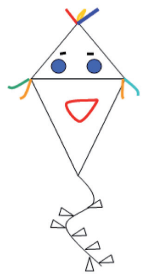
Uncle Robi would like to cover a parallelogram shaped area next to the sand pit of the children with 3,4 m by 3,6 m wooden plates. For this purpose he received congruent, symmetrical trapezoid shaped waste sheets of wood from the joinery in the street. The legs of the trapezoid are 6 cm long, the shorter base is 14 cm, and the longer one is 6 cm longer.
Will he be able to cover the parallelogram shaped area with trapezoid sheets?
How many sheets of wood does he need?

In this topic the authentic tasks require activities, where design, the follow up of the process and its planned checking and discussion get an accent.

Prepare a deltoid shaped kite. The symmetrical diagonal is 60 cm, the other diagonal is 40 cm. Design your kite. Measure how many laths, and how much paper you need. (You also need glue to finish it and rope to make it fly.)

You can also paint, decorate your kite.





Measurements

It is the task of the lower grades to lay the basis of length and area measurements. In grade 5 we are repeating and supplementing the acquired knowledge with the formula of the area of rectangular and square and with the formula of the volume of cuboid and cube.

Among the length measurement tasks the realistic problems require from the students to use their own experiences, or perhaps to follow trains of thoughts requiring occasional measuring tools.

Perimeter and area of a rectangle

Grandma has planted carrots in half of the rectangle shaped vegetble garden, radishes into ¼ of the land and spinnach in the remaining area. One side of the garden is 5 m, the other side is 8 m long.
Calculate how many m^2 *area is occupied by spinach in the vegetable garden?*
We had to fence the vegetable garden around with a low fence because of our dog named Buksi.
How many meters of fence do we need?

Volume of cuboid, unit conversion

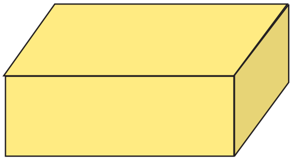
On our terrace there are 6 cuboid shape flower boxes. Their dimensions are: 100 cm by 30 cm by 40 cm.
How many m^3 *potting soil do we need if we fully fill all the boxes?*
Potting soil is sold in strong plastic bags. We bought soil in 50 liter bags.
How many bags do we need to fill the boxes?

Volume calculation, unit conversion

On April 20, 2010 a British oil production platform exploded in the Gulf of Mexico. During one week 795 thousand liter crude oil leaked into the sea. The spreading oil covered about 5000 km^2 *surface. This accident is a serious harm to the environment.*
Calculate how thick the oil slick can be if we consider the surface covered by oil of rectangular shape?

The problems based on the everyday situations children may experience mainly involve tasks in connection with cuboids, rectangles expecting active participation of the children.

In the school every child has a shoe box where the tools they need on drawing and mathematics classes are stored (paint, brushes, cups, cloth, and ruler, compass). The boxes should be put on top the each other so that they could be placed on the shelf. It was Klári's idea that everybody should cover the boxes by wall-paper so that they could look much nicer. Attila promised to buy the material. Wall-papers are sold in 10,05 m rolls with 0,53 m width.
Before making the calculations, give an estimatate whether this much of wall-paper would be enough for all the boxes if 24 children are learning in the class?
The size of a shoe box: 10 cm by 20 cm by 30 cm.

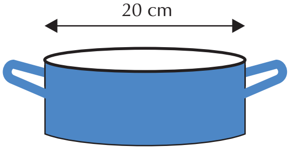


One of the taps is broken in the school's lavatory. Children measured how much water is wasted in 1 minute. They collected 60 drops of water into a measuring cup during 1 minute. The volume of dripped water was 13 ml.
Calculate how many liter of water is dripping away in 1 hour, 1 day, 30 days.
Estimate how many times one can take a shower with the water which is wasted during 30 days? Make calculation. (We generally consume about 75 l water when taking a shower.)

The following knowledge and skill elements can be diagnosed by this task: direct proportionality, unit conversions (time, volume units), computation skills (multiplication, division), and abilities for giving an estimation.

Volume calculation, unit conversion

Mix a little red pepper in 1 ml of cooking oil. Fill up with water a 20 cm diameter pot. Pour the painted oil on top of the water and observe the spreading of the oil. (The surface area of the water in the 20 cm diameter pot is about 314 cm^2.)
Calculate the thickness of the oil covering the water surface.



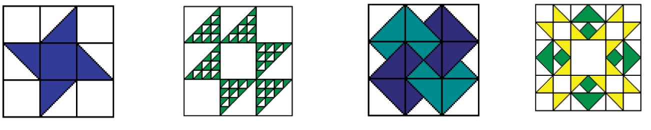
Transformations

Of the congruent transformations learners should know reflection around the axis and the shapes with axial symmetry (triangles, squares), as well as their construction.

The problems also contain enlargement, reduction tasks and the ratio of similarity is also present in the problems.

Axial symmetry

The favourite hobby of Aunt Zsóka is patchmaking. She made cushions decorated with patchwork for everybody as Christmas presents. Select the patterns with axial symmetry. Draw the symmetry axis.



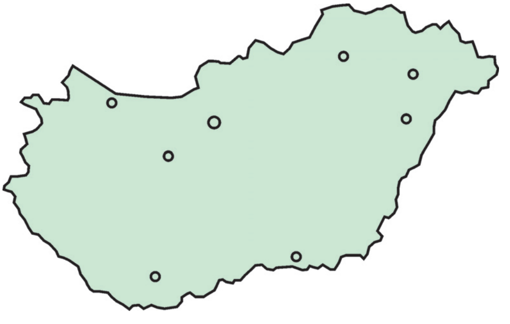
Reduction, proportion, area

The drawing shows the reduced floor plan of a medieval castle surrounded by a castle-ditch. One square side in reality corresponds to 10 meter.
About how many square meters could the ground surface of the castle be? Make a calculation.

Design patterns with axial symmetry and without it on a grid paper.

Similarity, enlargement, reduction, proportion

I have read on the internet that a miniature copy of Hungary would be built in Jakabszállás. It will be called Hungarian Garden and will occupy 93 thousand square meters. This area is about equal to the territory of 13 football fields. Check the territory of Hungary.
Calculate how many times the territory of Hungarian Garden is smaller than that of Hungary.
Enlarge the map on the picture to its double.



Orientation handled as separate geometrical field in grades 1-4 is not accentuated here. We present a sample task which can be regarded an extension of the assessment requirements of orientation in the lower grades, on the other hand it can also be listed to the authentic use of data pairs in the functions, relations topic.

Number the rows and columns in the classroom. Thus each seat is assigned a pair of numbers, where the first number shows the row, the second number shows the column.
Where do you sit?
Where is your desk neighbour sitting?
Who is sitting on chair (4; 4)?
Write down the index numbers of the boys.
Write down the numbers indicating girls in the second column.
Write down the index numbers of children with brown hair.
Write down the index numbers of children with blue eyes.

Combinatorics, Probability Calculation, Statistics

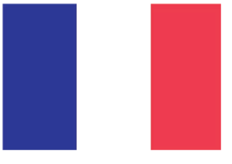
The most important feature of the realistic problems is that relevant role is given in the problem solutions to everyday life experiences, in certain cases to the specific knowledge. Starting from regular routine tasks we can for example arrive at realistic tasks if we attach characteristics to the actors, activities which have an impact on the possibilities to be taken into account during the solution.

A typical realistic problem:

Anna, Béla and Cili are siblings. Their parents ask them to do two sorts of housework every day: emptying the garbage can and watering the flowers. Make a plan that would split the chores between the three children in a fair way. In about how many days would the same child do the same housework?

In the field of combinatorics the examples requiring the computation of all the possible colouring versions of flags and maps are typical routine tasks.

The flags consisting of three different colour stripes are called tricolors, like for example the Hungarian or the French flags.



The stripes can be horizontal or vertical. How many types of tricolors can be prepared by using the red, white and blue colours? Which of them are actual flags of nations?

In the category of realistic problems the problems to be solved by using the pigeon-hole principle represent a typical class. In this age group we do not expect from the student the knowledge of the pigeon-hole principle in general, nevertheless the successful solution of the problem can be expected in case of familiar problems which can be modelled easily. In intuitively developing the principle we can proceed from the lower numbers to the numbers up to million.

In the class there are seven boys and all of them threw once with the dice after each other. Is it true that there will be at least two of them who threw the same number?
In one of the classes there are 20 pupils. How do we know for sure that there are some who were born in the same month?
The gymnastics teacher writesthe results of the small ball throwing contest rounded to meter. Why should it be certain that among the 200 senior class students of the school, there were some who achieved the same result in small ball throwing?

Regarding the topic of descriptive statistics the learners have a lot of opportunities for searching for models related to their everyday experiences.

In the interpretation of authenticity we follow the basic standpoint described in the introduction: we regard authentic a mathematical (word) problem if it describes a situation which can be regarded by the student realistic. In addition to the individual, paper-and-pencil type assessment methodology the demonstration of the reality of the problem situation often requires the generation of a problem context: everyday objects, text details, tables, etc. can be used as annexes to the problem text and the solution of the task is often made in group work. In the case of the paper-and-pencil assessment methods, or when using the individualized on-line diagnostic assessment method the surface characteristic of authentic problems is the longer, typographically varied and novel problem wording, while from deep-structural point of view as to the solution of the problem intransparency is a typical feature, that is the missing of an immediately applicable procedure leading to the solution. The authentic character of the task can be determined in a given historical-social environment, taking into account the majority of students belonging to a given age cohort. It is possible that for certain learners (or in other historical-cultural situation) an authentic problem changes into a routine task, what's more it may also happen that an authentic problem would not be regarded a routine mathematical problem from the point of view of certain learners or of certain historical-social context, but would be linked for example, to the assessment of critical thinking or of some kind of literacy domain.

A practical characteristic of the authentic problems is that the solution of the problem presumes the students' initiation. We can also say that in many cases the task of the learner is to create a mathematical task in the given problem sphere which requires the lower level application of the mathematical knowledge.

In the field of combinatorics the authentic problems require from the learners to recognize that a given everyday problem can be solved with the counting of the possible cases. A usual type of questions is the heap of problems starting with „How many different choices do I have?”.

The authentic version of the realistic problem presented earlier is the following:

Anna, Béla and Cili are siblings. Their parents ask them to do two sorts of housework every day: emptying the garbage can and watering the flowers. Make a plan which would split the chores between the three children in a fair way. What other data would you collect about the brothers and sisters and the housework, based on which you can prepare the best division of work? (E.g. age, difficulty or period of the housework, etc.)

In the field of probability calculation the authentic problems contain the description of activities related to the everyday experiences of the learners: lottery, tossing-up a coin, sports games, card games. A usual type of authentic problems can be the problems based on the question „When do I have the biggest chance?”. In the case of authentic problems belonging to the scope of probability calculation often linguistic-logical or game theory considerations lead to the solution.

Karcsi and Peti are throwing a rubber ball at a target in the middle of which there is a square with 20 centimeter sides. This square is just in the middle of another square with 30 centimeter sides the part of which outside the inner square makes the outer part of the target. Make a drawing about this peculiar target. One of the competitors has to hit the inner square, the other the outer part of the target. Karcsi has the choice which part of the target he wants to hit.
What do you suggest?

A student’s answer should contain the area calculation data belonging to the task description (according to this the outer part is bigger). At the same time the authenticity of the task allows to make other questions: how are the hits counted – are scores taken away if the ball lands on the wrong ground?

Is it true that if somebody tries to hit a certain point, his throws will more frequently go close to that point than further from it?

In the field of statistics the authentic problems expect from the learners the ability to plan and to implement some kind of data collection process. They should be able to formulate questions about the data of some kind of property, to represent data with the corresponding methods: column chart, pie and scatter plot.

Based on the idea of the American NCTM Standards consider the following problem:

Compare which paper plane flies further: the one which is made of a soft copy paper, or the one which is made of hard carton paper of the same size. Make both planes with the same folding technology.

This problem requires the planning of the data collection process. How many throws should we make in order to get a reasonable result? Where should we make the experiment? With what devices and what precision should we measure the distances? How should the data be represented? Based on what shall we make decision and give an answer to the question?

It can be seen that this last problem practically encompasses all the features of the authentic problems (individual setting of problem, making mathematical models rooted in the everyday life, several possible outcomes, and cooperative mathematical activity). It should be noted that these types of problems are time-consuming, in average conditions they can use half of the school lesson. A lot of signs show, however that what we loose on the swings, we can gain on the roundabouts: the time-consuming authentic problems can primarily take the resources from the drill flavour routine examples.

Content Areas of Mathematical Knowledge in the Diagnostic Assessment

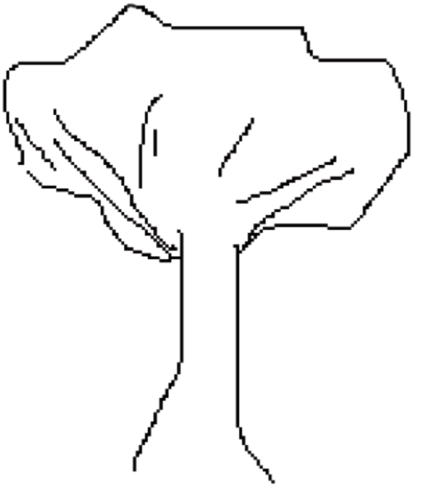
Detailed Assessment Frameworks of Grades 1-2

Numbers, Operations, Algebra

Numbers

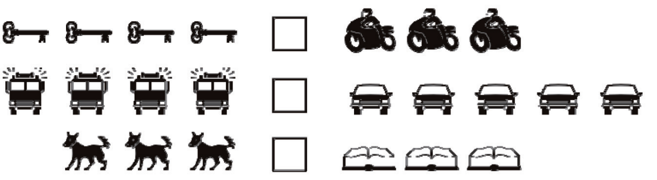
The development of number concept is closely related to "the same" relation. The understanding of the terms like more, less, the same, the learning and recognition of the different symbols (<; >; =), their filling up with correct content are basically assisted by the various pairing activities – using objects, pictures, drawings, words. In the course of the many different activities the learners gradually ascertain that if two groups, sets contain the same number of elements, their number is equal, that is – they are characterized by the same number, the same number relates to them – the number of the objects, things, living beings, etc. is the *same number of pieces.* The safe understanding, knowing of this relation, is a condition of the development of the correct number concept.

Draw three apples and two pears under the tree.



The problem focuses on the comparison of the number of pieces. The pupil can count, arrange in pairs, group the objects. The insurance of a lot of real experiences, the relating of number symbols to pictures, the use of number cards are very important. The writing of numbers (number symbols) can begin later.

Problem: On which side of the small squares do you see more objects? When you made a decision put the correct symbols (<; >;=) into the squares.



Problem: Which number is bigger? Put the < or > symbols between the numbers.

a) 8 – 2 9 – 1

b) 8 – 1 5 + 0

The development of the number concept is assisted by the empirical acquaintance with measurement index numbers. In grades one-two – as a continuation of the preschool preparation – the intelligent planning and direction of a great number of games (for example, filling space with cubes, with jug water, sand, bean, wheat, peas with the help of a glass, pouring them together, etc.) results that the pupils become capable of making comparisons (for example more, less, how many times the same) and besides the piece number they acquire the correct using of measurement index numbers. The experiences shall be understood, like for example: (1) to fill jug of the same size you need to pour more times with a smaller cup and less times with a bigger cup; (2) the same length can be made of more smaller units and of less bigger units; (3) using the same units the heavier objects can be balanced by more units, while the lighter objects by less units. In these measurements we can select the units freely and we can also select the official units without demanding their knowledge.

When comparing actual masses we use the traditional twin-pan balance, which demonstrate equalities, inequalities in an excellent way. (This experience will leave such imprints, memories in children what can be used by us later during the teaching of the principle of balance.)

In the case of measurements the varied selection of units (for example, use of colourful rods) supports the creation of more general, safe basis of the number concept. The many different measurement experiences contribute to the preparation of the idea of proportional changes.

The children often put their toys in order. The order and the ordinal number of the figures may change. This momentum includes the recognition that the ordinal number is not fixed to a figure but it depends on how we line up the figures and from where their numbering is begun. We will see that the number of figures will not change because of the order they are put in, or due to what direction the counting is started from. The many diversified arrangement of a given number of figures, the changing of the places of figures, the frequent repetition of the words first, second, third, … will promote the realization of the term of ordinal number, the understanding of the difference between number, ordinal number and will support the preparation of the concept of number line, too (for example, direction of the increase, decrease of ordinal numbers, looking for neighbouring numbers).

The assessment of the development level of the number concept is made by testing characteristics, properties observable by external experts. The basic condition of developing assessment is that we know the possibilities and process of the development of the appropriate level of knowledge and consequently of the diagnosis of the development difficulties.

By the end of the first grade children should learn the natural numbers at least until 20, and at least until 100 by the end of the second grade. This means that in this number circle the safe number concept shall be developed, number symbols should be learned and used appropriately in writing and reading. Further objectives can be listed as follows: neighbouring numbers, even or odd numbers, ordering according to size, their positions compared to each other (number line), division in different ways (for example, to the sum of tens and ones), rounding to tens (when buying in cash the sums in Forint to 5 or 10).

Some examples of the diagnostic assessment of the development of number concept:

Cross the numerals on the figure:

6 Z 9 F

4 12

M 3

7 ?

Cross the numerals on the figure:

6 Z 9 F

+ p 12

= M B 3

In these two grades activities for experiencing negative numbers (meeting with directional quantities (for example, warmer-colder; before, after 8 o'clock; to the right, to the left from me; etc.)), fraction numbers (cutting a whole into pieces, folding, etc.) also appear.

Zero is difficult for the small school children not only as a symbol, but the handling of zero as a number is also a big challenge. The following example is an illustration of the outstanding importance of zero as a numeral and zero as a number:

a) Which number is bigger? Circle it.

9 – 2 5 + 1

b) Which number is bigger? Circle it.

9 – 2 6 + 0

In these two grades we can already begin the preparation of the concept of number system and of the place-value system by the grouping of different objects, of smaller-bigger animated figures, which most often is made by tens, by the making the ten-hundred overstepping known. Being familiar with concrete numbers written in the decimal system, the knowledge of the concept of one and ten in the decimal number system is a minimal prerequisite knowledge.

Operations

The children learn the connecting role, the interpretation, and the use of parentheses also through examples (simple word problems, sums, taking away or multiplication of difference).

In the first two grades we expect skill-level verbal computation, addition and subtraction up to 20 together with the checking of results. In the first grade the breaking down of the learned numbers into the sum of two numbers, additions and the knowledge of adding three members is an expectation on practical level, while in the second grade this is a requirement up to 100 added by the safe knowledge of „little (multiplication) table". The „little table" means the table of multiplication and inclusion up to hundred.

While in the first grade the children get some kind of routine in complementing operations with missing members, in solving open sentences, and in the checking the truth of statements, in the second grade they make open sentences containing even two variables not only true, but also „not true". They formulate statements and find out if they are true.

Algebra

In the first two grades the introduction of symbols, their verbal expression and marking in writing in different relations, connections (for example, open sentences) can be regarded as basic elements of preparation of algebra. Below is an example of this.

Select natural numbers smaller than 20 which make the following open sentences true.

$13 + \square = 18$ Solution: $\square = 5$

$30 + \triangle + \triangle < 40$ Solution: $\triangle = 0, 1, 2, 3, 4$

In these types of examples the same symbols represent the same numbers, but different symbols can mark not only different numbers.

For example: *both* $\square = 3$, $\triangle = 3$ *number pairs are solutions of the* $\triangle + \square = 6$ *open sentence.*

Word problems which can be solved by one or two arithmetical operations, or where the understanding of the problem is mainly proved by a written open sentence represent an important field of the use of algebraic symbols in the school.

In the first grade the simpler word problems can be solved by the addition or subtraction of two data. In these types of problems it is not necessary to introduce a symbol for the unknown. The introduction of a symbol has a meaning if in the problem one of the member of the sum, or either the minuend or the subtrahend is unknown. The meaning of the symbols should be confirmed either verbally or in writing already in the case of these simple examples.

We can give simple word problems already at this age through the careful interpretation of which the learner can get rid of a lot of unnecessary work. Such is for example the next task.

Which number is bigger than 17, but smaller than 13?

Solution: *There is no such number.* (If we ask them to mark the partial solutions on the number line it becomes clear that there is no number which meets both conditions at the same time.)

These types of tasks first help to understand the problem instead of first trying to select the operation or to give an answer.

Putting down in writing, making models for the learned numerals, operational symbols, relation symbols, unknown symbols and later the parentheses requires rather high level of abstraction from the small child. The discussion of the relations discovered by the pupils – and their communication methods – contribute to manifold cognitive processes. A picture, a text can be approached from many different directions, they can evoke different thoughts, and the results of the thinking process can be appropriately shown in many different ways.

On Mother's Day Luci gave a bouquet of wild flowers to her mother. It contained 15 blow-ball flowers and by 10 more poppy flowers. Of how many flowers was the bouquet made of?

Solution: $15 + (15 + 10) = \triangle$, $\triangle = 40$; *there were 40 flowers in the bouquet.*

(In this case parenthesis means the coherence, but can be left, too.)

Formulate the following number problem by words. Write a word problem, too.

$4 \times (65\ Ft + 35\ Ft) = \triangle\ Ft$

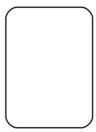
Solution for example: *The breakfast of our 4 member family was yoghurt for 65 Ft and a cheese biscuit for 35 Ft each. How much did a family breakfast cost?*

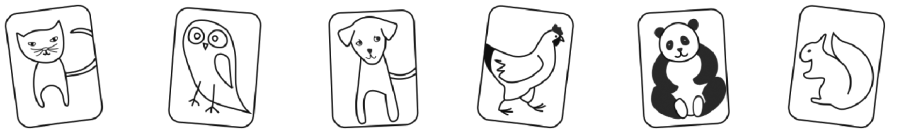
The writing down of word problems by symbols, making texts to the different notes, that is the frequent and factual practicing of the „to-and-back route” deepen the understanding of the content of concepts, makes the learner able to formulate in mathematical language simple word problems or to create adequate, simple texts to mathematical symbols.

A significant part of word problems relates to the open sentences. The verbal formulation, writing down of the open sentence based on a given text, making the contained unknown(s) concrete, or to replace them by actual elements can make the thus produced statement true or false. The scope of interpretation of the majority of open sentences is limited to the elements of the learned set of numbers, but elements of the basic set can be selected from many other fields, let’s say from the world of flora and fauna, from the world of tales, too.

In the next example we put cards with pictures of different animals on the table. The cards show the picture of a domestic animal or a wild animal. During the solution the selected cards should be actually put into the frame, and it should be stated after this if the decision was right. This envisages that it should be decided in advance about each element of the basic set if it is a solution or not.

From the cards below pick the ones which make the statement true.

On the cards put to  *- you can see a domestic animal.*



a) b) c) d) e) f)

During the solution of number problems the learners can collect experiences about the unnecessary use of parentheses, or about how their placing influences the result. The need for using parentheses should be demonstrated in connection with the word problems, too.

Aunt Juli buys 1 liter milk for 140 Ft and 1 kg bread for 160 Ft every day. How much money does she spend a week for milk and bread together?

Solution: $7 \times (140 + 160)$ *Ft = 2100 Ft. Aunt Juli spent 2100 Ft for milk and bread a week.*

For two years Aunt Kati bought one bar of chocolate for each of her four cousins and for her three friends for their birthdays. How many bars of chocolate did she buy during this period if she bought chocolate only for these children?

Mark the letter of the correct solution.

a) $2 + 4 + 3$ *b)* $2 \times (4 + 3)$ *c)* $2 \times 4 + 3$ *d)* $2 \times 4 + 2 \times 3$ *e)* $(3 + 4) \times 2$

Solution: b), d), e).

The children will check the correctness of the answers by the calculation of the results of operations and by „experimenting” with the results. Some of them will calculate the result by deduction and will look for the operations giving this result, others – probably less – will select the operations giving the good solution without knowing the end result.

During the first two grades the children should be able to formulate statements about simple activities, pictures, drawings, they should make a decision about their trueness, and they should make open sentences true by addition, and close them by replacement.

In order to develop the problem solving strategies it is important to establish double-direction relations between the things and relations in the problem and between the mathematical steps leading to the solution. To this end the pupils should be able already in grades 1-2 to find the correct word problem (or task consisting of an image) to a given mathematical structure. By the end of these two grades they should be able to formulate individually and collectively number problems, open sentences based on various activities and simple texts. As it was shown above they should be able to pick the ones matching to the text from given solution possibilities, and vice versa, they should select the right text to number problems, open sentences, or to create simple, clearly formulated texts.

Of the texts below select the ones matching to the following open sentence.

3 + 37 + 28 + ☐ + ☐ = 100

a) Aunt Bori lives on a farm and raises a total of 100 poultries. She has three cocks, 37 hens and 28 ducks, and as many geese as turkeys. How many geese does Aunt Bori have?

b) Évike was collecting the crop under their walnut tree. She collected 3 walnuts on Monday, 37 on Tuesday, 28 walnuts on Wednesday, the same number of walnuts on Thursday and Friday; and on Saturday the whole day she collected 100 walnuts. On Sunday she didn't pick any but counted the nuts she picked earlier. How many walnuts did she collect during the week?

c) Aunt Kati baked five different types of cakes for the birthday of her grandchild. She made "Gerbaud" cake, nut cake, chocolate balls, sour cherry pie and apple strudel. She took 50 pcs or 50 slices of each to the party. At the end of the party she counted the remaining cakes and said: Exactly 100 cakes are left. As I see the chocolate balls were the most popular, only 3 are left. The pie and the strudel were consumed equally. The nut cake was the least popular, 37 were left and they had not eaten 28 Gerbaud cakes. How many slices of sour cherry pie was left?

Solution: *Text b) does not correspond to the open sentence.*

By the end of the 2nd grade the learners know that comprehension is the first and most important step to the solution of a word problem. The understanding is made easier by playing, displaying, drawing, if necessary by rational re-formulation; understanding is followed by writing down with a number problem or with an open sentence, sequences, table of a problem and by computation, looking for the rule and by checking, relating to the initial problem, by comparison with data, real life, by preliminary estimation, and finally by the formulation, writing down of the answer. During assessment the steps of solution of the word problem are divided into separate problem units which can be evaluated independently ensuring by this that the eventual computation mistakes do not make invaluable the other, in principle correct steps of the solution of the problem.

Relations, Functions

At the age of grades 1-2 the following development problems and assessment requirements occur in connection with the content basis of the subject: continuation of sequences and looking for rules in the case of sequences consisting of objects or drawing symbols. The pupils should be able to generate sequences on the basis of a given rule. They should formulate in words the regularity determining the sequences.

By the end of grades 1-2 the following requirements can be set in the field of data pairs and data triads. The pupils should be able to recognize the relations between the matching members of two sets and based on the recognized rule set up appropriate pairs. Objects, persons, words and numbers known from their environment can all be used as elements of sets to be matched. They should be able to mark by arrows the relations between numbers and quantities. They should be able to arrange the related number pairs in tables and to recognize and continue the rule („computer game") in the case of number pairs arranged in table. In this age group the rule expressing the relations between the number pairs can be a simple, linear rule, or can be related to the sum of numerals, or to the formal properties of numbers. They should be able to represent in the Cartesian coordinate system specific points defined by coherent data pairs.

In the most typical cases of number triads it is about numbers of basic computation and the results of doing operations. For example in a subtraction operation three numeral data can be found the place of which cannot be changed compared to the operational symbols. We can arrange in a "machine-game" type table the thus relating number triads.

In grades 1-2 the typical examples of relations and functions include the continuation, addition of sequences about which the rule was determined.

The elements of sequences can be

- simple geometrical forms, for example, ◻◆❍◻◆❍◻◆…
- numbers, for example, 1 3 5 7 …
- symbols from different content areas, for example, a á b …

By the end of the 2nd grade the pupils have to be able to recognize the rule of quotient sequences within the 10x10 multiplication table.

The table arrangement is the typical form of problems built of the relations between data pairs, where we expect the continuation of the table after the recognition of the rule. Similarly to the sequences the data pairs also can have mathematical content or they can be connected to other symbol systems, and within the mathematical content geometrical and arithmetical phenomena occur typically.

Continue filling in the table.

□	❍	◇	▢	
■	●	◆		◗

5	11	3	4	14
3	9	1		

g	t		c	
gy	ty	ly	cs	zs

By the end of the 2nd grade, symbols representing data series have to appear in the table arrangement of data pairs (for example, the symbol of one data series is △, of the other is □) and the rule should be formulated by abstract symbols.

What can be the rule in the following table? What should be done with number in row of △ so that we get the numbers below in the row of □?

△	3	4	6	7
□	8	10	14	

The solution which can be expected from the pupil can be the following: „I add one to the number in row △ and I multiply this number by two and thus I get the number in row □." or „I take the double of the number in row △, then I add 2 to this number and I get the number in row □."

Geometry

One of the characteristics of teaching geometry is the learning-by-doing activities in grades 1-2. The experiences and knowledge gained during the great variety of activities give the basis to the conceptual building work in the lower grades and in the later years. In this age the priority of activities with spatial forms is evident, since taking the forms in hand, touching them, feeling things in general by hand belong to the first experiences of getting acquainted with the surrounding world. For this reason the constructions composing one pillar of the geometrical requirements begin with the three-dimensional (spatial) forms. Children of preschool age are already able to select from the toys the one which we ask from them by the often mentioned and heard words (for example, Give me the red cube.). In this phase the words, names are working as associations closely related to specific objects; the definition of cube as abstract concept is not the result of conscious school development. The main point of development – especially in the lower grades – is the active and conscious learning-by-doing approach, making the children discover through specific activities, and the use of definitions (and words) consequently. Playing games is a rightful demand and expectation of children coming from the kindergarten. All textbooks taking into account the age specificities, the psychological and mental developments offer plays, smaller competitions, humorous tasks which are by all means necessary to the healthy development.

The geometrical requirements can basically be put into four big groups: constructions, transformations, orientation and measurement.

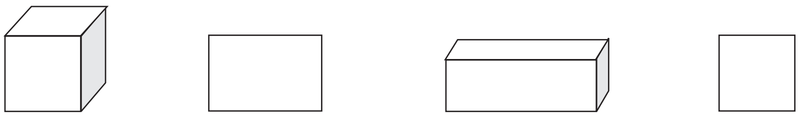
Constructions

Spatial and two-dimensional constructs and the studying of their properties are in the focus of this partial area of geometry.

We mainly examine the formal qualities of free works then of works connected to certain conditions and we lay the basis of the development of definitions. The series of varied, manipulative activities – cuttings, folding, gluing, copying to transparent paper, colouring, working with objects, drawings, building of cubes by adding new cubes or taking away cubes – mean the knowing and recognition of the properties of the finished forms. Children recognize identical and different characteristics and they formulate them in words with their vocabulary. The pupils will be able to identify the forms and to differentiate them based on the view or geometrical properties of forms; they are able to separate forms – setting simple, specific conditions – based on the geometrical properties.

On the basis of the general view they recognize the cube, cuboid, square, and rectangle. The filling with content of the geometrical system of symbols, the comprehension of relations develops gradually.

Name the forms on the picture.



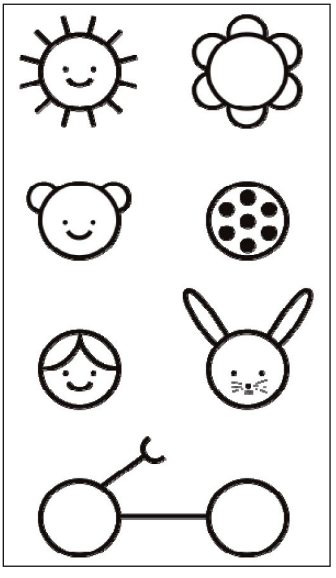
....

The more detailed discussion of the properties of bodies and plane surfaces (here squares, rectangles play greater role) generally comes in the second school year, where the conscious use of curved line, straight line, closing (closed) line, corner, sheet and edge, plus in given cases their numerical determination give help in the characterization of forms. After the detailed observation of forms we can build different bodies in simpler cases of specified layouts, and we can try the construction forms from the well-prepared silhouettes (projections).

1. *Put the cards in front of you containing a circle into separate groups.*
2. *Put those letter cards where the letters are only consisting of straight lines into different groups.*
3. *Take the box of colour rods. Put the red rod in the middle of the table, below that the smaller ones.* (Since the box contains several rods of every colour, we can give an additional instruction: „It is enough to put only one rod of one colour.")
4. *Build a decorative fence by using the colour rods. A castle with four towers. etc.*
5. *Build of colour rods the body on the table.* (First make cubes, rectangles and simple forms of them.)

We should choose tasks which create cheerful, playful atmosphere and at the same time serve a lot of different improvements.

1. *Supplement the big circle on the sheet freely so that we see a rabbit head. Colour it in.* (After visual checking we appreciate the ideas.)
2. *The circle can only be complemented by triangles so that at the end we can see a cat head.*
3. *You see circles on the paper in front of you. Supplement all of them freely.*



At the end of the work put the drawings on the big board and talk about them:

- What is common, what is different on the drawings?
- On how many pictures is there animal figure?
- Who made what of the second circle of the first row? And of the first circle of the third row?
- They should put questions in connection with the small exhibition.
- They tell true and false statements, opinions about the drawings.
- Which one do they like the best? Why?
- They have to try to find out what is shown on the different drawings.

The activities, constructions, observations similar to the above written make the pupils able to formulate short word problems themselves and to ask meaningful questions. The can put the same geometrical content into different „text robe”, too.

Children can develop in many different fields simultaneously, if the work is well organized. It may happen, however that there is no parallelism between the development of mental and verbal abilities. We may think that the child is not answering because he/she does not know the answer, whereas he/she is not fast enough in the formulation, or his/her vocabulary is not wide enough and cannot find the proper words to give an answer. The word problems of mathematics and within this of geometry – be they only simple thoughts which can be expressed by some words – are efficient means of the development of comprehensive, explanatory reading, understanding and creating of texts.

As a result of the development of acceptable self-confidence and language environment the children will be able to use the mathematical vocabulary correctly and to make accurate and distinct verbal formulations.

Build the following two buildings with the white cubes of the colour rod set or with sugar cubes. You can see their floor plan here.

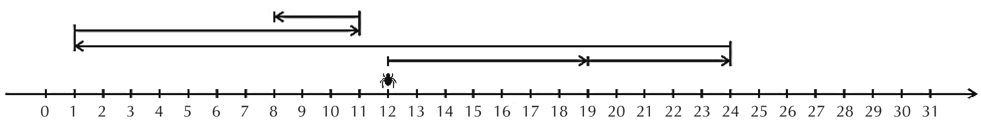
3	2	1
2	2	1
1	1	1

1	2	3
	2	3
		3

The numbers show how many small cubes should be put on top of each other.

Tools needed to the following task: a number line where whole numbers are indicated from 0 to 30.

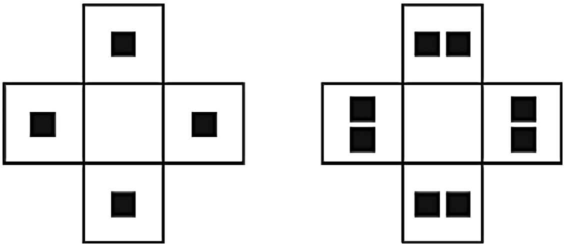
A flea is sitting on point 12 of the number line. Then suddenly, he starts jumping. First he jumps 7 units to the right, then again 5 units to the right and from here 23 units to the left, then again 10 units to the right, finally 3 units to the left. By now, how many units is he from the starting point?



The following can be a task for group-work:

Four children sit down on four sides of the table. They construct a collective building on the wrapping paper spread out on the table according to the given conditions. The building can be made of sugar cubes, white rods.

We draw a cross form of five big and congruent squares on a wrapping paper. The square in the middle remains empty, and on the four „protruding" squares we draw congruent plane squares in simple arrangement, for example as seen on the following figures.



Is it possible to create a construction which shows from four different directions exactly the view presented by the small black squares?

Solution:

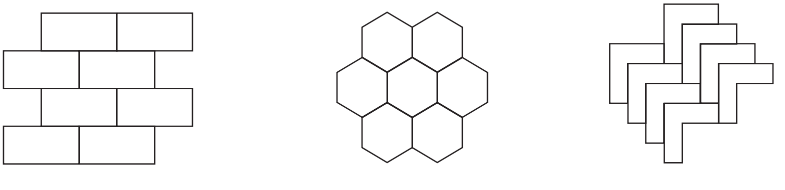
1. *Where only one-one small black square is drawn in each of the four bigger squares, the construction is made by only one cube placed well in the middle.*
2. *Where all the four projections are two squares next to each other the construction of several different forms gives a good solution. The diagonal placing of two cubes and the good placing of 3 and 4 cubes are equally good solutions.*

The children in general put four cubes in the middle, in this case we suggest them to try to take away from them so that their side view should not change.

Already at this age we can give tasks which have several possible solutions and the learners should become able to find all the good solutions. Here we always think of simple tasks corresponding to the children's age.

In the second grade the geometrical activity covers the enclosing of different plane forms by thread, string, the complete (tight and one layer) covering of planes by different units. *These activities serve the practical preparation of the definitions of perimeter, area.*

Such coverings are the following:

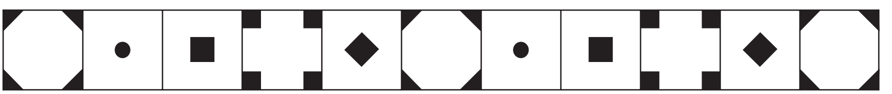


Transformations

Transformations include activities of moving the different drawings and shapes (reflection, translation, turning) and of the realization of the directions when the drawings or shapes move.

During the movement of the two- and three-dimensional drawings, in the course of moving them on different grids we can observe how the properties of the original and newly produced figures change compared to each other. Are there heritable (invariable) formal and dimensional properties? For example we enclose two adjacent squares on a quadratic grid, we copy this to another paper, cut it out precisely and move the received rectangle in a way that we determine a direction with the controlled connection of two grid points and we shift somewhat in this direction the cut rectangle. Or we glue the rectangle to a straw and mark a grid point around which we rotate the rectangle, or we fold the paper along the straight line connecting two grid points and mark the place of the reflection of the rectangle.

The geometrical transformation problems improve the creative fantasy, creativity, wit, aesthetical sense. We can produce beautiful line patters by reflection, translation. At this age children are already able to tell the difference between the reflected image and the shifted image based on the general view. It is very easy to make beautiful patterns by folding the small paper napkins precisely, make cutouts and cut ins in them. For example after folding the napkin twice we can cut out one of the corners, open it and the children can see the produced pattern. Encourage them to make different patterns. If we cut several napkins folded together, we can put on the board periodically changing their sequence (for example, the period has 5 elements) and we can ask a lot of questions, formulate many different word problems in connection with the beautiful view. (The black parts indicate the missing paper.)



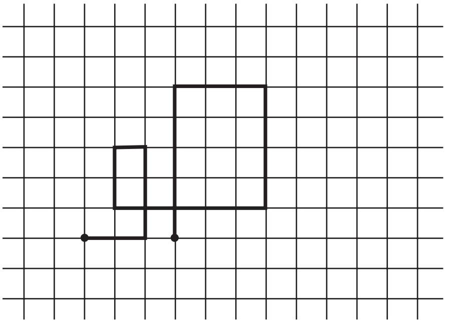
1. *Which pattern contains the least amount of paper?*
2. *How many holes are there on paper napkin number 5?*
3. *Do napkins number 3 and 7 have the same pattern? And of napkins numbers 3 and 8?*
4. *If we continued the pattern in the same way what type of patterns would napkins no. 20, 30 and 100 have?*

Orientation

The expression of position relations, showing the directions, finding places characterized by data will significantly contribute to the improvement of spatial orientation of the children, to the development of their correct orientation in space and plane. The elementary empirical level preparation of the definition of number line and later of the coordinate system (comparing the positions first by using terms like ahead, backward, under, above, next to, behind, farther, closer, between the two, by two to the right, by three to the left, and later expressing the relations by numbers, etc.) is supported by a series of activities related to this subject.

In the example below everybody has a squared paper, where the four cardinal points are indicated in the direction of the grid lines:

I start towards East from any grid point of a squared paper. I always follow the grid line and turn at the grid point. I always make a turn to the left. The length unit is the square side. The lengths of the routes taken are the following in sequence: 2, 3, 1, 2, 5, 4, 3, 5.
Draw my route.
How many units is the distance between the starting point and the end point of the route?



Solution: the starting point and the end point of the route is at 3 units distance from each other.

Measurements

The field of measurements consists of activities, requirements aiming at the examination of measurable geometrical properties.

The characteristics of spatial and plane objects contain – in addition to the formal properties – quantitative data which can be expressed by numbers. This group of activities and requirements can be connected to other fields of mathematics, too, for example it contributes to the development and strengthening of the number and operation concepts. Measuring length, perimeter, area, mass, volume, time should be made by many different optionally selected and by some standard units (for example, meter, kilogram, liter), the period of time, hour, day, week should be used correctly embedded in many different situations. Experiences gained during the well planned activities make possible the discovery of the relations between the units, quantity, index number, the empirical preparation of proportional changes. *By the end of grades 1-2 it can be expected that the pupil be proficient in practical measurements made with ad hoc units, should know the standard units and use them in practice.*

Fill in the missing numbers on the places of dots.

1 dm = ... cm	*6 cm + 2 dm = ... cm*
3 cm + 1 dm = ... cm	*2 dm – 15 cm = ... cm*
2 dm – 7 cm = ... cm	*.... cm + 1 dm = 12 cm*

The teaching of the four content domains within geometry requires a lot of tools. *During the assessment process, in addition to the written tests we also have to use manipulative and activity tasks in order to evaluate their practical solution, implementation.*

For the development and application of geometrical terms there are a lot of possibilities outside the school lessons, too. These are smaller tasks, projects which can be performed by the cooperation of the family, or friends setting shorter-longer deadlines and which in addition to extending the geometrical knowledge contribute to the improvement of division of labour between the member, to gaining shared feeling of achievement and to development of the other components of social competence.

In the field of measurement we can make important steps be putting into the children's hand the meter rule, the measuring tape, the cubic measures, twin-pan balance with the proper weights and make a lot of measurements. We can prepare for example a bag into which we can put sand, gravel, bean, corn, wheat, small fruits, sandwiches in order to compare them, measure their weight. The unit can be a lot of things. Measure the time needed for the completion of different activities. We propose the use of round face clock supplied with 1-12 number and well recognizable hands.

The practical knowledge, use in specific tasks of standard units (m, dm, cm, kg, dkg; l, dl; hour, minute, day, week, month, year) is a requirement. This does not exclude the use of occasional units.

Examples:

1. *Kati measured the ribbons bought to the packaging of Christmas presents. Of the golden colour there was 15 meter, of the silver 100 decimeter, of the green 250 centimeter. Kati also bought red ribbon thus she exactly had the 35 m ribbon needed to packaging. How many decimeters is the red ribbon?*
2. *Arriving at the forest clearing, mother took out a bottle of juice. Each of her five children drank two full glasses of juice and the bottle became empty. They recognized only then that Dani's glass was twice as large as the identical glasses of the other four children. How many small glasses of juice were in the bottle?*
3. Measure the distance between two trees with different length units. Write down the index number and the unit. Based on the experiences make the following open sentence true: the smaller unit I measure the same distance, the bigger will be the ....
4. *Fill up several plastic boxes of the same size with different materials. You can put sand, gravel, nails, lens, flour, etc. into them. With the help of a bascule compare their mass and put the boxes into increasing order on the basis of their masses. By writing down the order check their correctness by measuring with standard units.*

Put two questions during the measurements: What did you measure? What did you measure with? Can you show with your two hands how long 1 meter is? And 3 decimeter? And 5 cm? Can you put half kilo gravel on my plate? And 30 dekagram sand on this? Etc. And check every estimation by measuring and discussing the experiences. In this way we can achieve that the estimation skill of the pupil improves and that they become able to recognize the relation of units of different sizes.

The main point of measurement is comparison. At the beginning it is indifferent when, what and with what is the comparison made. For example we can use a piece of string, lath, the length of the step of pupils, the distance between two fingers of our stretched palm, etc. to measure length. When measuring volume we can use a cup or a paper glass of water, sand, bean, etc. We should select these so-called occasional units together with the pupils and should use in the task consistently.

Organize a competition consisting of 5-10 measurements. Before starting measurement estimate the probable result. Make a four column table on a sheet of paper where in the first column the measurement task, next to it the estimated result should be entered by the children, in the third column the actually measured value, while in the 4th column the difference of the estimated and measured values shall be written. The more times we organize such tasks the greater the probability is that the difference between the estimated and measured results will be small.

1. Select the odd one out.

a) cm m kg dm

b) minute year month dm hour

2. Let's play Twenty Questions game. (For example: the "logical set"[3]*, or the cards with different figures are in front of the children, one of them think of one of the papers or cards, the others ask questions about their characteristics (for example, is it hollow, not a triangle, red, not small, has a peak) and only yes or no answers can be given. After each answer everybody „screens" his/her own cards separately, that is they keep only those cards which can be a possible solution. The winner is the pupil who finds first the correct figure.)*

3 The "logical set" [logikai készlet] is a widely known and used set of coloured plastic plates with three types of shapes (circle, triangle, square), and half of the plates are holed.

Combinatorics, Probability Calculation, Statistics

During the development of combinatorial reasoning the teachers generally keep in mind the following stages:

a) Production of one of some cases meeting a given condition;
b) Production of as many as possible cases according to the given condition;
c) Finding all cases, ordering of the found cases and replacement of the shortages of the system;
d) Building of a system to the finding of cases of the given condition.

Of the above-listed four requirements a) and b) can be the means of developing induction reasoning. Assessing mathematical knowledge in a disciplinary sense we can mainly formulate c) and d) type requirements.

A typical problem in mathematical tests is the following:

How many two-digit numbers can you make of these number cards? Make all of them.

3 **4** **6** **2**

An equivalent formulation of the problem:

How many monogram can you make of these letter cards?

A **B** **C** **D**

We have to give assistance to the solution of word problems to our pupils so that they could find the model describing the problem well.

We present the following example of a combinatorics problem which at the same time measures students' systematization ability:

Funny Ferko has mixed the magnetic letters in the label "2. a osztály" (class 2.a).
This is what he made:

2. o asztály

How many letters are on the wrong place?

In connection with the next problem we show that rather different solution strategies may lead to the right solution. The equivalent use of the manipulative, image and conceptual level solution possibilities confirms the links between everyday experiences and mathematical concepts.

Anna, Béla and Cili organized a running competition. They passed the finish line in different points of time. How many ways could they pass the finish line?

One possible solution of the task is that the teacher invites 3 children in front of the class. Children sitting on their places instruct those standing out. They determine the possible orders since it is difficult to keep in head the cases already considered, it is obvious that they have to model the problem. For example in a way that they write the names on a piece of paper (the same name can be written on several pieces of paper) and solve the task by putting these paper here and there. Evidently such problems cannot be solved in written tests.

In the case of problems formulated during the assessment care should be taken that we present a good model with the help of which the problem will be understandable. We can start the writing down of cases what the children have to continue. In this way the task is limited to writing down as many cases as possible where the accent is on keeping the criteria.

Anna, Béla and Cili organized a running competition. They passed the finish line at different points of time. How many ways could they pass the finish line?
Continue the options.

A,B,C; A,C,B; B,A,C; _____; _____; _____;

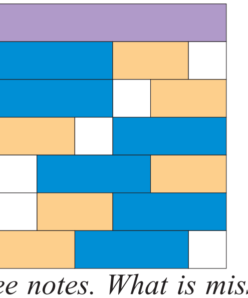
The other option is that we determine a system (for example a table), which makes the problem more clear-cut.

Anna, Béla and Cili organized running competition. They passed the finish line in different points of time. In how many ways could they pass the finish line?
Continue writing the options.

1.	A					
2.	B					
3.	C					

Already at this stage preparations are made for building up a system leading to finding all the cases belonging to the given conditions. For example examining the possible arrangements of the 3 elements it was observed in the classroom in how many ways the three stripe flag can be coloured using the red, white and green colours. Other examples:

1. In how many ways can the lilac rod be decomposed using different rods?



2. I prepared tunes of three notes. What is missing from the sequence?

do-mi-sol	**mi-do-sol**	**sol-mi-do**
do-sol-mi	**mi-sol-do**	

The task is getting more difficult when we increase the information noise.

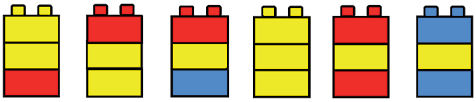
*3. Wh*at can be the last line of the „poem”?

Tirim taram turum
Tirim turum taram
Taram turum tirim
Taram tirim turum
Turum tirim taram
……………………

The structure of the flag colouring and the above three examples are the same, but their content is very different. In this age, however the structure is important for only very few children, that is why the different formulation of the same problem means a new challenge.

In the probability topic the separation of sure and not sure becomes important in the first two years of schooling. The problems formulated on the worksheets are preceded by many-many experiences.

I have built the following towers of red, yellow and blue Lego blocks. I have picked out one and made statements about the selected tower. Decide if the statement is for sure true.



	Surely true	Not sure that it's true
There is red block in it		
The middle element is yellow		
All three colours can be found in it		
There is no blue in it		
There are two identical elements		

Since children collected many everyday experiences about impossible events, we can make a try to ask about this difficult definition.

We put 5 red and 1 blue balls into a bag. Then we picked out two and made statements. Underline the statements which you think are false.

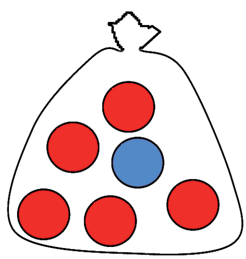
All of them are red

All of them are blue

There is blue among them

There is no red among them

There is blue among them



Detailed Assessment Frameworks of Grades 3-4

Numbers, Operations, Algebra

Numbers, set of numbers

Based on the reality content of numbers we extend the number concept up to 1000 in grade 3 and up to 10 000 in grade 4. Counting in the set of three and four-digit numbers has an important role, furthermore the estimation of piece numbers and measurement index numbers, the counting with approximation, and measuring with given precision with occasional and standard units becomes important in this age group. In the course of measuring practices the children will be able to define the relations expressed by measurements with different units, they will understand the conversion by units.

By using the different teaching tools the children acquire the command of numeral systems, gain experiences about grouping, conversion and exchange. The practical knowledge of the essential understanding of the decimal system and of the place value system makes for them the writing and reading of numbers safe, they will understand the system of numerals. Children are able to use reliably the formal, local and real values of numerals. They examine the numbers according to the familiar number properties or number relations (for example, parity, neighbouring numbers) and they get to know new number properties (for example, divisibility, number values rounded to tens, hundreds, thousands).

Circle the odd numbers.

1 2 4 5 6 8

Circle the neighbours of number two.

0 1 2 3 4 5

They recognize and are able to express the numbers in their different forms, they can judge the size of numbers and are able to put the numbers into increasing and decreasing order according to their size. They can place numbers on the number tables and on number lines with different scaling.

The children become acquainted with the concept of negative numbers in two interpretations. On the one hand negative numbers are interpreted as index numbers of vectored quantities (temperature, displacement, turning, time), on the other hand as deficits. To this end debt and asset cards are used. The numbers are compared by attaching specific content. The many different forms of the numbers are produced. They experience through activities that adding something does not always result increase of value, but taking away may result increase.

Word problems are often used to the assessment of computation ability. These problems which can be solved by one operation do not require data collection, they can be written down by a simple number problem and can be solved.

For example:
I had 750 Forint in my purse. I spent 480 Forint. How much money is left for me?
A bus ticket costs 320 Forint. What is the price of five bus tickets?

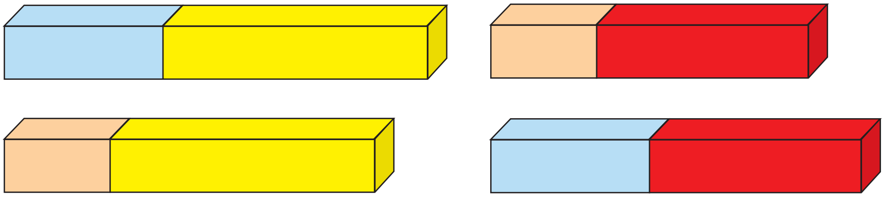
In the third grade children in many cases are computing with approximate values in the 1000 and in the fourth grade in the 10 000 number circle, the questions formulated by the problems also demand this.

For example:
Kati paid with a 1000 Forint banknote in the stationary shop. The cash-register showed 578 Ft.
How much is the return rounded to hundreds?

Operations

In the third and fourth grades the interpretation of the operations by objects, drawings, more abstracts figures and texts is necessary in the extended number circle, too. We pay special attention to the interpretation of operations with approximate numbers. The two-direction activities contribute to the understanding of mathematical models. On the one hand children read operations from displays, pictures, figures, on the other they collect examples to the given mathematical model, and they formulate problems. The interpretation of the addition, subtraction of bigger numbers is assisted by representation with segments or areas. This can be prepared by the use of colour rods.

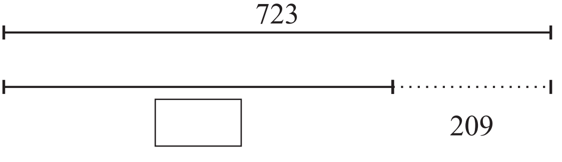
For example:
Let the white cube be worth 100. Which arrangement is close to the amount of 246 + 467?



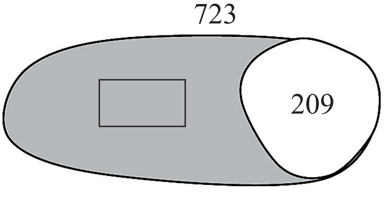
The settings, the readings about the settings can be followed by the use of segments, or areas. They are suitable for representing the numbers with approximations, for presenting the relations between the numbers.

For example:
One of the numbers is 723. This is bigger than the other by 209. What is the other number?

With segments:



With areas:

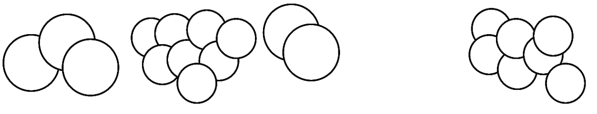


The verbal computation operations are made on the basis of analogies in the extended number circle. Their understanding is supported well by the use of tokens. The activities make the verbal computation ability of the children safe in the scope of round numbers. The computation procedures acquired by the children in the 100 system in the 2nd grade will be followed in the 3rd grade by round hundreds and tens in the 1000 number system, then in grade 4 by round thousands and hundreds in number system 10 000. In the computations the children use simplification procedures the basis of which is the unchangingness of the sum or of the difference. They can gain experiences about them by activities what they use during the computations.

How much is 380 + 270?

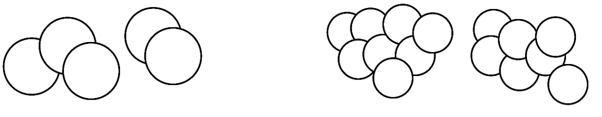
Presented by tokens:

Method 1: by division of the 2nd member:



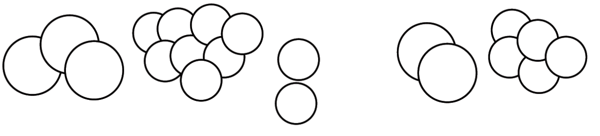
(380 + 200) + 70 = 580 + 70 = 650

Method 2: By adding the hundreds and tens:



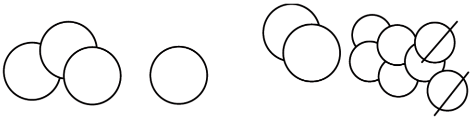
(300 + 200) + (80 + 70) = 500 + 150 = 650

Method 3: By placing from one member to the other:



(380 + 20) + (270 – 20) =400 + 250 = 650

Method 4: By increasing one member and decreasing the sum:



(400 + 270) – 20 = 670 – 20 = 650

The approximate calculations made with the rounded values will be necessary during the projection of the results of operations in writing.

The algorithms of written operations, the methods of checking the results of computations also build on the properties and relations of the operations. This also makes necessary the knowledge, purposeful use of the interchangeability or grouping of members, factors.

For example, in class 3, when the children have not yet learned multiplication in writing by two-digit numbers they are able to calculate the result of 26·24 by using multiplication in writing by one digit numbers. Some calculation options: (26 · 8) · 3 = (26 · 6) · 4 = (26 · 3) · 2 · 2 · 2. In grades 3-4 word problems get special importance in the development of problem solving ability. These are mostly complex tasks which cannot be solved directly. It is advisable to get to the solution of the problem by keeping some appropriate steps. The recognition of the problem is followed by its interpretation, by putting down the data and understanding their context. The children use many different models for the description of the relations between the known and unknown data. A number problem containing several operations can be a model for example. It is practical to use parenthesis in these descriptions even if you use them only for the indication of the coherence of data.

For example:

Peter's family organized a three days excursion by car. On the first day they travelled 160 km, on the second day 80 km more. On the third day by twice as much as on the first day. How many kilometers did Peter's family travel during three days?

Description of the relations of the problem by numbers:

160 + (160 + 80) + (160 · 2) =

Algebra

In addition to the word problems containing numerals, the open sentences appear and get greater emphasis. Continuing the activities began in grades 1 and 2 the finding of elements making the open sentences true or false is made by trial and error method, but the children also use the method of planned trial and error on finite basic sets in order to find the solution. Children are able to find (in case of simpler relations to create themselves) to the relations formulated between the known and unknown data the correct answer of the specified open sentences in the given situation.

I have thought of a number. I have subtracted 8 times of this number from 800 and received 12 times the number I thought of. What number did I think of?

With open sentence: $800 - \square \cdot 8 = \square \cdot 12$

The relations of problems – especially of the problems where the key words may be in contradiction with the arithmetic operation needed to be executed – are often written down in open sentences. For children of age group 8–10 it is often easier to recognize and write down operations to which the text refer, than reformulate it as inverse operation.

For example:
Csabi's school has 12 grades. The school has 160 pupils in the first four grades. These classes have twice as many children as the high school classes (grades 9 through 12). The number of children in the fist four grades is 40 more than the number of students in grades 5 through 8 combined. How many children study in the grades 5 through 8 and how many in the high school classes in Csabi's school?

We mark the number of high school children by: $\square$

The number of senior class children by: ∇

Using these symbols we can easily create the open sentences describing the problem:

$\square \cdot 2 = 160$ $\qquad$ $\nabla + 40 = 160$

The solution of the word problems can be assisted by the use of sequence, tables, simplifying drawings or diagrams even in the case that the problem has only one solution.

For example, the solution of this problem can be found by the children by filling a table, too:

I have only 20 and 50 Forint coins in my purse, 12 coins in total. The value of the coins amount to 360 Ft. How many 20 and 50 forint coins do I have in my purse?

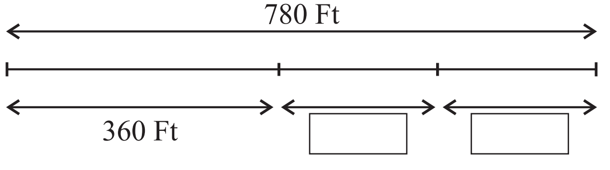
We can make to following table:

No. of 20 Ft coins	2	4	5	6	7	8
No. of 50 Ft coins	10	8	7	6	5	4
Value of 20 Ft coins	40	80	100	120	140	160
Value of 50 Ft coins	500	400	350	300	250	200
Total value of coins	540	480	450	420	390	**360**

A simplified drawing contributes to the solution of this problem:

In the stationary shop I bought two identical exercise-books and a pen and paid a total of 780 Forint. The price of the pen was 360 Forint. How much was one exercise-book?

The segment drawing can help to find the solution:



In the selected mathematical model of the problem the computations are followed by their checking. Checking can be made by comparison with the preliminary estimation, by inverse operation and we can use pocket calculator, too. If we select the inverse operation for checking this may confirm the relations between the operations.

Relations, Functions

The most important fields of development in the domain of relations, functions in grades 3-4 are:

- comparison, identification, ability of differentiation, observation;
- abilities of selection, sorting, systematizing and highlighting the importance;
- collection, recording, sorting of data;
- abstraction and materialization abilities;
- recognition of correlations, discovery of casual and other relationships, recognition, following of analogies;
- expression of experiences in different ways (by presenting, drawing, sorting of data, collection of examples, counter-examples, etc.), formulation by own vocabulary, in simpler cases by using mathematical language of symbol system.

Children are able to formulate in the language of mathematics the recognized relations, to express them by words, symbols, rules (in the case of function by arrow, in case of relations by open sentence). They are able to continue the commenced pairing according to the specified or recognized relation.

In grades 3-4, a new element in the treating of correlating data pairs is the graphical representation of relations in the Cartesian coordinate system. Since during the representation of data pairs the order of the members of the data pair is important, it is advisable to make exercises where we represent in a shared coordinate system data pairs produced by the exchange of the prefixes and suffixes.

The learners are able to arrange data, numbers in sequence according to their content or size, to formulate guesses as to the continuation. The express the recognized correlation by the continuation of the sequence or by words. They can continue the sequence on the basis of the formulated rule, they are able to check the compliance of the rule and the data. They look for different rules to the sequence started by some elements.

What can be the rule in the following table? What shall we do with numbers in the row of symbol △ in order to get the number in the row of symbol □?

△	3	4	6	7
□	8	10	14	

The solution expectable from the learner can be the following: „I add one to the number in row △, then I multiply this number by two and get the number in row □." In connection with this table it is possible to formulate a closed problem, where we ask the children to select the rule matching with the Table's data.

What can be the rule in the following table? Encircle the letter of the relation which is true for the table, and cross the one which is not true.

△	3	4	6	7
□	8	10	14	

a) □ = (△ + 1) · 2
b) □ = (△ – 1) · 2
c) □ = (△ + 2) + 3
d) □ = △ · 2 + 2

Geometry

In the field of geometry in grades 3 and 4 the same four sub-domains give the frameworks as in grades 1 and 2. The (1)constructions, (2) transformations, (3) orientations and (4) measurement domains cover all the learning objectives that we define in these grades in the field of geometry.

Construction

Similarly to grades 1-2 the requirements contain here, too the recognition and construction of cuboids, cube, rectangle and square. The learners learn the definitions of edge, base and lateral face.

The learners learn the expression of body mesh, specifically the typical two-dimensional nets of cuboids and cube.

Of the geometrical properties – during the practical activities – they learn the following terms: form, vicinity, direction, parallelism, perpendicularity.

The learner will become able to group bodies and plane figures on the basis of certain geometrical properties. Other typical characteristics observed while grouping objects: angularity, holedness, symmetry, identity and difference of dimensions.

The concept of reflection (symmetry) can be improved on the one hand by paper folding activities, on the other by building the reflected image of spatial forms.

Besides the priority of spatial forms the activities with plane drawings also get greater emphasis. Students will become able to copy bodies and plane figures, to create the reflective image of a plane figure or a body. The copying is primarily made by bodies, rods which can be taken in hand, but in grades 3-4 we increasingly utilize the abstraction possibilities offered by drawing.

The learners are able to use the compasses and the ruler. The basic level use of the compasses is implemented for example when the learner takes a 5 cm distance into the span of the compasses.

Transformations

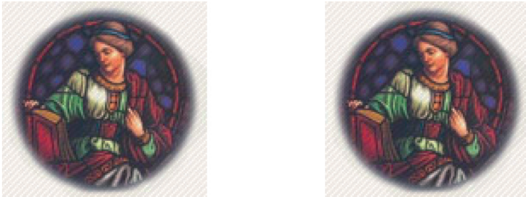
Built on the experiences acquired in grades 1-2 the manipulative and pictorial level components of the concept of congruence and similarity are developed. Students are able to recognize if two figures or their images are congruent or similar. They can confirm the identity or difference of formal features. In the case of the difference of figures they can formulate by words the type of difference (for example, longer, more oblique).

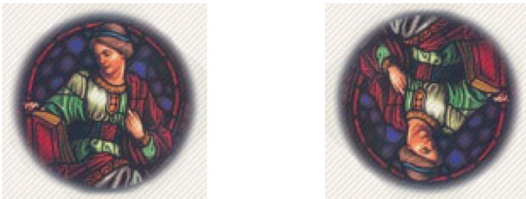
In the case of 3D shapes they are able to reduce or enlarge forms of the elements of the original body, on case of plane shapes with the help of the quadratic grid. Students can reflect plane figures along axis and rotate them with the help of a copying paper.

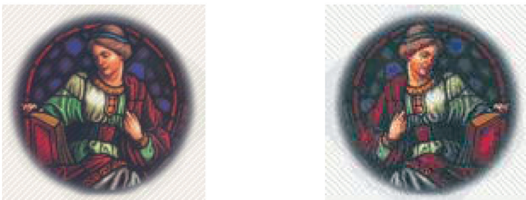
They can make difference between figures produced by translation and by reflection along axis, even in case of complex forms.

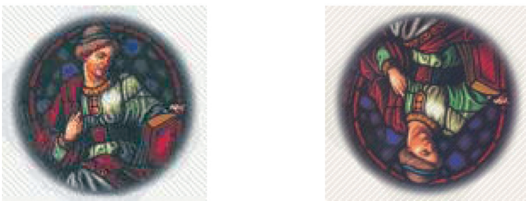
The next task evaluates the making of difference between reflection along axis and translation. The content of the problems is basically optional, there is no reference to (and there is no need for) the use everyday experiences.

In this example you have to decide about two matching figures if they can be transferred into each other by reflection along the axis or by translation. Write the letter of the figures in the corresponding row.

a) 

b) 

c) 

d) 

Can be transferred into each other by reflection along the axis: ....
Can be transferred by translation: ....

Orientation

We have to mention that the major part of orientation in the geometrical sense is related to the orientation recognized as a category of geographical discipline. The connection of the two fields can be interpreted in a way that the development of orientation abilities is made in a different context. The orientation ability developed in mathematical context prepares the learning of the coordinate system as a universal mathematical means, during which we use definitions known from the everyday life. As we have indicated at the requirements of grades 1-2 the two, independent data typical in the case of the use of the plane coordinate system used as arranged data pairs give the basis of the orientation in the everyday meanings.

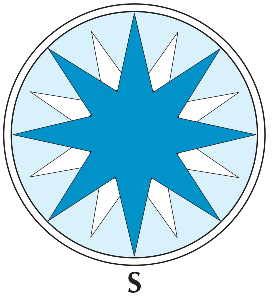
Orientation starts from the experiences collected during motions in the three dimensional environment. Learners of grades 3-4 are able to orientate on the basis of one, two or three data. Orientation on the basis of three data, which represents the mathematical model of spatial orientation is in practical life many times replaced by orientation on the basis of two data. The orientation ability of the learners include that they are able to receive and understand the relating information (for example, „if you step five ahead and two to the right you arrive at the destination") and they are themselves able to formulate the information needed to the orientation.

The construction of pictorial elements of orientation, for example the making of simple map drafts is discussed in another volume of this book series, in the geographical chapters of the science framework.

The next task might have even been included in the tests of the geographical or natural sciences subjects. In our opinion this does not question the validity of the task, since the context of the problem, the words of mathematics or natural sciences included in the title of the test make an influence on the performance of the learner. We consider it desirable that both the imaginary and verbal knowledge system creating the basis of orientation develop on good level both in mathematical and in other context.

On the figure you can see a compass rose where the four main cardinal directions are indicated. We go from the middle of the circle to the north. We turn back and go to the middle of the circle. Which cardinal direction is to the right from us in this case?

N

W  E

S

Measurement

Measurement is included in the subject of geometry in the Hungarian mathematical didactical traditions, while in the American „Principles and Standards for School Mathematics" which is regarded as an important reference basis for us, measurement appears as a separate chapter. The reason for this can partly be found in the different cultural traditions (for example, differences in the use of the metric system), it partly expresses our approach which considers measurement as an activity related to the well-known geometrical shapes. Since according to a much more general approach, which in the world of sciences is widely accepted, measurement is defined as the assignment of numbers to objects, events, properties according to a set of rules. Although there are efforts that this latter, general approach to the measurement of geometrical shapes also enter into the school (requirements of making measurements with the so-called „occasional units" in grades 1-2), the school practice is still characterized by the fast switch-over to the standard units, then by the immersion in the arithmetic operations of conversion of units.

In grades 3-4 the students should know the definitions of unit, quantity and index number. During the measurement activities the measurement of perimeter is made by enclosure, the measurement of area by overlapping, and the measurement of volume by occasional units („small cubes"). The subjects of perimeter, area and volume measurements should be rectangle in the case of plane shapes and cuboid in the case of bodies.

The learners should know the following units of measurements: mm, cm, dm, m, km, hl, l, dl, cl, ml, t, kg, g. They have to be able to convert into each other the "neighboring" units. The conversion should mainly be connected to practical activities, that is after the measurement made by one of the units we make a repetition measurement with the adjacent unit. In order to measure the time they have to know the hour, minute and second and to convert the neighbouring units into each other.

The greatest part of the practical exercises in connection with measurement is related to the conversion of units.

How many deciliters of milk was consumed today if we drank three one liter bottles of milk?
If the step of a child of grade 4 is 60 centimeters, how many steps does he need to make 12 meter?

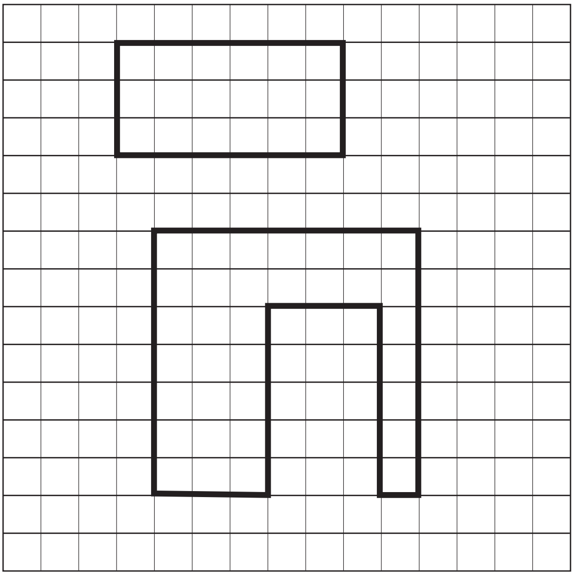
In the radio, 5 minute long musical pieces were played for two hours. How many musical pieces were played during this time?
In England, mile is often used to measure distances. One mile is equal to 1 km plus 609 meters. How many meters is one mile?
The weight of a small box of butter is 100 gram. How many boxes should we buy if we want to buy 3 kg?

In addition to the conversion of units we can formulate simple perimeter and area calculation problems as world problems. From the manipulative activities of the children we get to the image level problem solution with the following tasks.

On the square grid table a small square means one unit area. Calculate the area of both plane figures combined framed by the bold line.

a)

b)



Area of plane figure *a)* is: ________

Area of plane figure *b)* is: ________

Combinatorics, Probability Calculation, Statistics

The teaching of combinatorics, probability calculation and statistics mainly aims at gaining of experiences also in grades 3-4.

Students' combinatorial reasoning is further on mainly shaped by making the importance of systematization understood. In grades 1-2 the children are not primarily interested in how many different possibilities are there, since it is the process of finding and producing different options that is important for them. In the case of production of a set of small number of elements the endeavour for completeness is already a realistic requirement in this age group. We have to continue to assist the children in finding an ordering principle, since this is important for finding all cases. We can give up providing stronghold to the commencement of the task in the case of very low number of elements.

The aim of the probability games in the classroom is to present that things which more often happened are more probable. In this case the teacher is a real participant of the experimentation of the learners and hopes that the outcome of the game will bring the „expected" result. In these years it can also be observed during the analysis of the games, that it can be considered more probable what may occur in different forms (even if this is not confirmed by the actual experimental data). Thus in the course of assessment the intuitive determination of smaller, or bigger probability is a requirement.

In all probability the terms "sure", "not sure", "probable", "possible" were built into the children's vocabulary during grades 1-2. The curriculum requirements formulate the separation of deterministic (sure or impossible) and non-deterministic (possible) events. Thus we can certainly ask only indirectly to what extent they consider the given event probable.

Observation, collection, recording, ordering of data appear in the curriculum requirements, too, which helps, besides the deeper understanding of statistical subject the making probability decisions.

In general the probability activities and tasks are not independent development targets, but are connected to other fields (for example, computation, geometry, and combinatorics). Let's say we throw with two dices and tips should be made about the parity of the multiplication. The solution of the problem requires from the children knowledge of the number theory, perhaps their computation ability, plus their ideas about probability. These should be taken into account during the preparation of test sheets.

The finding of the missing elements of an existing complete system is a different task for the searching of all cases, for the ordering of the found cases and for the addition of the deficit sin the system.

In this task the finding of all the missing elements can be a legitimate claim, since the pre-planned systems show the solution. Plus the task improves the orientation ability in the table.

The more simplified version of the above tasks can be:

Make 3 digit numbers from numbers 5, 2 and 7. Write down all the possible solutions.

At this grade the children regard important not how many possible options there are, but finding and producing the options are interesting for them. If in the activities they recognize the already tested similarities, they made a big step towards generalization. Consequently, in the combinatorics tasks (including their correction key) not only the number of all possible options is important, but the regularities manifested in the partial solutions, or in the listing of options can also be and should be evaluated.

Certainly, there is a possibility for the transformation to word problems of the formerly played and experienced activities. We still cannot give up providing help either in the form of a table or of a commenced tree-diagram. It is still important that the main question to be answered is not in how many different ways a thing can occur, but we should ask for the production of all the possible options from the children.

For example regarding the following two tasks the offering of a table solution seems effective.

Grandma is preparing to fill up her pantry, so she bought apples (A), pears (Pe) and plums (Pl) on the market. She would like to put the fruits on three shelves. She puts one sort of fruit on one shelf. In what order can she put the fruits?

Using the initial write all the possible solutions into the table indicating the shelves.

	Options					
Shelf III	Pl					
Shelf II	Pe					
Shelf I	A					

Put together a three-course menu (soup, main course, dessert) for the Falánk family so that none of the family members get the same three courses.

Menu

Soups:
- **m**eat soup
- **f**ruit soup

Main courses:
- **s**pagetti
- **r**oast a la Brasssó
- **p**otato casserole

Dessert:
- **d**umplings somlói

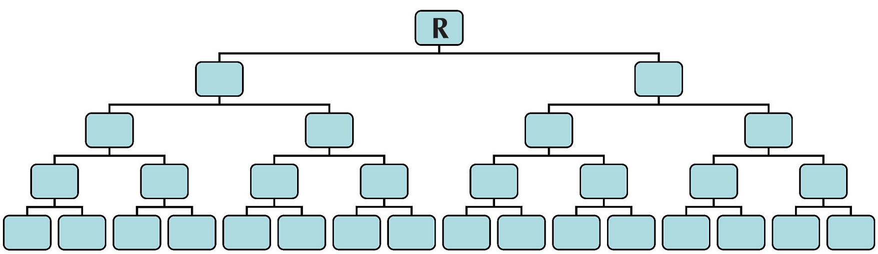
Complete the table with the initials of the foods.

Soup								
Main c.								
Dessert								

Maximum how many members can the Falánk family have if there were not two family members who ordered the same three courses?

In another case the tree diagram is of help:

Évi is threading pearls. She has yellow, red and blue pearls. She can put five pearls on a string. First she always threads the red pearl. Same colour pearls cannot be put next to each other. In how many different ways can Évi thread the pearls? Draw into the figure the initials of the colours, showing what options she has for making the row of pearls.



Detailed Assessment Frameworks of Grades 5-6

Numbers, Operations, Algebra

At the beginning of grade 5 the mathematical preparedness of the learners in this domain should be evaluated carefully. The most reliable data about the preparedness of the learners can be received if the assessment of their skill is preceded by repetitions through manifold, varied activities.

Numbers, number systems

By the end of grade six children learn the rational numbers. The extension of the number circle (wholes, fractions, decimal numbers), within this the interpretation of the negative numbers, two types of interpretation fractional numbers (for example, 2/3 may mean that we divide a whole cake into 3 equal parts and we take 2 pieces of these parts, or we take 1 third of two whole cakes – equal to the above), the learning of the definition of opposite, absolute value, examination of the properties of numbers (for example, parity, neighbouring numbers, divisional options, etc.) make the children capable to write down and read the learned numbers in the correct way, they understand and are able to use fractions, decimal numbers and negative numbers.

At these grades the learners are interpreting the concept of rounding in extended number circles and they use the rules of rounding. They learn the definition of percentage, base, interest rate. The discussion of experiences collected from other fields of literacy (for example, subjects of natural sciences) is also important because it serves the enrichment of these definitions. Similarly, the natural sciences build on the knowledge of mathematics, and through knowledge transfer the computation skill has a decisive role in the biological, chemical, physical and geographical calculations, too.

During the teaching of number systems (decimal base and binary system which is only demonstrated in grades 5-6 and is not a requirement in the curriculum) it can be made clear to what extent the placing (place value) of numbers (form value) influences the real value of the number. Here there is again an occasion for the introduction of the role of 0 as replacement of a place value. In these grades the reliable knowledge of decimal system is already a requirement.

Fill in the table based on the example.

Decimal system									Writing of numbers	Writing of numbers with words
...	10^3	10^2	10	1	1/10	1/100	1/1000	...		
	2		1			3			2010.03	Two thousand ten point three hundred
									207.8	
										Seven thousand eight hundred seven

During the studying of the properties and division possibilities of whole numbers we can learn the simpler rules of divisibility. *The knowing, and the application of divisibility in problems by 2, 5, 10, 4, 25, 100 is formulated as a requirement.* It is important to deal with parity, divisibility of 0, too. Tasks related to the number theory are able *to develop, improve the need for proving* (for example, How can we prove that divider of the odd numbers is odd?).

During the years *the many different representations, denotations of numbers also improves the combinatorial reasoning of learners* (for example, 6 = 3 + 3 (can be divided into two equal parts, therefore the whole is even) = 2 × 3 (there is 2 in the division to members, therefore the whole is even) = 4 + 2 = 7 – 1 = 4 – (–2) = etc.).

The task below also proves that the representation of numbers in different forms plays a great role in the understanding of the subtracting of negative numbers.

Operations

We extend the arithmetic operations to the growing circle of numbers, the properties of which are inherited, the operation concept is deepening, and the operational algorithms are recognized. The root of the apperception of operations with wholes, fractions, and decimal numbers of different prefixes, be they made verbally or in writing, is the correct understanding of decimal system (place value, formal value, real value).

In connection with the operations mention should be made of the role of numbers 0 and 1 in the operations. For example the following examples show the consequences of the use of 0 as a multiplier factor:

Calculate the result of the following operations.
2 · 3 · 2 · 7 · 5 · 0 · 4 · 6 = ?
Solution: *The result of the multiplication is 0, since if one factor of the product is 0 the result is 0. Use fewer factors in case of paper-and-pencil tests.*

5	**13**	**9**	**8**	**7**
0	**7**	**4**	**11**	**22**
3	**32**	**0**	**6**	**18**
27	**2**	**4**	**0**	**9**
8	**12**	**19**	**5**	**3**

We have written whole numbers in the fields of the 5 × 5 grid shown on the figure.
Draw a thick continuous line along the grid lines so that it starts from the grid point of one of the border line of the quadratic grid and arrive at the grid point of another border line. The product of numbers on one side of the thick line be equal to the product of numbers on the other side.

Solution: *On both sides of the thick line there should be a 0 number. There are several solutions.*

During the years the role of 0 and 1 in the operations is gradually recognized, the knowledge of operational properties and their use become aware. The preliminary estimation, calculation and checking of results and the comparison of the results of computation, discussion of the possible causes of differences improves the computation skills, the algorithmic reasoning, the estimation ability and the need for self-checking. The correct keeping of the sequence of operations requires consistency, concentration.

The children should be able to multiply and to divide positive fractions by positive whole numbers, they should understand the basic operations and operational properties in the circle of rational numbers, and they should know and use the correct order of operations.

Put operational symbols and parentheses between the numbers so that you get the specified result.

3 7 3 3 = 4 Solution: 3 · (7 – 3) : 3 = 4
12 3 9 99 = 43 Solution: 12 · (3 + 9) – 99 = 44

Algebra

In the extended number circle besides problems containing numerals the equations and inequalities also appear in the open sentences (open sentences where the predicate is equal, smaller, bigger, smaller or equal, bigger or equal). The unknowns are in general marked by a letter. When making operations by letters we formulate conditions concerning the numbers to be written in the place of the letters (for example, in the case of 5/b the b cannot be 0). With the expressions received by using numbers, letters (unknowns) we formulate operations (for example, 3a; –2b; c/4), relations between operations and search for the solutions (solution set or truth set). These activities prepare the subject of algebra being an independent topic in the higher grades.

Solve the equation by trial and error method on the basic set consisting of numbers 1; 3; –2; 0; 5; –4.
2a+(–4)=6

Think of a number. Add 7 to it. Subtract the double of the result. Take the double of the result. Subtract 14 from it. From the received result subtract the originally thought number.
If you computed well you got the thought number as a result.
Why?
Solution:
We can confirm the validity of the above statement if we exactly follow the instructions in the language of mathematics.
Mark the thought number by x.

The sequence of instruction is: $(x + 7) \cdot 2 - 14 - x = x$*,*
and the equality is true.

Relations, Fractions

The proportionality tasks have practically been present since the beginning of schooling. The concept of fraction, the comprehension and practicing of multiplication, comparison, certain number theory questions, measurement, conversion of units, determination of perimeter, area are all based on proportional reasoning. Tasks connected to sequence often occur since the repeated execution of certain operations results in a row of numbers containing some kind of regularity. The recognition of regularities, their use serve from the beginning the improvement of the ability to follow the rules and to make inferences. In other subjects the natural, physical phenomena, the studying of timeline changes in processes, the mathematical description of the cause-effect relation, modeling of the everyday life give the basis to the development of the concept of functions.

As to direct proportionality there are several possibilities for the selection of the task. Every unit conversion, shopping, uniform motion, work, sale, interest rating, enlargement, scale of map, comparison of areas, etc. are suitable for the formulation of routine tasks. Examples:

A petrol tank of a car can receive 47,5 l petrol. We fill the petrol by a 2,5 l can. How many cans do we have to pour so that it be full?

We bought 8,5 kg of apples for 340 Ft. How much does 12 kg of this type cost? What is the relationship between the price of the apple and its weight?

Zoll (inch) is a German unit of length, 10 inches=254 mm. How many mm is the diagonal of the computer screen if it is 15 inch long?

On a map of scale 1 : 30 000 000 the distance between Budapest and London is 7 cm. In reality, how many km is the aerial distance between the two cities?

The population of a city has grown by 15% during one year. How many inhabitants lived in the city at the beginning of the year if the growth was 7500 persons?

The relations used in the case of direct proportionality in a different formulation are appropriate for the use and testing of the concept of inverse proportionality. The textbooks mainly contain tasks about work, distribution of costs-profits, relations between time and speed needed to making a given route, length of lateral face of a rectangle of specified area.

A family preserves raspberry juice for winter. If they fill the juice into half liter bottles, they need 21 bottles. How many would they need of 7 dl bottles?

An express train travelling at an average speed of 80 km/h takes the distance between two cities in one and a half hour. How long does the route last between the two cities by a local train if this train travels at an average speed of 45 km per hour?

4 people can finish a work in 12 days. With the same pace of work in how many days can 6 persons finish the work?

How long can the the sides of a rectangle with area of 24 cm² be, if its sides expressed in cetimeters are integers? Write in the table.

a (cm)											
b (cm)											

When studying the different effects and events the children can record the recognized, collected data in different ways (by text, formula, table, diagram, graph). The different solutions can be converted into each other. They are able to determine locations in practical situation, in specific cases. At this age mainly problems relating to motions, change of temperature, standing of water level are suitable for representing relations, connections on diagrams, graphs.

Mark on the number line, that:

- *it is warmer than –2 ºC*
- *it is not colder than –4 ºC, but the temperature is below freezing point.*

We are cooling water of 40 ºC. The temperature is decreasing by 6 ºC per minute. Make a table and a graph about the change of water temperature. Formulate in mathematical language how the temperature of water (T) depends on the time passed (t).

Geometry

In grades 1-4 we lay the foundations of geometrical definitions and knowledge. Here the procedures connected to actions, experiences are dominating. As a continuation of the work started in the lower grades the introduction, comprehension, ripening of concepts is accompanied by a lot of carefully planned activities in the upper grades.

Grades 5-8 are connecting the years of approach formulation, activity and discovery inducing works of lower grades and the work of grades 9-12 developing, teaching deductive reasoning. In the teaching of mathematics in the upper grades great emphasis should be placed both on the specific, practical activities in the integration of the children's experiences into teaching and on the development of abstract reasoning. Although the accents are gradually moved from the specific activities to abstraction, this dual approach is present parallel through the upper grades.

In the upper grades in teaching geometry and measurement topics it is always important that children could always return from the abstract concepts to the specific, practical meanings and certainly vice versa and that they could discover the general in the world of specific effects. Using the terms of the realistic mathematical movement: geometry is an excellent domain for the development of horizontal and vertical mathematical activities.

The geometrical topics of the lower grades get a role in the upper grades, too and we continue to use the methods of the lower grades. Diversified collection of experiences, use of tools, playfulness and games assist the building of concepts from the specific to the general. The children's reasoning is primarily inductive, but the need for generalization gradually comes into the foreground.

During teaching special attention should be paid to the development of the ability of regular trying, estimation, checking, pre-planning of solutions, and to the comprehensible description of the sequence of solution.

In the description of the geometrical frameworks of grades 5-6 we follow the partial content domains learned in the lower grades. In line with the traditions of the mathematical curriculum the children meet with the different definitions several times during their school years, thus in many cases seemingly the requirements of the lower grades are repeated. On the verbal level of the geometrical concepts this is the case, on the other hand the manipulative activities and visual memories playing a decisive role in the development of geometrical concepts make possible that the already acquired concept build of abstraction the mathematical knowledge of the learners further on a higher level. Of the four geometrical partial areas of the former grades orientation is not playing a role as a separate field in grades 5-6. The knowledge elements which could be listed in that category has no differentiating role in the assessment in this age group or can be put into the measurement category.

Constructions

A list of requirements: getting acquainted with spatial geometric elements, their relative positions, the concepts of parallelism, perpendicularity, the properties of rectangle (square), cuboids (cube), nets of cuboids, the pictorial view and properties of polygons, characteristics and categorization of triangles, rectangles.

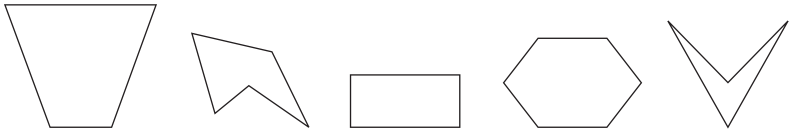
Children learn the concept and measurement of distance and angle. The looking for points with given specificities evokes the definitions of perpendicular bisector of the segment, circle and sphere, solution of construction tasks. Children are able to create plane shapes and bodies on manipulative and pictorial levels. Convexity appears among the known geometrical properties. Learners are able to group plane shapes and bodies on the basis of the learned geometrical properties.

They should know the properties and body nets of cube and cuboid and the basic properties of triangles and squares. The development of the concept of circle and sphere, and knowledge about basic properties is also a requirement.

As to the use of compasses and ruler it is required from the learners to be able to copy a segment, to draw parallel and perpendicular lines with two rulers, to copy angles and to construct median perpendicular to a segment.

The learners have to know the definition of angle, the different types of angles and learn how to use the angle-meter. In grades 5-6 learners are able to use precisely the terms point, straight and segment.

Group the plane shapes below according to their being convex or concave. Write the corresponding letters on the dot line.



a) b) c) d) e)

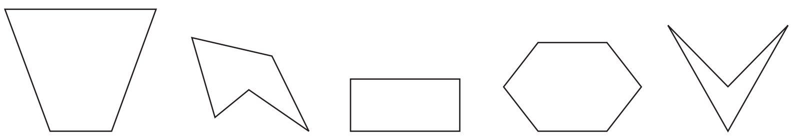
Convex polygons: ....

Concave polygons: ....

Transformations

Students will be able to construct the mirror-image of well-known shapes. They have to recognize shapes symmetrical around the axis. Symmetry should be recognized on specific examples taken from everyday life and from art. They are able to formulate by words the properties of projection on the axis.

Of the plane figures below which one has mirror axis? Circle the letter of the one which has minimum one mirror axis and cross the letter of the one without mirror axis.



a) b) c) d) e)

Measurement

In grades 5-6 the measurement index numbers can be used in the extended number circle. On the one hand this means that when making conversion of units it is required to convert not only the neighbouring units, but also the more distant ones, but thus we deal not only with measurements which are known or can be reconstructed from the everyday life, but many tasks of conversion of units turn into simple computation tasks. The extended number circle at the same time means that fraction numbers are included in the perimeter, area and volume calculations, as well as a new operation; the squaring is also applied in the geometrical computations.

At this age group children are able to compute the perimeter of triangles and squares, the surface and volume of cubes and of cuboids. Not the knowledge and the use of non-general formulas are required, but they should be able to make computation with specific, known or to be determined number data.

The children should know the standard units of measurement of length, area, mass, cubic content, volume and time. They have to make unit conversions in the number circle up to million. They should be aware of the volume and cubic content units and should convert them into each other.

They use the knowledge gained in grade 5 to the calculation of volumes and surface areas, they determine the surface and volume of bodies built of cuboids and cubes. In grade 6 they use knowledge acquired in the previous grades in area computation tasks which can be led back to the area of rectangle or get acquainted with the calculation of the area of right-angle triangle and mirror triangle, of convex and concave kite, rhombus, squared.

In connection with measurement activities we connect the area of measurements through preliminary estimations to the everyday experiences.

The simplest measurement tasks where the learned mathematical terms and symbols are checked typically belong to the following basic types:

Conversion of units

125 cm = ... mm
40 hl = ... cl
117 000 cm = ... km

Area and perimeter calculations

Calculate the perimeter of the rectangle the shorter lateral face of which is 2 cm, while the longer face is 3 cm.

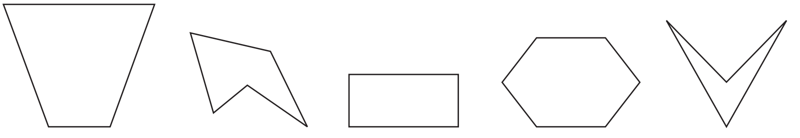
What is the length of the sides of the square with area 49 m^2?

Volume calculation

What is the volume of the cuboid the height of which is 6 cm, the other two edges are 8 and 10 cm?

In the case of simple word problems of measurements the wording of the problem determines the method of measurement, or the consecutive operations to be made with the received numbers.

With the help of your ruler measure the perimeter of the below shapes. Put them in descending order on the basis of their perimeter.



a) b) c) d) e)

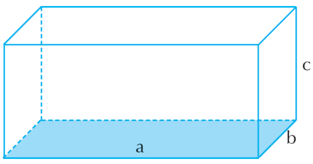
Letter mark of the shapes according to the size of their perimeter in descending order: ....

Perhaps in connection with the next task we have to explain why we regard it a simple, routine word problem and why we do not consider it a realistic problem. The key is that the numbers and geometrical terms contained in the problem lead to the solution without their comparison with the everyday experiences. It is not necessary to make a model with the help of mathematical symbols and concepts about the problem situation, but we mainly look for the mathematical definitions and operations contained in the text of the problem. Although the dimensions of the pool contained in the text of the problem can be compared with the everyday life, but it can be seen that the given dimensions can be varied optionally in the known number system and the task will not be easier for the majority of learners if we give the dimensions of a standard garden pool or swimming pool.

The recently built pool for children in the community center for water sports is 0,5 meter deep, 10 meter wide and 15 meter long. How much water is needed to fill up the pool?

It should also be considered to what extent the drawing prepared to the problem, or the drawing expected as part of the solution modify the difficulty of the task. If we give a draft drawing to the previous problem, where we assign data to the three edges of the cuboid, we will remain in the routine word problem category in the same way, as when the learned rules have to be applied in the framework of mathematical concepts and symbol system.

The recently built pool for children in the community center for water sports is 0,5 meter deep, 10 meter wide and 15 meter long. How much water is needed to fill up the pool?



Compared to the problems written only by symbols the simple word problems of unit conversion can test the comprehension of texts. It is possible that not the same learners can solve correctly the following to tasks:

Version 1:

32 dm^3 = ... liter

Version 2:

Determine the cubic measure of the vessel the volume of which is 32 dm^3.

The above problems highlighted the assessment problems of the unit conversion skills. Part of the unit conversion tasks can be solved on the basis of cognition based knowledge, the other part requires the use of computation skill. It is also possible that it depends on the character of the problem setting how the learners handle the conversion problem, as a computation problem, or as a problem intending to check his/her knowledge about units of measurements.

Combinatorics, Probability Calculation, Statistics

During the past decade there was significant change in the Hungarian mathematical education since the probability calculation and statistics topics were included in the maturation exam requirements, thus they had a reactive influence on the high school education and had significantly reshaped it. Nevertheless, it should be underlined that in the international educational surveys made since the 1960s, the descriptive statistics has been present from the beginning even in the assessment of the youngest age group, of children around 10 years of age. In the Hungarian education system the corresponding knowledge elements are related to the mathematical education, but also to the integrated subjects of education of sciences (environmental, natural studies).

Students – in the course of the solution of diversified problems and tests – learn terms which have the same use in mathematical and everyday contexts: case, event, and experiment.

They are able to determine and to represent on event tree or in table format the possible outcomes of the different probability tests. They use the terms of certain events and impossible events. The learner are aware of the terms of mutually exclusive and mutually non-exclusive events. They are able to represent the frequency of events in tables on different figures – frequency bar diagram, circle diagrams. They are able to determine events of the lowest and highest frequency and are able to calculate the arithmetic mean of some numbers. They are able to sort a disordered data set according to the frequency of occurred events or according to other criteria in the form of list, table and diagram.

At this age the probability concept is interpreted by the learners by the fraction „positive event / all events" and they can also formulate some remarkable probability values, too: the probability of an impossible event is 0%, the probability of a secure event is 100%, and the probability of two events with equal chances is 50-50%. They also meet with problems which can eliminate certain misconceptions. This can be for example the following: in a family with two children (supposing 50-50% probability of the birth of a boy or a girl) the probability of the case of one boy – one girl is not 1/3, but ½, or when we throw up a coin twice the probability of heads or tails is not 1/3, but 1/2.

Students know the dice as a visual tool of representation of unintentional events. They try by simple tests that if we throw by the dice many times, the six possible values on the dice will occur in approximately the same number.

Of the methods of computing all possible cases the children know in empirical way (without formal formula) the method of counting of options received with permutation without and with repetition in case of sets of less than ten elements; how to count the options gained by the repetition and without repetition variant if the end result remains in the hundred number system; the method of counting options received by the without repetition combination in the case of selection of partial set of the maximum six element set.

We present an example where the linguistic elements exclusively serve the mediation of the mathematical structure of the problem.

How many two-digit numbers can be made of numbers 1, 2 and 3 if we can use each number only once?

This task can be regarded a routine word problem if one of the numbers is changed to 0.

Parts of the requirements of descriptive statistics belong to the topics of fractions, relations, too. Analysis of graph, figures, reading of the most frequent value, scope of observable values are the required knowledge elements.

Combinatorics has traditionally been an integrated part of the Hungarian mathematical education. There have been refined traditions of setting tasks on manipulative, pictorial and symbolic levels where the number of options has to be determined. The major part of these textbook problems can be regarded routine tasks, since of the mathematical structures dressed in text form appear, but the everyday knowledge and experiences do not have real, relevant role. In such cases a typical problem setting strategy is the problem starting with „Anna, Béla, Cili and Dani…", where for example four different, equal activities can be associated to the children's names. Another typical solution is when topics alien to the children's experiences appear in the problem texts: water pipe systems, phone line networks, managerial appointments, etc.

Anna, Béla and Cili are siblings. One of them empties the garbage bin every day, the other waters the flowers. In how many different ways can they share the household chores?

As a consequence of the characteristics of this age group the teaching examples will be underrepresented compared to the realistic problems in the field of combinatorics. The solution without formula of the basic counting examples can be expected if the comprehension of the problem is assisted by memories or for example by drawing models.